ALTERNATING-CURRENT MACHINES

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PREFACE TO SECOND EDITION

In the six years since the appearance of the first edition a number of methods of analysis pertaining to alternating-current machines have crystallized into standards or have come into general use. Notable among them have been the new methods of calculating alternator regulation and the concepts of direct- and quadrature-axis synchronous reactance. Accordingly, the sections on alternators and synchronous motors have been rather completely rewritten to include such theory.

In the same period the manufacturing industry has gone forward in the process of standardization on ratings, machine dimensions, and characteristics. While the inclusion of such material comes under the onerous classification of "current practice," the subject is too important to be ignored. This serves to acquaint the reader with the general idea that the machines with which he deals usually are not built with dimensions, characteristics, or ratings chosen at random, but that they must adhere to rather rigid standards.

In general, the scope and character of this book have not been changed in the revision. As in the previous edition, only the steady-state phenomena have been covered, with the exception of the analyses of hunting under various conditions of synchronous-motor operation. This is a common and important problem, as may be witnessed by the many pages which various association standards devote to the subject.

The reception given the first edition and the suggestions received for modifications in the rewriting have been very gratifying. The inclusion of many practicing engineers among those making suggestions is looked upon as significant of the place which the text has found in industry as well as in the classroom.

Several matters of controversial nature deserve comment. As was to be expected, the old question of cross-field versus double-revolving-field theory for single-phase motors drew forth adherents to each school of thought who advised that the other presentation be omitted. Our reaction to this, as well as to near-duplicate methods of analyses in other classes of apparatus, is that so long as industry has not adopted one, to the exclusion of others, and so long as constant reference is being made to certain classical theories, the groundwork of these theories should be presented in a book of this type.

The authors are especially indebted to Professor Robin Beach, of Brooklyn Polytechnic Institute, for many valuable suggestions and a careful reading of the manuscript.

Space does not permit a complete list of those who made useful suggestion for this revision, but contributions were made by the following: R. M. Kershner, Kansas State College; D. D. Ewing, Purdue University; Melville S. Munro, Tufts College; H. M. Crothers, South Dakota College; Otto Meier, Jr., Duke University; A. Naeter, Oklahoma A & M College; George F. Corcoran, University of Maryland; and R. E. Brown, formerly of New York University.

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July, 1942

PREFACE TO FIRST EDITION

A study of the catalogs of engineering colleges reveals wide differences in the time devoted to the subject of alternating-current machines. Inasmuch as a course is usually built about a textbook, the length and thoroughness of the textbook are of great importance.

It is obvious that a thorough book with bibliography and suggestions for further reading and investigation is of more value to the student than the present type of textbook. It is our belief that it is desirable to place such a volume in the hands of electrical engineering students even though only a fraction of the material can be covered in the classroom. This procedure intimately acquaints the student with sources for future reference.

The attendant problem of deciding what can be omitted in a college course has been simplified in this text by the arrangement of material. Each subject is treated in somewhat the following order:

Construction.
Discussion of operating characteristics.
Calculation of operating characteristics from tests.
Discussions and analyses of various related phenomena.

Some of the material comprising this book is appearing for the first time in textbook form. However, we base any claim to value for this text upon its method of presentation and the arrangement of material; the aim is to conserve the time and labor of the student so far as the difficulties of the subject permit.

Particular emphasis is laid upon methods of calculation and the physical principles on which the analyses depend. The material selected is well within the scope of fourth-year students, but it is hoped that others will find much of it readable and useful.

An acknowledgment of appreciation is due to the various electrical companies for their cooperation in furnishing illustrations, and to the Engineering Department of the General Electric Company for various machine data. Professor G. V. Mueller of Purdue University has kindly furnished a number of oscillograms; Professor George McC. Porter of

the Carnegie Institute of Technology, and Professor A. G. Conrad of Yale University have made many helpful suggestions in the preparation of the manuscript; and Mr. Carl Sipe contributed greatly to the preparation of the illustrations, bibliography, and problems.

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CHAPTER I

CONSTRUCTION AND VENTILATION

1. Chapter Outline.

- Construction.
  - Field.
    - Frame.
    - Winding.
    - Insulation.
  - Armature.
    - Frame.
- Ventilation.

2. The Elementary Alternator. The alternating-current generator in its simplest form consists of a single turn of wire, rotated in a magnetic field. Consider the coil side *a* of Fig. 1. As it cuts through the flux, the voltage generated in it will be in the direction indicated by the arrow. After the coil has rotated through 180° from the initial position

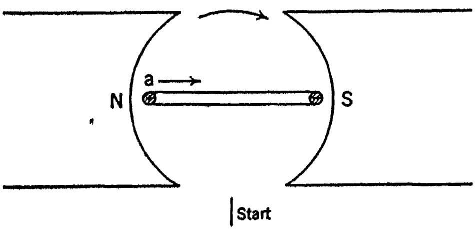


Fig. 1.

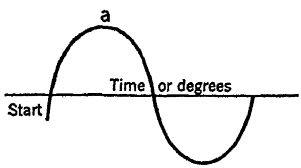


Fig. 2.

it will be cutting down through the flux and the direction of the emf will then be reversed. A complete revolution of the coil through 360° produces the sine wave of emf if the field is uniform.

Every direct-current generator, with the exception of the homopolar machines, generates a similar alternating emf in its coils each time the coils pass under a pair of poles. The action of the commutator on the

d-c armature is to rectify this emf into a unidirectional value which becomes more steady the greater the number of segments and coils used. The a-c generator needs no such rectifier; the emf and the current it forces through the connected load are allowed to vary in magnitude and direction as indicated in Fig. 2.

3. Alternator Construction. Because the alternator needs no rotating commutator to rectify its alternations, it is not necessary that its armature be the rotating member. Practical considerations of design (especially the problem of insulation) make for a construction embodying a rotating field structure and a stationary armature, called the *stator*, in all except some of the smaller low-voltage a-c generators in which the armature rotates within the stationary field structure.

Among the advantages of the stationary armature construction are the following:

(*a*) It permits sturdy mechanical bracing of the armature coils and better insulation than would be possible on a rotating armature. The coils and insulation are not subjected to centrifugal stresses, and are less exposed to mechanical vibration.

(*b*) The high voltage (13,200 volts, and more recently as high as 33,000 volts) generated in the armature windings need not be brought to the external circuit through slip rings and sliding contacts, but direct connection to the terminals can be made. Only the comparatively low voltage, usually 125 or 250, necessary for excitation, need be supplied through slip rings to the rotating field.

4. Speed and Frequency. One cycle is generated each time the moving member passes under a pair of poles, i.e., each time it passes through 360 electrical degrees. The frequency is therefore dependent upon the number of poles and the speed. The rated speed, in turn, depends upon the type of prime mover used. Engine-driven alternators run at slow speed and require many poles to give commercial frequencies of 25 or 60 cycles. Steam turbines operate most efficiently at high speed, and turbine-driven alternators are usually designed with 2, 4, or 6 poles. Hydraulic turbines and waterwheels operate at various speeds from low, in wheels of low-head developments, to high, in high-head developments.

The relationship is given by the equation

$$f = \frac{P}{2}\frac{\text{rpm}}{60} \qquad [1]$$

$$= \text{pole pairs} \times \text{rps}$$

where f is the frequency in cycles per second and P is the number of poles.

5. Field Construction. The rotating fields or rotors of alternators embody two types of construction: (1) those with salient or projecting poles for the slow-speed machines; and (2) those with non-salient poles.

The salient field poles would cause an excessive windage loss if driven at high speeds and would be very noisy. In addition, salient-pole construction could scarcely be made strong enough so that the poles could withstand the greater stresses to which they are subjected at high speeds.

FIG. 3. Rotor of a vertical water-wheel generator rated at 2500 kv-a, 12,000 volts, 25 cycles, three phase, 300 rpm. (*The Allis-Chalmers Mfg. Co.*)

Salient poles are sometimes constructed of solid pieces or of stampings, dovetailed and bolted to the central spider through which the shaft projects. In some cases the shaft is forged integral with the spider.

The non-salient-pole field construction embodies three distinct types.

(*a*) The cylindrical field is of solid steel, slotted for the field windings. If the field is not too large, the projecting shaft stubs and central portion of the field are of one piece. In the construction of larger fields three pieces are used, the central cylinder and two shaft stubs with flanges which are bolted to the cylinder. This reduces the size of the individual pieces and consequently the cost. Flaws in the material are less likely and more easily discovered.

(*b*) The field is built of steel plates, approximately 2 in. thick, bolted together. This construction also uses a shaft in two pieces, bolted to the end plates.

(*c*) The shaft is a solid forging, passing completely through the central cylinder of the field which is built up of steel plate punchings, keyed to the shaft.

Either radial or parallel slots may be cut in the cylindrical field core. Radial slots, illustrated in Fig. 4, are to be preferred, as the stresses in the teeth are radial and the bending due to the tangential component of

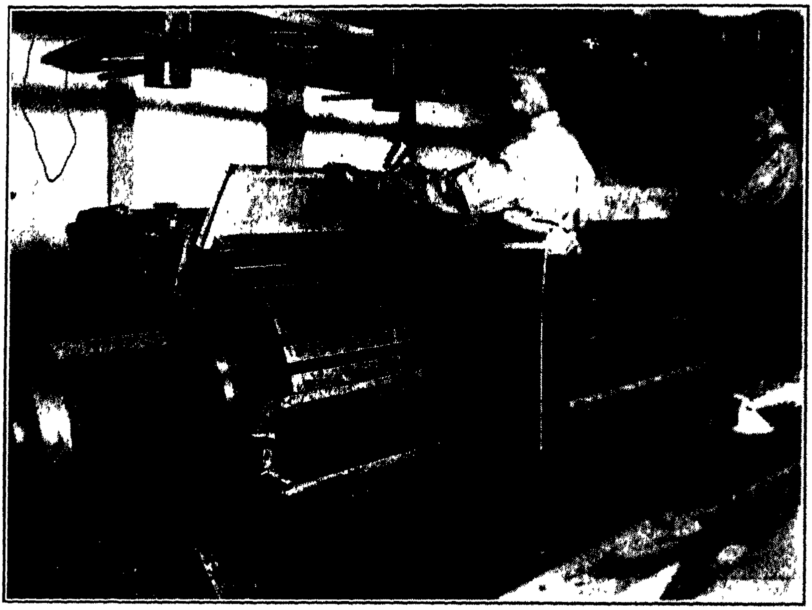


Fig. 4. Winding the rotor of a 12,000-volt, three-phase, 60-cycle, 1800-rpm alternator rated at 70,600 kv-a. (*The Allis-Chalmers Mfg. Co.*)

the centrifugal force from the winding is avoided. However, it is then more difficult to insert the field coils.

6. Field Windings and Insulation. Fields are wound for 125 or 250 volts. For that reason the dielectric strength of the insulation used is not so important as the mechanical strength and heat-resisting properties. In salient-pole machines the field core is covered with an insulating spool over which flat copper strap is wound edgewise. This is sometimes bare copper with insulating paper or asbestos between layers. An insulating fiber collar is put on the outer edges of the winding. In smaller machines requiring less current for excitation, double cotton-covered wire is often used instead of the strap.

The crushing forces exerted on the insulation of non-salient-pole, high-speed machines are very great. (Thus the centrifugal force on 1 lb at a

2-ft radius, at a speed of 1800 rpm is 2210 lb.) They are further increased by the magnetic forces occurring on short circuit. In order to withstand these forces and the heat generated in the field core, mica and asbestos are used almost exclusively as insulation. The slots into which the field winding of bare copper strap is placed are lined with asbestos or mica, and asbestos or mica sheets are placed between the layers. Aluminum

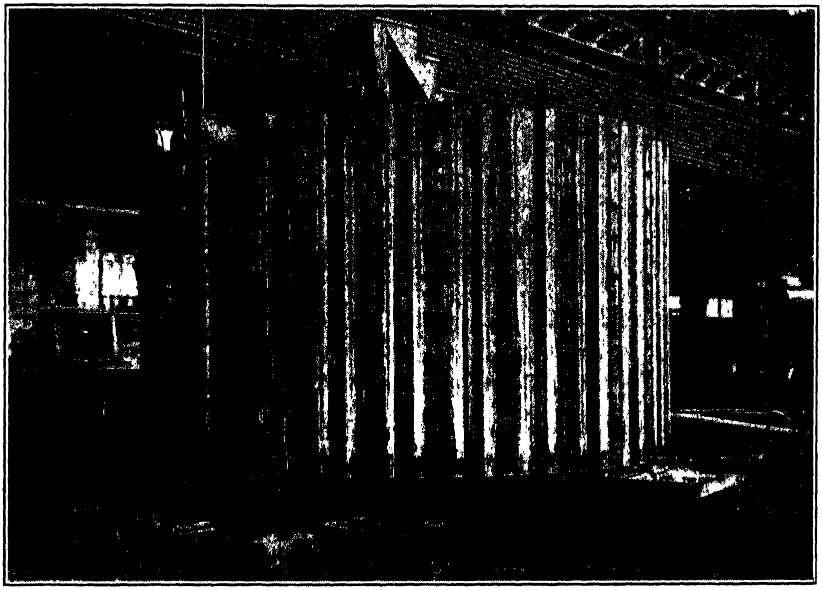


FIG. 5. Rotor, showing pole mounting for a vertical water-wheel generator, rated at 35,000 kv-a, 257 rpm. (*The Allis-Chalmers Mfg. Co.*)

or other saddles are placed over the ends of the winding for protection, and steel rings hold the completed coil ends in place against the action of the centrifugal forces. To reduce the windage loss and to hold the rotor winding in place, heavy non-magnetic metal wedges are placed in the top of the slots, finished flush with the field core surface.

7. Armature Construction.[1] The stationary armature of alternators can be considered as being made up of two parts.

(*a*) *The armature yoke or frame support* as shown in Fig. 6. This is usually of cast iron or of cast or welded structural steel.

(*b*) *The armature stampings* built up of laminated sheet steel dovetailed into the yoke. The inner edges of the laminations are slotted for the armature winding. These stampings frequently have a series of

[1] *Gen. Elec. Rev.*, July, 1927, p. 330.

holes through them which form axial passages for the flow of cooling air. In addition, spacers are placed between stacks of the laminations to provide radial air ducts for ventilation.

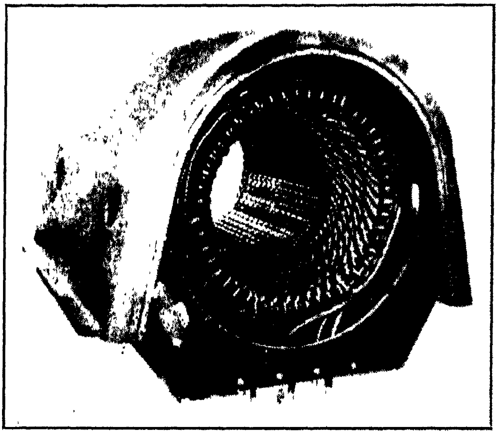


FIG. 6. Wound stationary armature for a-c turbine generator. (*The General Electric Co.*)

8. Ventilation.[2] Slow-speed, salient-pole-type alternators are cooled sufficiently by the natural movement of the air around their field coils and armature winding. The high-speed, non-salient-pole alternators of large capacity all require forced ventilation. Because of their compact design, a great amount of heat energy must be dissipated from a small area. The rate of heat dissipation by forced ventilation depends upon the surface exposed to cooling, the volume of air passed through the machine, and the difference in temperature between the cooling air and the surface to be cooled. The volume of air required is very great in large machines, values of over 100,000 cu ft per min being common. This is forced through the ventilating ducts at high velocities. For this reason all turbo-alternators so cooled are totally enclosed, with a fan system for circulating the air. The requirement is 100 cu ft per min per kw loss when the outgoing air is 19 C hotter than the incoming air.

Assume that a 100,000-kw generator has an efficiency of 98 per cent. Neglecting the slight amount of heat dissipated through the outer shell, 2000 kw is to be carried off by the ventilating system. Air at constant pressure has a specific heat of 0.238. Each pound of air per minute carries off

$$0.238 \times (\text{temperature increase}) \text{ calories per minute} \qquad [2]$$

[2] Lamme, *Trans. A.I.E.E.*, January, 1913.
Lamme and Newbury, *Trans. A.I.E.E.*, November, 1916.

This corresponds to a rate of

$$0.238 \times 453.6 \times \text{(temperature increase) gram-calories per minute}$$

The weight of air, and hence the heat which can be carried off by it, varies with the humidity, temperature, and barometric pressure. As a basis of calculation it is convenient to use "standard air," which is defined as having a temperature of 68 F (20 C) dry bulb, a barometric

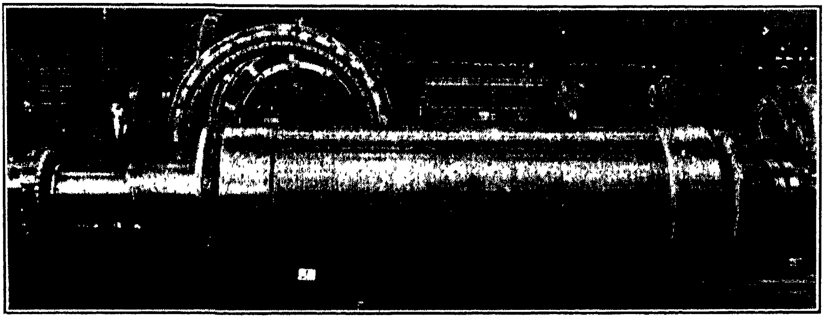


FIG. 7. Non-salient type of rotor for a turbo-alternator rated at 30,000 kw, 0.80 pf, 12,000 volts, 60 cycles, 1800 rpm. (*The Allis-Chalmers Mfg. Co.*)

pressure of 29.92 in. of mercury, and 50 per cent relative humidity. Standard air weighs 0.07488 lb per cu ft.

Since 1 watt = 14.34 gram-calories per minute, each cubic foot of air per minute dissipates

$$\frac{0.238 \times 453.6}{14.34} \times 0.07488 \times \text{(temperature increase in C) watts}$$

or

$$0.563 \times \text{(temperature increase in degrees centigrade) watts} \quad [3]$$

Assuming that the temperature of the air increases only 15 C in passing through the machine, each cubic foot of air per minute dissipates

$$0.563 \times 15 = 8.45 \text{ watts}$$

A total of 237,000 cu ft per min would be required to cool this alternator.

Suppose, instead of using standard air, the following conditions held: input air, 30 C; barometer, 29.82 in. of mercury; 86 F, dry bulb; 83 F, wet bulb.

The weight of air under these conditions would then be 0.07145 lb.[3]

[3] *Weight of Air Tables*, General Specifications for Building Vessels of the United States Navy, Appendix 7, Navy Department, U. S. Government Printing Office, 1929.

Recalculating, this air would carry off 8.05 watts per cubic foot, requiring a total of 248,450 cu ft per min to cool the alternator.

Hydrogen [4] offers some advantages as a cooling medium, chiefly because its specific heat is about $14\frac{1}{2}$ times greater than air, and its thermal conductivity is about 7 times greater. Since the problem of designing large generators involves difficulties in carrying off the heat and in providing enough air to do so, the use of hydrogen offers the opportunity of

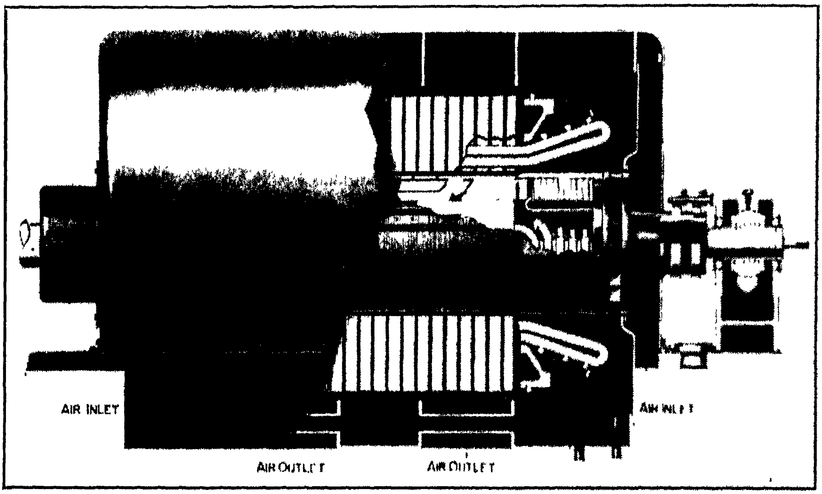


FIG. 8a. Synchronous a-c turbine generator, 60 cycles, longitudinal semisection with air inlets and outlets indicated. (*The General Electric Co.*)

circulating a cooling medium which is less dense, and therefore is circulated with less windage loss and noise. Its increased conductivity assures a transfer of more heat units for a given temperature gradient, thereby permitting more effective heat dissipation.

To prevent an explosive mixture of air and hydrogen from occurring in the machine, the hydrogen is kept under slight pressure so that all leakage is outward. Special oil-sealed glands are used between shaft and casing, with the oil purified periodically.

Hydrogen cooling was first used on large synchronous condensers, but more recently on turbo-alternators. On a typical installation of 25,000

[4] C. J. Fecheimer, "Hydrogen Cooling of Large Electrical Machines," *Elec. J.*, p. 127, March, 1929.

R. W. Wiesemann, "Outdoor Hydrogen-ventilated Synchronous Condensers," *Trans. A.I.E.E.*, Vol. 48, p. 1221, October, 1929.

M. D. Ross and C. C. Sterrett, "Hydrogen-cooled Turbine Generator," *Elec. Eng.*, January, 1940.

D. S. Snell, "Hydrogen-cooled Turbine Generator," *Elec. Eng.*, January, 1940.

kw, 800 cu ft of hydrogen are used to charge the unit under a gauge pressure of 0.25 in. of water. The loss is about 45 cu ft per day under operation, 20 cu ft when shut down. The hydrogen is cooled by water in fin-type radiators.

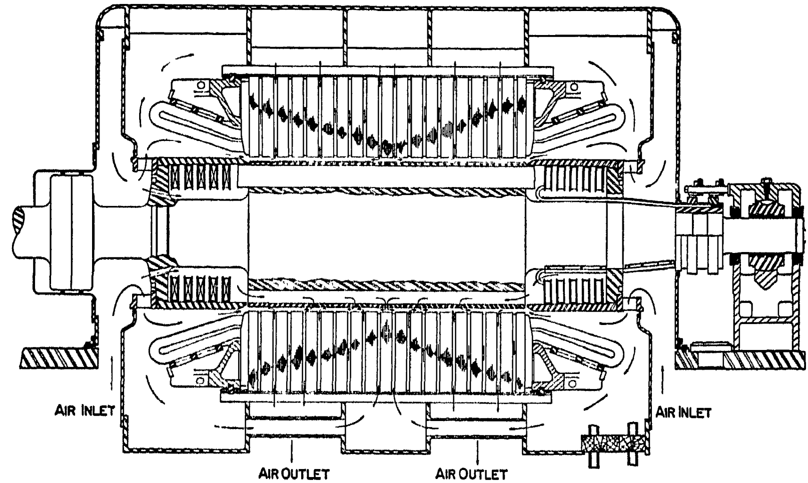


FIG. 8b. Enclosed a-c turbine generator, longitudinal sections with arrows to show ventilating air paths. (*The General Electric Co.*)

9. Ventilating Methods.[5] Forced ventilation cooling systems fall roughly into two classes.

(*a*) *Air Gap and Radial.* The air is introduced at each end of the alternator housing, through the air gap and axial ducts and out through radial ducts in the stator. A modification of this method forces part of the air through the rotor longitudinally, thence along the air gap to the stator ducts. In long alternators of large capacity, the pressure drop is so large as seriously to interfere with the efficiency of this system.

(*b*) *Radial and Circumferential.* Regardless of the length of the alternator, sufficient air can be forced through the stator by dividing it off into a number of longitudinal sections through which air is forced radially in, toward the air gap, circumferentially along the air gap and axial ducts, and out through another part of the stator. A modification of this method circulates the air through the stator only, the openings in the stator not extending entirely to the air gap. The rotor is then cooled by forcing additional air through the gap.[6]

[5] M. D. Ross, *Elec. J.*, Vol. 21, p. 540, December, 1924.

[6] For a discussion of air washing and cooling systems see:
E. Knowlton and E. H. Freiburghouse, *Gen. Elec. Rev.*, April and May, 1918.
J. Christie, *Elec. Rev.*, p. 1088, June 27, 1913.

CHAPTER II

FACTORS AFFECTING ALTERNATOR ELECTROMOTIVE FORCE

10. Chapter Outline.

Emf Equation.
- Flux Distribution.
- Fractional Pitch.
  - Harmonics.
  - Pitch Factor.
- Distribution of Winding.
  - Harmonics.
  - Distribution Factor.
- Form Factor.

11. Generated Electromotive Force. One conductor in making 1 revolution on a generator armature cuts $\phi_m \times P$ lines of force if ϕ_m is the number of lines per pole and P is the number of poles. If the speed is given in revolutions per minute, the number of lines of flux cut per second becomes

$$\phi_m \times P \times \frac{\text{rpm}}{60}$$

Since 1 volt is generated when 10^8 lines of flux are cut per second, the average voltage generated in this one conductor becomes

$$E_{\text{avcr}} = \phi_m \times P \times \frac{\text{rpm}}{60}\, 10^{-8} \text{ volt} \tag{4}$$

If the total number of conductors on the armature is Z and they are connected into a paths, the voltage between terminals becomes

$$(E \text{ average per conductor}) \times \frac{Z}{a}$$

This analysis holds for any d-c generator, but in dealing with alternating current the emf depends not only upon the total number of flux lines cut per second but also upon the way in which the lines of flux and the conductors are distributed. A change in distribution of flux changes

the relative values of the maximum and effective emf's. In addition, the emf built up in any one conductor, when considered vectorially, cannot always be added directly to that of another as there may be a phase displacement between them. The instantaneous values can always be added algebraically, but in adding effective values it is necessary to consider the phase differences between the different emf's to be added.

In order to take these factors into account the flux distribution and winding types must be discussed in more detail.

Refer to Fig. 9. Assume that the flux passing through a coil from the pole of an alternator varies as some function of the coil position from the

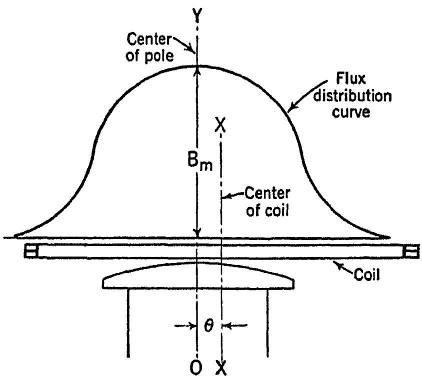


FIG. 9.

vertical reference axis O–Y. As a coil passes across this flux, the voltage generated in it will vary according to the following relationship:

$$e \propto \frac{d\phi}{dt}$$

where ϕ is the instantaneous flux embraced by the coil.

When the flux distribution is as shown, ϕ varies as the cosine of θ for all values of coil pitch. Since ϕ is a function of the angle θ, if the coil has N turns, the instantaneous value of the induced voltage is

$$e = -N \frac{d}{dt} f(\theta, \phi) \tag{5}$$

For practical units divide equation 5 by 10^8.

For the sake of simplicity it will be assumed that the flux variation is sinusoidal. With the starting position shown at O–Y and with the angular velocity of the coil taken as ω radians per second, the angle θ then becomes ωt.

t = the time required for the conductor to move from O–Y to X–X
$\omega = 2\pi f$.

Then

$$\phi = \phi_{\max} \cos \omega t$$

Equation 5 becomes

$$e = -N \frac{d(\phi_{\max} \cos \omega t)}{dt} \qquad [6]$$

$$= \omega N \phi_{\max} \sin \omega t \qquad [7]$$

This formula is significant in that it shows, among other things, that with the flux assumed to be a cosine wave the voltage is a sine wave; that is, the two are displaced by 90°.

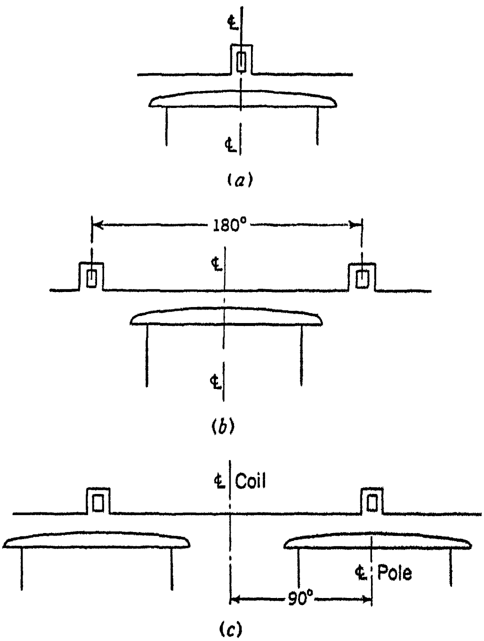


FIG. 10. Contrasting flux and emf relationships based on (a), single conductor; (b) and (c), coils.

The equation for instantaneous values can be made to yield an effective value by substituting unity for the sine or cosine term and dividing this maximum value by $\sqrt{2}$. Using ω as $2\pi f$,

$$E = \frac{2\pi}{\sqrt{2}} f N \phi_{\max}\, 10^{-8} \text{ volt} \qquad [8]$$

$$= 4.44 f N \phi_{\max}\, 10^{-8} \text{ volt} \qquad [9]$$

This equation holds for sine-wave distribution of flux in the air gap. It is based on the voltage generated in a *coil* rather than in a single conductor, as this treatment lends itself more readily to further analysis.

12. Flux Distribution.[1] In salient-pole machines the shape of the pole face and the ratio of the pole arc to pole pitch are the principal deter-

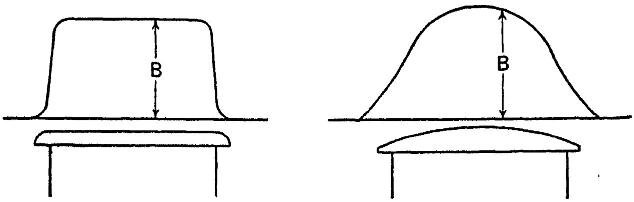


FIG. 11.

mining factors in the distribution of flux in the air gap. Figure 11 shows several pole shapes and the resulting flux waves. Figure 12 shows that the resulting emf generated in the armature coil is of the same shape as the flux wave, though displaced by 90°.

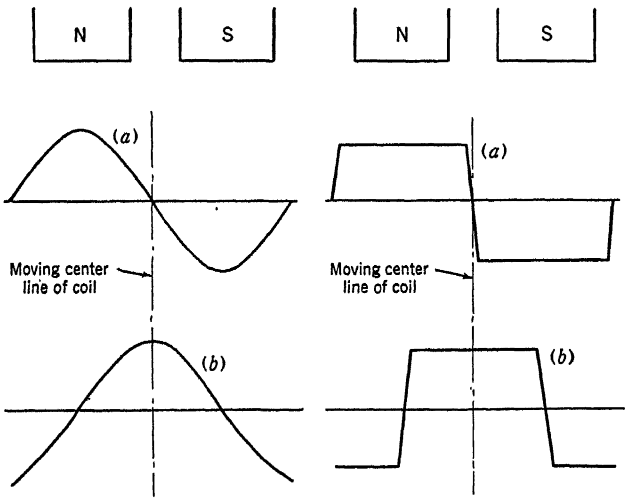


FIG. 12. (*a*) Flux density in air gap. (*b*) Emf of full-pitch armature coil.

In the above analysis the flux distribution was assumed to be sinusoidal, but in actual alternators this is not usually the case. However,

[1] S. P. Thompson, "Dynamo Electrical Machinery," Vol. II, p. 206.
C. A. Adams, *Trans. A.I.E.E.*, Vol. 33.

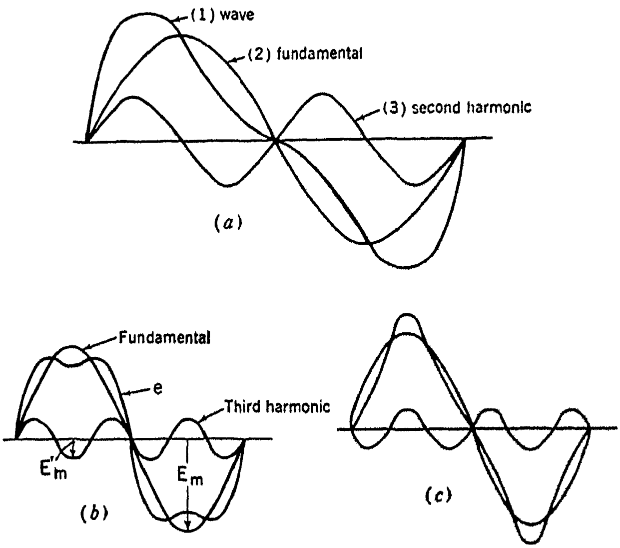
any single-valued periodic wave can be expressed as a series of sine and cosine waves by Fourier's theorem:

$$y = A \sin x + B \cos x + A_2 \sin 2x + B_2 \cos 2x + A_3 \sin 3x + B_3 \cos 3x \cdots \quad [10]$$

The cosine terms can be replaced by displaced sine terms and so [10] becomes

$$y = A' \sin x + A'' \sin (2x + \theta_2) + A''' \sin (3x + \theta_3) + A''''(4x + \theta_4) \cdots \quad [11]$$

The first term of equation 11 is called the *fundamental*, and the others are called *harmonics*. Thus $A'' \sin (2x + \theta_2)$ is called the second harmonic, and so on. As a rule, each of these harmonics decreases in magnitude, and so the higher harmonics are of less consequence. Since the waves of the emf generated in rotating machinery are of similar shape above and below the axis, no even harmonics can appear. The type of wave distortion caused by a second harmonic is shown in Fig. 13*a*. It can be seen from the curves that the relative displacement of the harmonics is important in determining the final wave shape.



$e = E_m \sin 2\pi ft + E'_m \sin 6\pi ft$

$e = E_m \sin 2\pi ft + E'_m \sin (6\pi ft + 180°)$
$e = E_m \sin 2\pi ft - E'_m \sin 6\pi ft$

Fig. 13. Wave shapes.

In non-salient-pole machines the distribution of the windings in the face of the rotor determines the flux and emf wave shapes. Figure 14 shows a spiral field winding and the resultant wave shape which it builds up. A properly distributed field winding is capable of producing a wave so nearly sinusoidal that it can be treated as such, usually with little error.

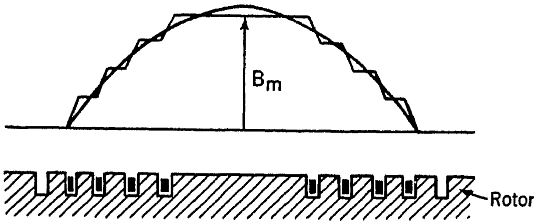


FIG. 14. Stepped curve shows field built up by a non-salient pole-field winding, approximating a sine wave.

13. Coils of Fractional Pitch. The pole pitch is the distance between the center lines of adjacent north and south poles, measured along the circumference at the armature surface. If the two sides of a coil on the armature are apart a distance equal to the pole pitch, the armature winding is *full pitch.* Since a pole pitch is always 180 electrical degrees, the coil pitch can be given in electrical degrees or merely as a fraction of the pole pitch. If the coil pitch is less than 180 electrical degrees, the winding is called a *fractional-pitch winding.* In such a case the voltages generated in the two coil sides will be out of phase (other than 180° or 0°) and must be treated as vector quantities. Figure 15*a* illustrates a coil pitch of 160 electrical degrees. This is 20° less than full pitch. If E_1 and E_2 are the respective values of the effective voltage built up in each coil side, the resulting voltage of the coil is the vector difference of E_1 and E_2, 160° apart, or the vector sum of the two if they are considered as 20° apart. Assume that the flux wave is so distorted as to be equivalent to a fundamental and a third and a fifth harmonic. Since the third harmonic repeats itself three times in one cycle of the fundamental, 20 electrical degrees' displacement (between the coil sides) of the fundamental is equivalent to 60° displacement for the third harmonic, 100° displacement for the fifth, etc. Figure 15*b* shows the vector additions of the fundamental and third and fifth harmonics in the two coil sides.

A ninth harmonic could not be present in the voltage wave of the above coil because the 20° displacement in the fundamental would be equivalent to 180° of the ninth harmonic. That is, a ninth harmonic

in one coil side would neutralize the ninth harmonic in the other coil side as they would be 180° out of phase. This is shown in Fig. 15c. If a coil pitch is shortened by $1/n$ of the pole pitch, the nth harmonic will be balanced out.

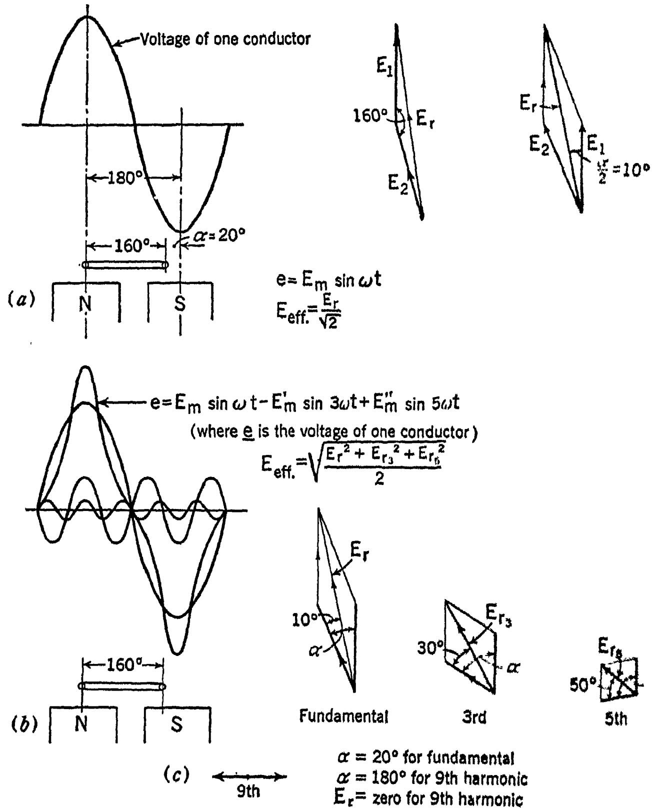


Fig. 15.

14. Pitch Factor. In a full-pitch winding the voltages generated in the two coil sides add directly. A fractional pitch gives a vectorial addition which is always less than the arithmetical sum. For that reason a factor must be introduced in the emf equation to account for the reduced resultant voltage. This is called the *pitch factor,* and it is computed as follows:

Refer to Fig. 15a. The voltage E_r, built up by the two coil sides, is the vector addition of E_1 and E_2. Since the effective values of the two voltages are the same:

As vectors:

$$E_1 + E_2 = 2E_1 \cos 10° \qquad [12]$$

$$E_r = 2E_1 \cos \frac{\alpha}{2} \qquad [13]$$

or, as it is sometimes written,

$$E_r = 2E \sin \lambda \frac{\pi}{2} \qquad [14]$$

where λ = coil pitch ÷ pole pitch

α = angular difference between pole and coil pitch

The term $\sin \lambda\, \pi/2$ is the pitch factor, and is abbreviated k_p. It can be defined as the ratio of the voltage generated in a fractional-pitch winding to that which would be generated if the winding were full pitch with an equal number of turns.

The winding pitch cannot be chosen independently, but its values depend upon the number of slots per pole. Thus, if there are 6 slots per pole, the pitch can be only such values as 7/6, 6/6, 5/6, 4/6. Windings of more than full pitch are seldom used. Common pitch factors are shown in Table I.

TABLE I

PITCH FACTORS, k_p

(For Fundamental Component)

Slots per Pole	2	3	4	6	8	9	10	12	Pitch Factor
	1	1	1	1	1	1	1	1	1.000
								$\frac{11}{12}$	0.991
							$\frac{9}{10}$		0.988
						$\frac{8}{9}$			0.985
					$\frac{7}{8}$				0.981
				$\frac{5}{6}$				$\frac{10}{12}$	0.966
λ							$\frac{8}{10}$		0.951
Possible pitch						$\frac{7}{9}$			0.940
ratios			$\frac{3}{4}$		$\frac{6}{8}$			$\frac{9}{12}$	0.923
							$\frac{7}{10}$		0.891
		$\frac{2}{3}$		$\frac{4}{6}$		$\frac{6}{9}$		$\frac{8}{12}$	0.866
					$\frac{5}{8}$				0.831
							$\frac{6}{10}$		0.809
								$\frac{7}{12}$	0.793
						$\frac{5}{9}$			0.766
	$\frac{1}{2}$		$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$		$\frac{5}{10}$	$\frac{6}{12}$	0.707

15. Distributed Windings. Usually all the winding of one phase is not concentrated in one slot under each pole. That is, there are several slots per pole for each phase. Consequently, the voltage generated in the various parts of one phase are displaced from each other and must be added as vectors. Figure 16 shows a three-phase alternator with 6 slots per pole. There are then 2 slots per pole for each phase. These 6 slots cover 180 electrical degrees, and the center lines of slots are 30 electrical degrees apart. The voltage waves built up in the two parts of

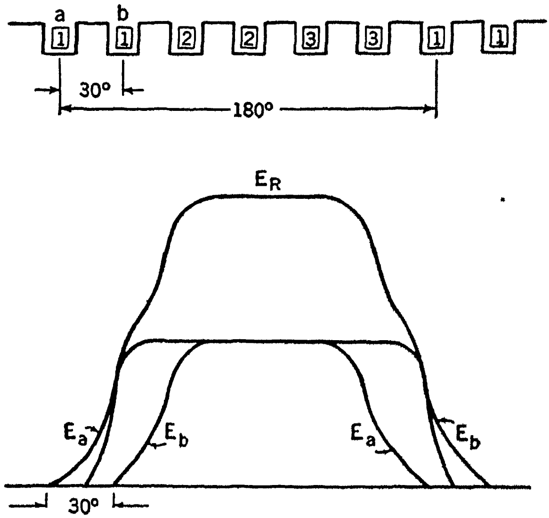


FIG. 16. Effect on the wave shape brought about by distributing a winding.

the winding are therefore 30° apart, and the resultant voltage wave for the winding is the sum of the individual waves. This is shown as E_R, and the effective value of this resultant voltage is always less than the voltage which would be produced if the coils were in the same slot. The ratio of the resultant voltage E_R to the arithmetical sum of the effective values E_a and E_b is called the *breadth* or *distribution factor* (denoted as k_d).

Dealing with the above example,

$$E_R = 2E \cos \frac{30^\circ}{2} \qquad [15]$$

Where E is the effective value of the voltage built up per coil,

$$E_R = 1.932E$$

Such analysis holds only for sine waves or for sine components of the waves, and, as the waves of Fig. 16 are obviously not sinusoidal, the analysis just made applies to their fundamental components.

The arithmetical addition of the voltages in the two slots would be $2E$, and the distribution factor is

$$\frac{E_R}{2E} = \frac{1.932E}{2E} \quad \text{or} \quad 0.966$$

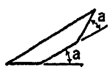
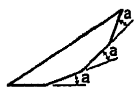
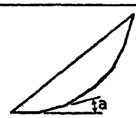
There are many different methods of deriving the value of k_d. One of the simplest ways is to assume that an effective value of 1 volt is induced in each coil side. Then, if the slots are equally spaced, as usual, the phase difference, α, between neighboring slots is 180° divided by the slots per pole.

In the diagrams shown in Table II the distribution factor is

$$k_d = \frac{\text{length of long chord}}{\text{sum of lengths of short chords}} \qquad [16]$$

TABLE II

DISTRIBUTION FACTORS k_d

Total slots per pole	2	4	6	8	12
2 phase, α=		45°	30°	$22\frac{1}{2}$°	15°
Slots per pole per phase	1	2	3	4	6
Vector diagram	→				
Formula	$\frac{1}{1}$	$\frac{2\cos 22\frac{1}{2}°}{1+1}$	$\frac{1+2\cos 30°}{1+1+1}$	$\frac{2\cos 11\frac{1}{4}°+2\cos 33\frac{3}{4}°}{1+1+1+1}$	$\frac{2\cos 7\frac{1}{2}°+2\cos 22\frac{1}{2}°+2\cos 37\frac{1}{2}°}{1+1+1+1+1+1}$
k_d=	1	.924	.911	.906	.903

Total slots per pole	3	6	9	12	
3 phase, α=		30°	20°	15°	
Slots per pole per phase	1	2	3	4	
Vector diagram	→				
Formula		$\frac{2\cos 15°}{1+1}$	$\frac{1+2\cos 20°}{1+1+1}$	$\frac{2\cos 7\frac{1}{2}°+2\cos 22\frac{1}{2}°}{1+1+1+1}$	
k_d=	1	.966	.960	.958	

16. Effect of Distribution on Harmonics. If the flux distribution in the air gap is not sinusoidal, the voltage waves built up in the armature turns in the various pairs of slots can be analyzed into their component harmonics. Assume a flux distribution as shown in Fig. 16. When applied to a three-phase generator with 6 slots per pole, the instantaneous values of the resultant voltage are shown on curve E_R in Fig. 16. This is the sum of the ordinates of curves E_a and E_b. Since these curves are shown 30 electrical degrees apart, the fundamentals are displaced 30°. This is equivalent to a displacement of 30° × 3, or 90° for the third harmonic, 150° for the fifth harmonic, etc.

The resultant wave E_R is made up of the vector sum of two fundamentals displaced 30°, two third harmonics displaced 90°, and two fifth harmonics displaced 150°, etc.

The net result of distributing the winding in the separate phases of an alternator is to reduce the voltage and the relative effect of the harmonics; the wave form of a distributed winding is more nearly sinusoidal.

The effect just discussed can be considered numerically as follows.

Let the flux-space distribution in the air gap contain a third and a fifth harmonic so that the voltage induced in each armature turn would be:

$$e = E(\sin wt + \tfrac{1}{3} \sin 3wt + \tfrac{1}{5} \sin 5wt)$$

If there are 8 turns per pole per phase and they are concentrated in one pair of slots, the resultant voltage is

$$e = E(8 \sin wt + \tfrac{8}{3} \sin 3wt + \tfrac{8}{5} \sin 5wt)$$

The relative magnitudes are

Fundamental	100%
Third harmonic	$33\frac{1}{3}$%
Fifth harmonic	20%

Suppose the 8 turns are distributed in 4 slots per pole; i.e., there are 12 slots per pole on this three-phase generator, or 4 slots per pole per phase. The angular displacement between each slot is 15 electrical degrees. Let us write the equations for the emf of each coil, referred to the center of the phase belt. (Two coils per slot.)

$$e_1 = 2E[\sin (wt - 22.5°) + \tfrac{1}{3} \sin 3(wt - 22.5°) + \tfrac{1}{5} \sin 5(wt - 22.5°)]$$

$$e_2 = 2E[\sin (wt - 7.5°) + \tfrac{1}{3} \sin 3(wt - 7.5°) + \tfrac{1}{5} \sin 5(wt - 7.5°)]$$

$$e_3 = 2E[\sin (wt + 7.5°) + \tfrac{1}{3} \sin 3(wt + 7.5°) + \tfrac{1}{5} \sin 5(wt + 7.5°)]$$

$$e_4 = 2E[\sin (wt + 22.5°) + \tfrac{1}{3} \sin 3(wt + 22.5°) + \tfrac{1}{5} \sin 5(wt + 22.5°)]$$

The resultant voltage is the vector sum of these, or:

$$e = E(7.65 \sin wt + 1.74 \sin 3wt + 0.328 \sin 5wt)$$

The relative magnitudes are now

Fundamental	100%
Third harmonic	22.7%
Fifth harmonic	4.3%

In short, distribution of the winding has minimized the effects of the harmonics, producing more nearly a sinusoidal-resultant emf wave. A more direct solution of the relative values involved in the above equations could be obtained by the use of the factors shown in Table IV.

The effective values of the voltages used in this example can be obtained as follows:

Concentrated:

$$E = \frac{8E}{\sqrt{2}} \sqrt{(1)^2 + (\tfrac{1}{3})^2 + (\tfrac{1}{5})^2} \quad \text{or} \quad 8.56 \frac{E}{\sqrt{2}}$$

Distributed:

$$E = \frac{E}{\sqrt{2}} \sqrt{(7.65)^2 + (1.74)^2 + (0.328)^2} \quad \text{or} \quad 7.98 \frac{E}{\sqrt{2}}$$

17. Form Factor. The ratio of the effective or root-mean-square value of a wave to its average value is called the *form factor*.

Thus for a sine wave with a maximum value of unity: The effective value is 0.707, the average value is 0.636, and the form factor for a sine wave is then

$$\frac{0.707}{0.636} = 1.11$$

In the elementary voltage equation for generated emf the factor 4.44 occurs. This is 4 × (form factor) since the analysis was based on a sine wave. Had the wave been of different shape this factor would have been another value. For non-sinusoidal waves the generated emf equation can be written:

$$E = 4(\text{form factor})fN\phi_{\text{max}}\, 10^{-8} \text{ volt}$$

Obviously, if E is desired in terms of phase voltage the member to the right in this equation must be multiplied by the number of coils per phase, since N is the number of turns per coil. Allowance must also be made for fractional pitch and for distributed windings.

18. Summary. In dealing with sine waves the elementary voltage equation can now be modified by the previously described correction factors, thus:

$$E = 4.44k_pk_df\phi_{\text{max}}N\, 10^{-8} \text{ volt} \qquad [17]$$

in which k_p and k_d are the pitch and distribution factors, respectively.

If non-sinusoidal waves are generated, the factor 4.44 is changed somewhat in the above equation. If the wave is analyzed into its harmonic components, the voltage equations are then written for the fundamental, using the fundamental pitch and distribution factors; for the other harmonics, using their corresponding pitch and distribution factors: and for the effective voltage wave obtained from these components.

$$E = \sqrt{E_1^2 + E_3^2 + E_5^2 \cdots}$$

wherein E_1 applies to the fundamental, E_3 to the third harmonic, etc.

The use of fractional-pitch windings enables various harmonics in the space distribution of the flux wave to be reduced or eliminated. It reduces the effective voltage.

The use of distributed windings reduces or eliminates various harmonics, results in a lower effective voltage, but also yields a lower leakage reactance for the winding and a better distribution of the heat from the armature coils.

TABLE III

PITCH FACTORS FOR VARIOUS HARMONICS

Harmonic	Pitch		
	$120° = \frac{2}{3}$	$144° = \frac{4}{5}$	$150° = \frac{5}{6}$
Fundamental	0.866	0.951	0.966
Third	0.000	0.588	0.707
Fifth	−0.866	0.000	0.259
Seventh	−0.866	−0.588	−0.259
Ninth	0.000	−0.951	−0.707
Eleventh	0.866	−0.951	−0.966

TABLE IV

DISTRIBUTION FACTORS FOR VARIOUS HARMONICS

(Three-phase Machines)

Total Slots per Pole	3	6	9	12
Fundamental	1.000	0.966	0.960	0.958
Third harmonic	1.000	0.707	0.667	0.653
Fifth harmonic	1.000	0.259	0.218	0.205
Seventh harmonic	1.000	0.259	0.177	0.157
Ninth harmonic	1.000	0.707	0.333	0.270
Eleventh harmonic	1.000	0.966	0.177	0.128

CHAPTER III

ARMATURE WINDINGS

19. Chapter Outline.

Armature Winding Types.
  Single and Double-layer.
  Lap, Wave, and Spiral.
  Fractional-slot.
Y and Delta Connections Contrasted.
Elimination of Harmonics.

20. Polyphase Windings. So far some of the effects of different placement of armature coils have been pointed out, but the types of armature windings in general use have not been considered. The following paragraphs include brief mention and description of armature windings.

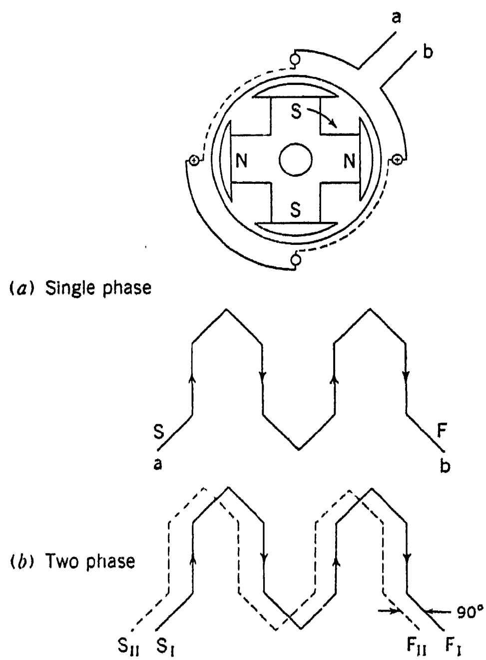


FIG. 17. Fundamental single-phase and two-phase winding diagrams.

The fundamental single-phase diagram of one slot per pole is shown in Fig. 17*a*. By adding a second winding 90 electrical degrees from the first, a two-phase winding is obtained. Figure 18 shows a three-phase winding in which the conductors of each phase are necessarily separated by 60 electrical degrees, but connected so that their emf's are 120° apart in time. On these figures *S* and *F* signify start and finish, respectively, of the winding. Connections are made as shown in *b* for a three-phase Y, and as shown in *c* for a three-phase Δ-connected winding.

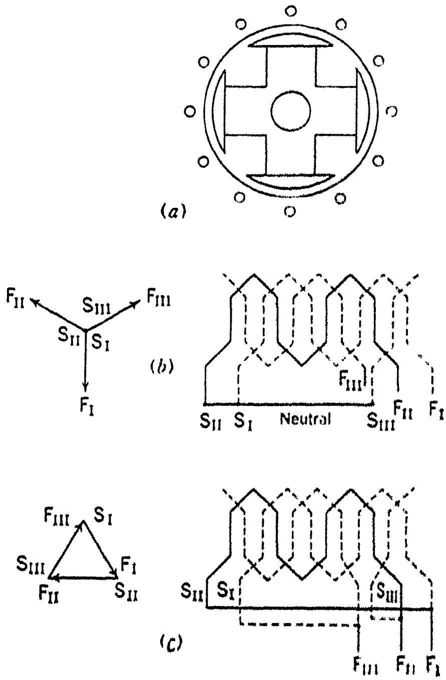


FIG. 18. Fundamental three-phase winding diagram.

21. Winding Types. The fundamental diagrams of Figs. 17 and 18 are shown with single conductors, placed in 1 slot for each phase under each pole. Actually, in most cases, more than 1 turn per coil is required to give the correct member of conductors on the winding. These conductors are wound in coils which are placed in the proper slots, and groups of them are connected in various series, parallel, or other combinations.

It is possible to use *single-layer* or *double-layer* windings as shown in Fig. 19. The double-layer windings are more common and result in a number of coils equal to the number of slots; there are 2 *coil sides* per slot. The number of conductors per slot is always even.

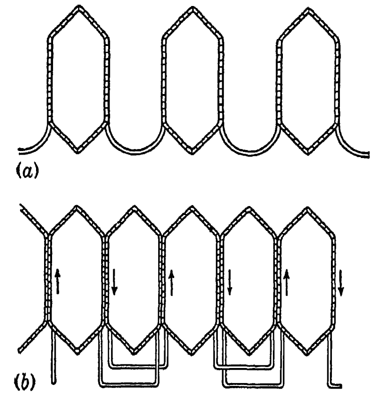


FIG. 19. (a) Single-layer or half-coiled winding. (b) Double-layer or whole-coiled winding. This is the common type in which the number of coils and the number of slots are equal.

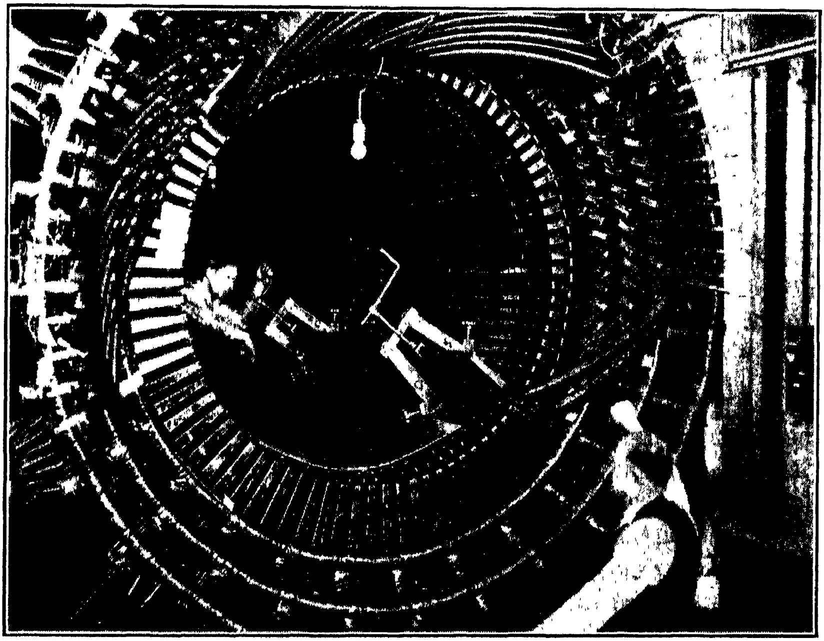


FIG. 20. Constructing a double-layer winding in a stator rated at 121,000 kv-a, 0.95 pf, 18,000 volts, 60 cycles, three phase, 1800 rpm. (*The Allis-Chalmers Mfg. Co.*)

A further subdivision of winding types is possible by the consideration of the manner in which the end connections are made. Examine the three drawings of Fig. 21. *Single-layer*, single-phase windings are indicated for simplicity. The types shown are (*a*) *spiral*, (*b*) *lap*, (*c*) *wave*. The last two types are similar to the lap and wave windings used on d-c armatures. Since the pitch, distribution, and number of conductors

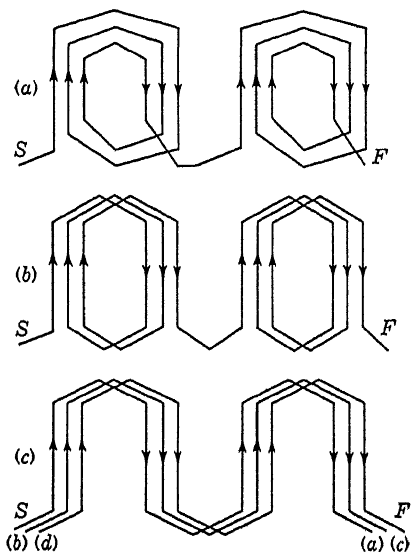


FIG. 21. Elementary single-layer, single-phase windings. (*a*) Spiral. (*b*) Lap. (*c*) Wave.

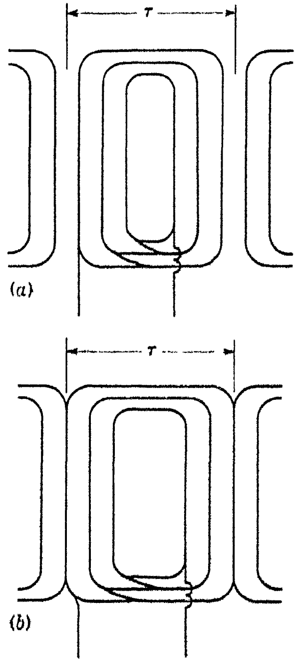


FIG. 22. Two spiral or concentric windings in which (*a*) all coils are short pitch or (*b*), the outer coil is full pitch.

shown on all three windings are the same, the three are electrically identical. Although these are shown as single-phase windings, the addition of similar coils, properly displaced, would result in the correct polyphase construction. All can be used *double layer*, although the form of *spiral* winding most frequently used does not lend itself practically to such construction.

The largest application for the *spiral winding* is on single-phase induction motors. In its most common form it is also called a *concentric winding* and is made up of short pitch coils, single layer, as shown in Fig. 22.

An elementary three-phase *chain* or *basket* winding is shown in Fig. 23. Note the varying sizes of the coils to prevent end-coil interference. This

type of winding is sometimes used on large, high-voltage alternators, and the most practical method of distributing the winding into more than 1 slot per pole per phase is to use concentric coils. In this way the ends of the coils are not required to overlap, and hence the chain or basket construction becomes a type of spiral winding.

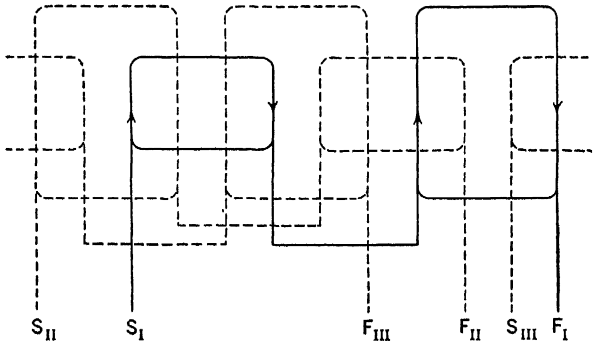


FIG. 23. Three-phase chain winding, one slot per phase, per pole.

22. Fractional-slot Windings. In the interest of economy, manufacturers do not design new stator laminations for each rating of generator which they build. Consider an 8-pole generator having 9 slots per pole, or 72 slots total. To provide a three-phase winding, there will be available 3 slots per phase per pole, and this will determine the distribution factor.

$$\frac{\text{Total slots}}{\text{Poles} \times \text{phases}} = \frac{72}{8 \times 3} \quad \text{or} \quad 3$$

Because this division results in a whole number, the winding will be symmetrical.

Consider next a 10-pole winding to be constructed in the same stator laminations.

$$\frac{72}{10 \times 3} = 2\tfrac{6}{15}$$

There are now $2\frac{6}{15}$ slots per phase per pole, and it is obvious that the usual uniform spacing of the winding is not possible. The resultant winding is of the *fractional-slot* type. Under favorable conditions such windings can result in balanced voltages on all phases.[1] The determination of pitch and breadth factors requires special consideration.

[1] E. M. Tingley, "Two-phase and Three-phase Lap Windings in Unequal Groups," *Elec. Rev. and Western Elec.*, Vol. 66, 1915.

M. G. Malti and Fritz Herzog, "Fractional-slot and Dead-coil Windings," *A.I.E.E. Paper*, pp. 39–134, May, 1939.

23. Winding Diagrams. Because of the great variety of winding diagrams which would be required to cover all voltages, poles, and capacities, manufacturers usually construct a given winding after referring to a *winding specification* and a *connection diagram*. The former refers to the type of insulation, number of coils, size of wire or strap, and number of turns. The latter indicates the connections and grouping of coils. Thus, by a variety of written specifications and a comparatively few drawn diagrams, most types of windings can be indicated for correct construc-

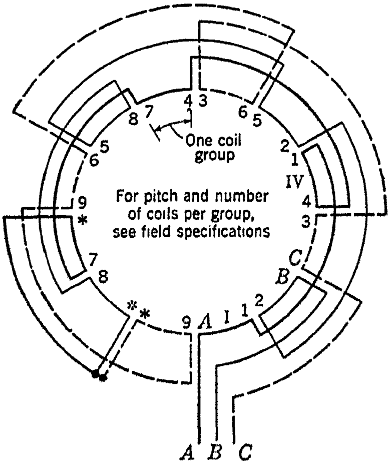


FIG. 24. A winding diagram for a 4-pole, three-phase, single-circuit alternator armature. Points in the winding of terminal potential are indicated by letters; the neutral points are indicated by asterisks.

tion. The usual type of connection diagrams are shown in Figs. 24 and 25. Note in Fig. 24 that the arc from A to 1 represents one phase belt, made up of a number of coils fixed by the slots and indicated on the winding specification. The beginning of this coil group is A, provided with a terminal lead. The finish of this coil group connects to the finish of coil group IV, the apparent reverse being necessary to keep the emf's additive, considering that group IV is under a different pole from group I. The expansion of this diagram into a double-circuit Y connection is shown in Fig. 25*a*.

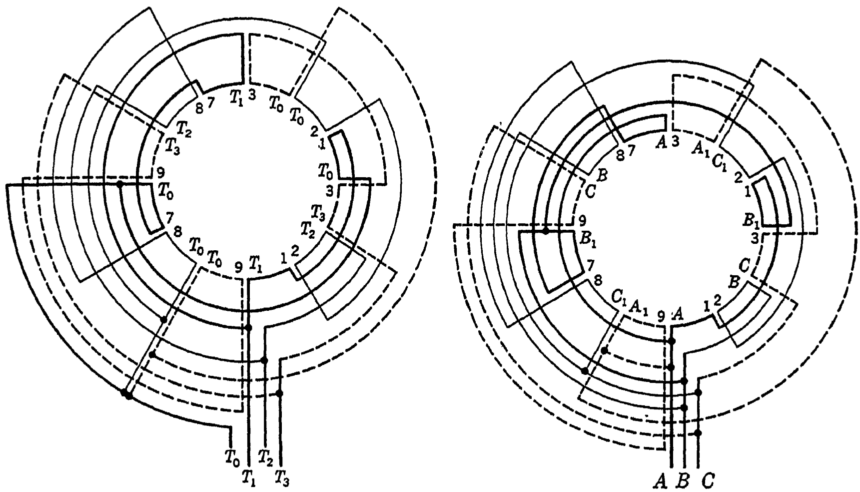
24. Y and Delta Connections. Harmonics. The majority of three-phase generators have their windings connected in Y. It offers the following advantages:

(*a*) To obtain a desired voltage between outside terminals of a generator, the voltage built up in any one phase need be only 58 per cent of the terminal value. Hence only 58 per cent of the turns required for a

Δ-connected armature are necessary, with a consequent lowering of insulation cost.

(*b*) A Y-connected winding [2] offers the advantage of a fourth or neutral lead, making possible the advantages of a four-wire system, with or without grounded neutral.

(*c*) The wave shape of a Y-connected winding is improved, owing to the elimination of third harmonics and all multiples of the third harmonic from the terminal voltage.



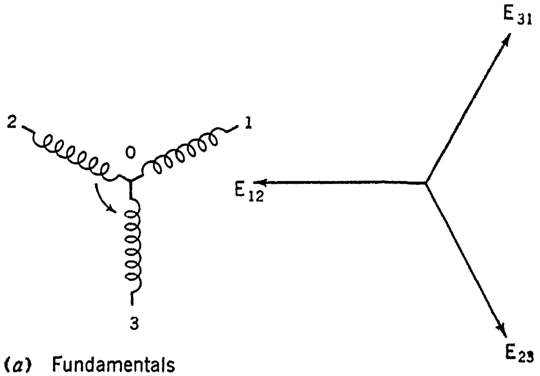
(*a*) (*b*)

Fig. 25. (*a*) Winding or connection diagram for a 4-pole, three-phase, 2-Y armature. (*b*) Winding diagram for a 4-pole, three-phase, 2-circuit, Δ-connected armature.

Figure 26 shows a Y-connected armature. The potential differences between the terminals are E_{12}, E_{23}, and E_{31}; where $E_{12} = e_{02} + e_{10} = e_{02} - e_{01}$, etc. These terminal voltages are 120 electrical degrees apart. The third harmonics are then 3×120, or 360 electrical degrees apart. When the third-harmonic emf's of any two legs are vectorially subtracted, as above, to obtain terminal voltages, the resultant is zero. Hence no third harmonic appears between terminals, although the third harmonic in any phase may distort the voltage wave between one terminal and neutral, causing it to be larger than $E/\sqrt{3}$, where E is a line voltage.

Figure 27 shows the diagram for a Δ-connected armature. With a pure sine wave, the connection of the windings with equal voltages in a closed delta causes no circulating current, as the vector sum of the three

[2] *Trans. A.I.E.E.*, Vol. 33, Part 1, p. 803, 1914.

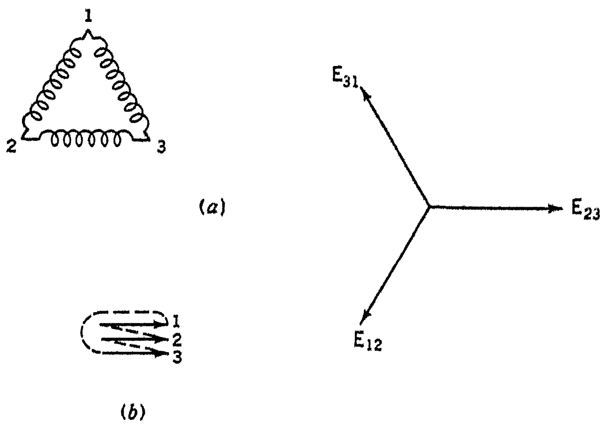


$$E_{12} = e_{02} + e_{10} = e_{02} - e_{01}$$
$$E_{23} = e_{03} + e_{20} = e_{03} - e_{02}$$
$$E_{31} = e_{01} + e_{30} = e_{01} - e_{03}$$

$e_{02} - e_{01} = 0$, etc.

(*b*) Third harmonics

IG. 26. Diagrams showing that in a Y-connected winding, the third harmonics are balanced out with respect to the terminals.



IG. 27. Diagrams showing (*a*) that the vector sum of the fundamentals in a Δ-connected winding is zero; (*b*) that the vector sum of the third harmonic is the arithmetical sum of the individual harmonics.

voltages is zero. But, if the wave is so distorted as to have a pronounced third harmonic, the voltages of the third harmonic add directly in the three legs as shown in Fig. 27*b*. This is the equivalent of a short circuit, permitting the circulation of a current through the windings. This current is of the magnitude

$$I_3 = \frac{3E_3}{3Z_3}$$

where E_3 is the effective value of the third harmonic voltage, and Z_3 is the impedance of one leg of the winding at triple frequency.

Similarly the ninth harmonic, if present, would cause its circulating current:

$$I_9 = \frac{3E_9}{3Z_9}$$

The value of the impedance changes, of course, with the change of frequency.

The reason for the use of Y-connected armatures can be seen from the above statement. The circulating currents from the third harmonics can readily cause unnecessary losses and dangerous heating in Δ-connected armatures. In addition, the use of a $\frac{5}{6}$-pitch winding in three-phase, Y-connected generators reduces the fifth and seventh harmonics, if present, to almost nil, so the lowest harmonic that can be present is the eleventh. This is generally inconsequential in power systems.

25. Summary. The preceding information on armature winding is by no means a complete picture of the art. It brings out only the more important considerations and types.

From this information and from that covered in the preceding chapters the student should be able to realize how a nearly pure sine wave can be achieved in a three-phase generator, and how a winding can be subdivided into groups which may be connected in series or in parallel.

(*a*) The shape of the pole shoe can be made such as to give a nearly sinusoidal flux distribution at some definite load.

(*b*) If the distortion in the generated emf wave contains a third harmonic, it can be eliminated by Y connection in a three-phase armature. (This applies also to the ninth, fifteenth, etc.)

(*c*) The selection of the proper pitch can be used to eliminate a certain harmonic. Thus a pitch of 144 electrical degrees eliminates the fifth and reduces the seventh; a pitch of 150 degrees greatly reduces the fifth and seventh; and so on.

(*d*) The distribution of the winding further reduces the effect of the remaining harmonics, and the resulting voltage wave will be very nearly sinusoidal at all loads.

CHAPTER IV

FACTORS AFFECTING ALTERNATOR REGULATION

26. Chapter Outline.

Factors Affecting Alternator Regulation (discussed qualitatively).
- Effective Resistance.
- Armature Reactance.
- Armature Reaction.
- Pole Leakage Flux.

27. Alternator Regulation.[1] The load supplied by an alternator can vary in power factor so that its current can lag, be in phase with, or lead

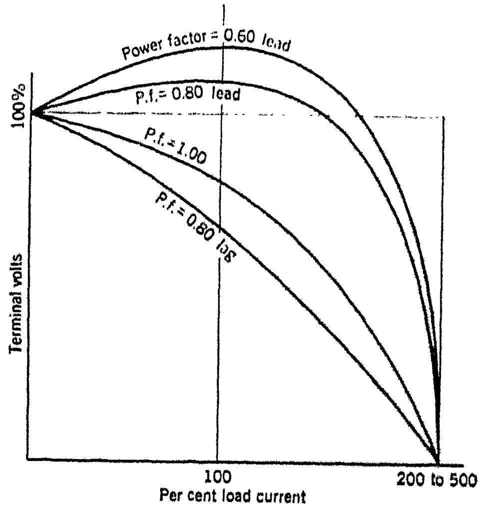


FIG. 28. Load curves of an alternator showing effect of pf. Field current and speed are maintained constant.

the terminal voltage. The constants of the load determine the power factor at which the alternator operates. With the speed and the excitation of the alternator maintained constant, the terminal voltage will vary with load as shown by the curves of Fig. 28.

[1] A. Still, "Inherent Regulation of Synchronous A. C. Generators," *J.I.E.E.*, Vol. 53, p. 587, 1915.

A numerical expression for voltage regulation is obtained by the formula:

$$\text{Regulation (in percentage at stated pf)} = \frac{V_{\text{no load}} - V_{\text{full load}}}{V_{\text{full load}}} 100 \quad [18]$$

Briefly stated, regulation is the percentage change in voltage as the load is reduced on an alternator from full-rated value to zero, speed and excitation being assumed constant. At leading pf, the terminal voltage is apt to decrease as the load current is reduced, thus making the regulation negative.

The understanding of the reactions which cause this voltage change involves a number of factors which are explained in the following pages.

28. Effective Resistance. The effective resistance of the armature winding is greater than the conductor resistance as measured by direct current. This is because additional energy over the purely I^2R value is expended inside and sometimes outside of the conductor, owing to the alternating current. The chief sources of this additional energy loss are

(*a*) Eddy currents in the surrounding material.
(*b*) Magnetic hysteresis in the surrounding material.
(*c*) Eddy currents, or unequal current distribution in the conductor itself.

In many cases it is sufficiently accurate to measure the resistance of an armature by direct current and increase it to a fictitious value, called the *effective resistance*, large enough to take care of these extra losses. The exact value can vary widely from 1.25 to 1.75, or more, $\times$ the d-c resistance, depending upon design. Extreme accuracy is not necessary in this factor for calculating regulation.

29. Armature Leakage Reactance. The load current, flowing through the armature winding, builds up local flux which on cutting the winding generates a counter emf. This effect gives the armature a *reactance* which is numerically equal to $2\pi fL$. L, in henries, is the leakage inductance of the winding. This armature reactance is called the *leakage reactance* (X_l) since the flux which causes it is around the armature turns only and does not affect the field flux directly. This leakage flux is proportional to the armature current since the magnetic path it covers is not normally saturated. X_l varies somewhat with the position of the armature and the field poles.[2]

[2] V. Karapetoff, "Variable Armature Leakage Reactance in Salient Pole Synchronous Machines," *J.A.I.E.E.*, Vol. 45, p. 666, July, 1926.

30. Components of Armature Reactance.[3] For convenience in calculating, the armature leakage flux can be divided into two parts:

(*a*) The flux around the end connections of the windings, which has an air path.

(*b*) The flux around that part of the conductors which is embedded in the slots. This flux is further divided into two parts, giving rise to slot reactance and tooth-tip reactance. These correspond to the flux systems ϕ_s and ϕ_t in Fig. 29.

End-connection Reactance. The flux surrounding the end connections of the armature coils is shown in Fig. 29*b*. It has an air path. This

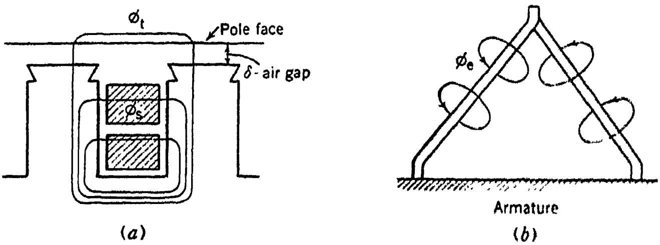


FIG. 29. (*a*) Slot leakage flux. (*b*) End-connection leakage flux.

total flux will vary with the length of the end connections, and hence the type of winding enters as a factor in its determination.

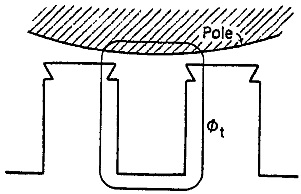
Slot Reactance. Consider the 2 bars in the slot of Fig. 29*a* to be 2 coil sides of a double-layer winding. It is evident that the reactance of the lower bar will be greater than that of the upper since the lower bar is surrounded by more flux. This yields equal reactances for the various coils because each coil will have one side on the top and the other on the bottom of the slots. This unequal distribution of flux not only increases the reactance of the bottom conductor but also gives a reactance gradient through the cross-section of the conductor itself. The current no longer distributes itself uniformly through the conductor, but is crowded to the top, giving rise to what is known as the skin effect. It forms one component of the difference between conductor and effective resistance. This *skin-effect factor* varies from 1.1 to 1.25 or higher. It can be reduced by subdividing and transposing the positions of the conductor in the slots.

Narrow, deep slots result in high armature reactance. To reduce the slot reactance an increased width of slot can be used. This increases the reluctance of the path over which the flux travels. A wide, open slot may produce irregularities in pole-flux distribution which cause in-

[3] P. L. Alger, "Calculation of Armature Reactance in Synchronous Machines," *Trans. A.I.E.E.*, Vol. 47, p. 493, April, 1928.

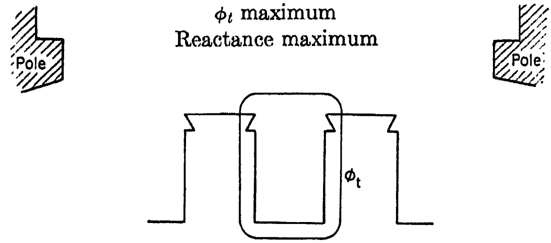
creased losses in the pole faces, or it may under certain conditions cause a pulsation of the pole flux as a whole.

Tooth-tip Reactance. Consider the flux path ϕ_t of Fig. 29. The magnitude of this flux depends upon the reluctance of its path, and hence is affected by the shape and width of the teeth and the length of the air gap. Since the flux goes across the air gap and into the face of the pole, the shape of the pole is a factor in determining the value of this flux and the



(*a*)

Reluctance minimum
ϕ_t maximum
Reactance maximum



(*b*)

Reluctance maximum
ϕ_t minimum
Reactance minimum

FIG. 30.

reactance due to it. In salient-pole machines the tooth-tip reactance will vary from a maximum to a minimum value as the field rotates, depending upon whether the slot is under a pole or midway between poles.

An additional variable is introduced by the pf. Imagine that a conductor in the slot of Fig. 30*a* is carrying current of unity pf. The voltage, current, and tooth-tip flux will all be at maximum values. Should the pf be zero, the current and tooth-tip flux could not be a maximum until the field had moved to a position *b*. The flux built up now would not have the same maximum value as would be built up at *a*, owing to the increased reluctance of the air path. Hence power factor will vary the value of tooth-tip reactance for a given position of the armature. The net average effect on the entire armature reactance is slight, however. In addition, there is some doubt as to the feasibility of using tooth-tip

reactance as a component of leakage reactance since it crosses the gap, and its effect might already appear in armature reaction as described below.

31. Armature Reaction. The previous discussion has dealt with a portion of the armature flux which was built up around the armature conductors but which did not pass through the magnetic circuit of the main field. In addition to this purely local effect, the ampere turns of the armature conductors comprise a magnetomotive force which aids or opposes the mmf of the field, depending upon whether the pf is leading or lagging, respectively. This effect is known as armature reaction. It is fundamentally similar to armature reaction in d-c machines.

Armature reactance and armature reaction arise from the same source: the flux built up around the armature conductors as they carry load current. Armature reactance, however, as a local effect gives the winding an IX drop. It is therefore commonly considered as a voltage vector. Armature reaction is customarily dealt with as a flux or mmf effect.

32. Effect of Power Factor on Armature Reaction. Consider a full-pitch armature coil of a single-phase winding shown in Fig. 31*a*. The

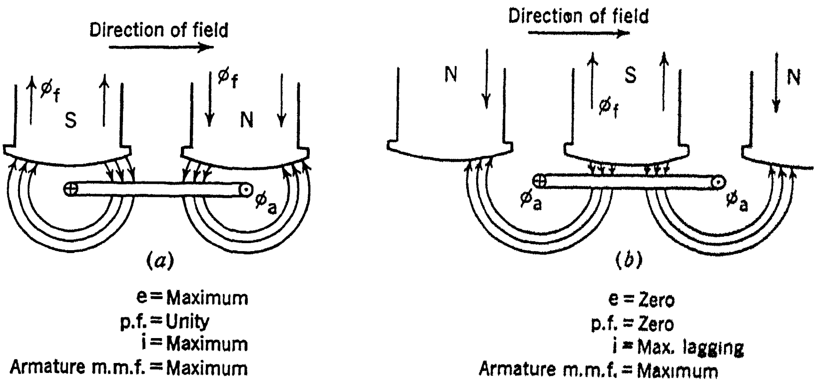


FIG. 31. Armature reaction.

voltage generated in this coil will be a maximum when the coil center is displaced 90 electrical degrees from the pole center. For a unity power factor load, the current will then be a maximum as will also the armature mmf. The effect of this armature mmf can be seen from an examination of the flux ϕ_a, which it would tend to build up. This armature mmf would strengthen the flux of the lagging pole tips and weaken that of the leading pole tips.

If the alternator were operating at zero pf lagging, the condition would be as shown in *b*. The coil sides would be midway between the

poles when the armature current had reached its maximum, since the current is lagging the voltage by 90°. At this position, shown in *b*, the mmf of the armature is in the best position for directly opposing that of the field. The result is a reduced effective field mmf and decreased field flux. Should the current lead the voltage at zero pf, the flux ϕ_a would be reversed in direction and would add to the field flux.

Zero pf loads are never encountered in practice. At lagging pf's of 0.60 to 0.90, the net effect of armature reaction is a combination of the

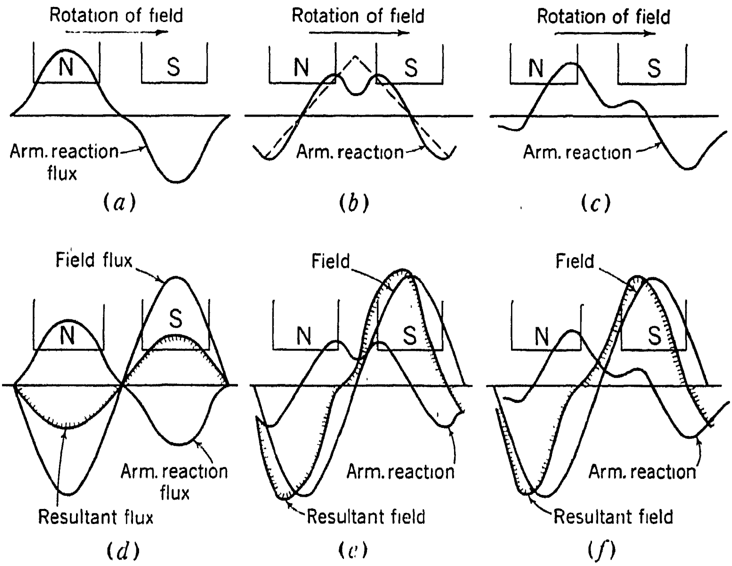


FIG. 32. Effect of pf upon armature reaction, and the resultant field flux. (*a*) and (*d*) Zero pf, lag. (*b*) and (*e*) Unity pf. (*c*) and (*f*) 80 per cent pf, lag.

the two effects discussed above. It both distorts and weakens the field flux. We shall see later how these two effects are covered, by considering two components.

Owing to the unequal reluctance around the air gap, the distribution of the main flux from salient poles will be about as shown in Fig. 32. On this figure, *a*, *b*, and *c* represent the armature flux at zero pf lagging, unity pf, and 80 per cent pf lagging, respectively. The resultants of the armature and field fluxes are illustrated by *d*, *e*, and *f*. Actually, a flux is the effect of net mmf's and hence the mmf's should be considered as the components. The method used here is to express the general idea.

It can be seen from this that the resultant field is not sinusoidal, and as such, strictly speaking, cannot be represented as a vector quantity on generator diagrams. In spite of this, the analysis of armature

reaction, based on sine waves and vectors, developed from non-salient-pole machines, is often applied to salient-pole generators with but little error.

For the present purposes it is sufficient to grasp the idea that armature reaction varies in magnitude proportionally with the armature current, and that it also varies in its effect with the pf. These changes make it possible to replace the effect of armature reaction on the generated voltage (through flux reduction) by a fictitious reactance drop, an approximation made for the purpose of applying certain methods of analysis.

33. Pole-leakage Flux.[4] In addition to the main flux of the field, passing across the air gap and through the complete magnetic circuit, some of the field flux passes directly between the poles without crossing the air gap. This is called the pole-leakage flux, and its magnitude is expressed by the pole-leakage factor or coefficient, which is equal to

$$\frac{\text{Useful flux} + \text{pole-leakage flux}}{\text{Useful flux}}$$

This coefficient varies from 1.1 to 1.4 The pole leakage is greater for large loads than for small loads, and is larger for low lagging pf's than for high pf's.

This coefficient depends upon design, and varies as mentioned above. Unfortunately it is not a constant for a given machine but varies with the load.

34. Summary. In dealing with alternator operation two mmf's have been named:

(*a*) Field mmf.
(*b*) Armature mmf.

Three main divisions of the flux have been named:

(*a*) Field flux, crossing the air gap which generates the effective voltage of the alternator. It is caused by the resultant of field and armature mmf's.

(*b*) Armature-leakage flux around the armature conductors, giving rise to a reactance of the armature circuit.

(*c*) Pole-leakage flux: The flux leaking between the poles and not passing through the air gap.

Three factors which influence the terminal voltage of an alternator with load increase (excitation and speed being considered constant) have been dealt with:

[4] Theodore Schou, *Elec. Rev.*, Vol. 77, p. 281.

(a) Effective resistance of the armature winding, causing an armature voltage drop IR_e.

(b) Leakage reactance of the armature, causing an armature voltage drop IX_l.

(c) Armature reaction, causing a change in the effective air-gap flux, which in turn affects the voltage. The effects of this armature reaction will be calculated in various ways.

In Chapters VI and VII, methods are given for calculating the results produced when all the principal factors are included.

CHAPTER V

VECTOR DIAGRAM OF THE ALTERNATOR

35. Chapter Outline.

Vector Diagram of the Alternator.

Table of Abbreviations Used on the Alternator Diagrams.

E_0 = the no-load voltage of the alternator, or the voltage built up by the field flux only

E_g = the generated voltage after the armature mmf has affected the field flux

V_t = the terminal voltage

E_x = the voltage self-induced in the armature by its own leakage flux

E_a = the voltage built up by the armature flux (or armature mmf) if it were acting alone

M_f = the field mmf in ampere turns per pole

M_a = the armature mmf in ampere turns per pole

M_r = the resultant of the above mmf's in ampere turns per pole

ϕ_f = the flux of the field in lines per pole

ϕ_l = the armature leakage reactance flux

ϕ_a = the flux built up as a result of M_r in lines per pole

θ = the terminal pf angle, usually in degrees

R_e = the effective resistance of the armature in ohms per phase

X_l = the reactance caused by the armature leakage flux in ohms per phase

36. Vector Diagram of an Alternator. Consider one phase of any one-, two-, or three-phase alternator with a distributed winding and with non-salient poles. A sine-wave distribution of flux in the air gap may be assumed. The mmf of the field is represented by the vector M_f. At no load this mmf sets up a flux represented by the vector ϕ_f. As sinusoidal distribution of flux is assumed, these quantities can be represented as vectors. In Fig. 33, the voltage generated in one phase of the armature winding by cutting this air-gap flux is shown by the vector E_0. It is 90° behind the flux. (See Article 11.) Assume that a load is connected

to the alternator armature winding so that the phase current is I (Fig. 34). In a polyphase machine the effect of armature reaction is fixed in direction with respect to the field. The maximum value of the armature mmf occurs at the same time as the maximum value of the armature current. The vector M_a of armature mmf is in phase with the current. The magnitude of M_a is determined by the sum of all phases, as the effect on the field is independent of the number of phases in which the conductors are grouped so long as the values of k_d and k_p are the same for a given current and pf.

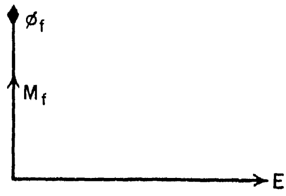


FIG. 33. No-load vector diagram of an alternator.

The resultant of M_f and M_a is M_r, the mmf effective in forcing flux around the magnetic circuit. This flux ϕ_a causes a voltage to be generated in each phase of the armature winding. It is represented by E_g. It is convenient to take this voltage as lagging the field flux by 90°.

It can be concluded that loading the alternator changes the generated voltage from E_0 to E_g, owing to the effect of armature reaction. This

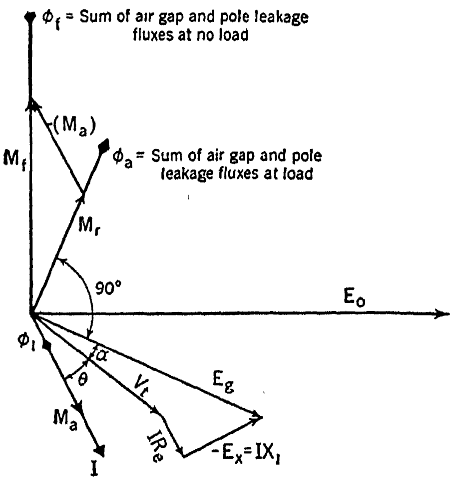


FIG. 34. General or Potier vector diagram of an alternator. The terminal voltage is V_t, the load current is I, and the pf is cos θ.

does not show, however, the influence of saturation; the exact relation is shown later.

The leakage flux of one phase is shown by ϕ_l. It may be considered in phase with the armature current and directly proportional to it,

although it is affected by the saturation of its path. The leakage flux cutting the armature conductors builds up a voltage drop, E_x, 90° behind it. This voltage drop is numerically equal to IX_l, where X_l is the leakage reactance. As vectors, IX_l is equal to *minus* E_x.

In addition to this drop in voltage the terminal voltage will be still further reduced by the armature IR_e drop. A resistance drop here is always considered in phase with the current. R_e is the effective resistance of one phase of the armature. The resultant terminal voltage is then V_t, and the angle θ which it makes with the current I is the pf angle of the load.

The terminal voltage, V_t, per phase, is less than the generated voltage, E_g, by the vector subtraction of the leakage reactance voltage and the effective resistance voltage.

For the purposes of analysis these reactions have been explained in reverse order to that in which the calculations of regulation are usually made. It is customary to begin such calculations with a required terminal voltage, pf angle, and load current, as these values can readily be measured or taken as the rated values of a given machine. Such procedure makes it more convenient to carry through the computation. To calculate the regulation, find R_e, X_l, and M_a. Use the given saturation curve, and assume values for I, V_t, and θ. Then lay off successively to scale: I, θ, V_t, IR_e, IX_l, and E_g. With this value of E_g read the ampere turns M_r from the saturation curve. Generally the saturation curve with no-load leakage factor is used, but the saturation curve with full-load leakage factor is more accurate. Lay off M_r as shown, 90° ahead of E_g. Then lay off M_a in phase with I, to find M_f. With this value of M_f read E_0 from the no-load saturation curve. Fairly accurate results can be obtained on salient-pole machines by this method, which is useful in predicting the regulation to be expected from a design before the machine is built. See also Blondel's diagram in Chapter VIII.

37. Sources of Error in This Analysis. In order to represent the mmf of the field and armature as vectors, it was assumed in the above that the field flux, voltages, and currents dealt with were sine waves.

As the load was applied to the above machine, the field-leakage coefficient was assumed to be constant. Actually, increased load and armature reaction cause more pole-flux leakage and consequent lowering in the air-gap flux. This effect is usually small. Less flux is built up per unit of M_r than per unit of M_f.

It might also be pointed out that the vectors on this diagram representing flux and mmf are *space* vectors. That is, their relative directions are derived from their positions in space. The voltage and current vectors (E_0, E_g, V_t, I, etc.) are *time* vectors, rotating in space but reach-

ing their maximum values at different times, coresponding to the angles between them. With one system in space vectors and the other in time vectors, it is not strictly necessary that the displacement between them be the 90° shown between E_0 and M_f. By some writers this relationship is not maintained, but it is most convenient here to do so.

CHAPTER VI

REGULATION FROM NO-LOAD TESTS

38. Chapter Outline.

Regulation from No-load Tests.

I. Synchronous-impedance Method.
  Theory.
  Tests Necessary for Its Use.
  Example.
  Errors in This Method.

II. Saturated Synchronous Impedance.
  (Old A.I.E.E. Method.)
  Theory.
  Tests Necessary for Its Use.
  Example.

III. Potier Triangle.
  Potier Reactance.

IV. Load Characteristic Curves.

V. Magnetomotive-force Method.
  Theory. Brief Outline.

VI. American Standards Association Method.
  Theory.
  Example.

39. Methods of Predicting Regulation. Many methods have been devised for determining the regulation of an alternator from other than direct readings and actual load tests. They fall into two groups: (1) the prediction of regulation by means of simple no-load tests made upon a completed machine; (2) the prediction of regulation from design data available before the machine is actually built.

Three methods based upon no-load tests will be given here. They are:

(*a*) The synchronous-impedance method and the use of saturated synchronous reactance.

(*b*) The magnetomotive-force method.

(c) The American Standards method, making use of the Potier reactance.

It will be understood that it is usually impracticable to determine the regulation of a large generator by actual load tests, owing to the lack of adequate loading devices and the great expense involved.

40. The Synchronous-impedance Method. In Fig. 35 is shown the vector diagram necessary for the calculation of regulation by the synchronous-impedance method. V_t is rated voltage per winding phase,

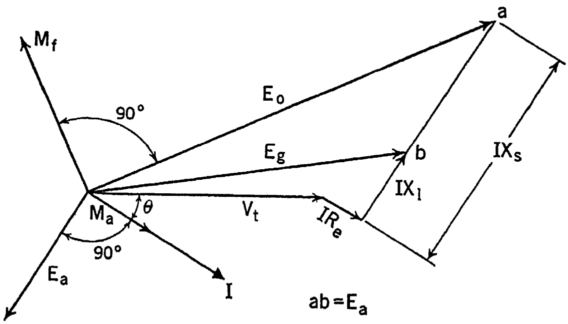


FIG. 35. Simplified vector diagram for explaining regulation by the synchronous-impedance method.

regardless of whether Y or Δ connections are used. I is the rated current per phase, and the angle θ is the pf angle for which the regulation is to be calculated.

M_f is the mmf of the field, and the voltage which its flux would generate when M_f is acting alone is E_0. M_a is the mmf of armature reaction, and the voltage it would produce if acting alone is E_a. The reducing effect of armature reaction upon the voltage at lagging power factor is shown by adding E_0 and E_a vectorially, resulting in E_g. *Armature-reaction effect is treated as a voltage drop by this method.* The reactance drop (IX_l) and the resistance drop (IR_e) for one phase are subtracted vectorially from E_g, leaving the terminal voltage V_t.

Since E_a (or its equal ab) varies with the current and is in phase with IX_l, regardless of the power factor, it is replaced by a new fictitious reactance drop. That is, to the actual leakage reactance of the armature is added a fictitious value and the sum is called the *synchronous reactance*, X_s.

$$IX_s = IX_l + E_a \text{ (by definition)} \qquad [19]$$

We have now reduced the vector diagram to voltage vectors, and its solution, with V_t, θ, and I known, depends upon evaluating R_e and X_s.

With V_t as the rated terminal volts per phase, the voltage at no load with the same excitation will be E_0. Hence the regulation becomes

$$\text{Regulation (in percentage)} = \frac{E_0 - V_t}{V_t} 100 \qquad [20]$$

The regulation predicted by this method is usually worse than the actual value. X_s is affected by both the pf and by the point on the saturation curve at which the field current is taken, and cannot actually be constant as assumed. This is the simplest form of applying the method.

41. Test Data Necessary for the Use of This Method. The data necessary to calculate the regulation by the synchronous-impedance method and the methods of obtaining them are given below.

Effective Resistance of the Armature. For a rough approximation, the effective resistance of the armature circuit can be taken as 1.6 times the resistance as obtained by d-c measurement. The skin effect in the armature conductors can increase the effective resistance as high as six times its d-c value. In such a case corrective measures (transpositions and subdivisions of conductors) are resorted to until the value does not exceed 1.25 times the d-c resistance. The other losses (listed in Article 28) sometimes combine to increase the effective resistance until it can be as high as 2 or more times the d-c value. In general this factor reduces with the size of the machine and its frequency. We will use 1.6 as an average factor for 60-cycle alternators. The results for the regulation as a rule are but little affected by the value taken for R_e.

It must be kept in mind that the resistance per phase of a three-phase alternator is not the same as the resistance between terminals. In the Δ connection the resistance per phase would be $1\frac{1}{2}$ times the resistance between terminals. A convenient method in dealing with Y connections (in which the resistance per leg is one-half of that measured between terminals) is to measure the resistances t_{1-2}, t_{2-3}, t_{3-1}; add them; and divide the result by 6. This gives an average phase resistance which when multiplied, say, by 1.6 can be used as the effective resistance per phase.

An experimental method of determining effective resistance is to apply an external alternating voltage to the armature winding, and to measure the power and current input to one phase (or one pair of terminals with corrections).

The effective resistance is then

$$R_e = \frac{\text{power input}}{I^2} \qquad [21]$$

Results obtained by this method are usually high because of abnormal iron losses.

Synchronous Reactance of the Armature. If the armature of the alternator (the vector diagram of which is shown in Fig. 35) were short-circuited, the vector diagram would reduce to that of Fig. 36. That is, under short-circuit conditions there is no terminal voltage and all the generated voltage is used up as IZ_s. I is the armature current, Z_s is the *synchronous impedance*. In order to determine this synchronous impedance (Z_s), or the synchronous reactance (X_s), the short-circuit characteristic must be obtained. The alternator to be tested should be run at or near rated frequency (though a considerable departure is often permissible) with reduced excitation. The armature leads should be short-circuited. The excitation is then increased from zero until from 1.5 to 2.0 times normal current flows through the armature. Readings are taken of armature amperes versus field amperes. Examination of Fig. 36 shows that the alternator is operating at very low pf. Under these conditions the armature mmf strongly reacts against the field mmf, and the magnetic saturation of the machine will be low. This introduces a source of error, but it also means that the armature current will be directly proportional to the excitation. Hence the short-circuit characteristic is a straight line except at high currents. Except at low speeds for which X_s approaches R_a, the short-circuit current is nearly independent of speed and frequency.

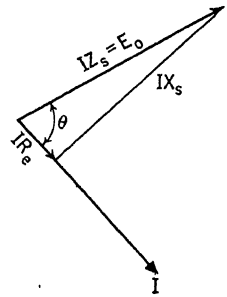


FIG. 36. Vector diagram of a short-circuited alternator by the emf or synchronous impedance analysis.

A second test run is necessary (in the determination of the synchronous impedance) in order to obtain the open-circuit characteristic. This is the same as the saturation curve for d-c dynamos. The alternator is operated at normal frequency and its external voltage read at no load with field excitation varied. This gives the open-circuit characteristic shown as *b* in Fig. 37. It is possible to calculate this curve from design data without testing the finished machine.

42. Calculation of Synchronous Reactance. Reference to Fig. 36 shows that the entire voltage generated by the short-circuited armature is used up as IZ_s drop. If the machine were run open-circuited at the same speed and excitation, the voltage generated would be a value on the no-load characteristic curve. Or, if no saturation occurred in the magnetic circuit, this voltage would be represented by a point on the air-gap line. This is obtained by drawing a straight line through the origin and tangent to the lower part of the open-circuit saturation curve.

Dividing the open-circuit voltage by the corresponding short-circuit current gives the synchronous impedance at that point. From Fig. 37:

$$Z_s = \frac{FE_0}{FI} \approx X_s \qquad [22]$$

Since X_s is approximately equal to Z_s, the value obtained above is frequently used as the synchronous reactance. The error is noticeable

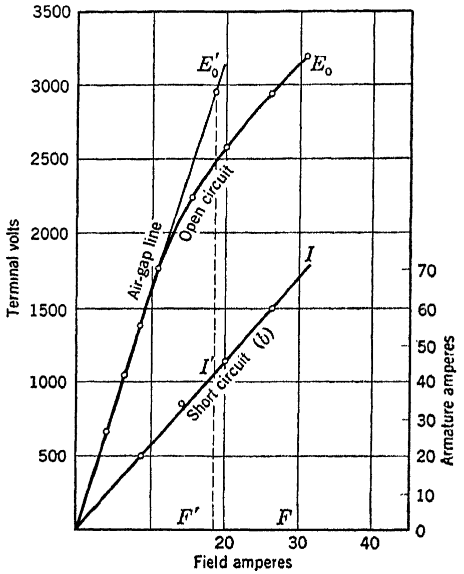


FIG. 37. Alternator test curves. 164 kv-a, 2200 volts, 3600 rpm, 60 cycles, three phase.

only in small alternators with appreciable resistance in the stator winding. More exactly

$$X_s = \sqrt{Z_s^2 - R_c^2} \qquad [23]$$

A source of confusion arises from the fact that a different value of X_s is obtained at different points on the curves. This depends upon the degree of saturation and raises the question as to what (constant) value of X_s should be used in the calculations. One method for considering saturation effects will be shown later.

The term *unsaturated synchronous reactance* is used in the technical literature. This is calculated as follows:

Note E'_0 and I' on Fig. 37.

$$Z_s \text{ (unsaturated)} \approx X_s \text{ (unsaturated)} = \frac{F'E'_0}{F'I'} \qquad [24]$$

This makes use of the air-gap line instead of the saturation curve. When it is so used, later corrections are usually applied to the calculations, based on the degree of saturation present for the conditions investigated.

43. Example of Calculation by the Synchronous-impedance Method. A three-phase alternator with the open-circuit and short-circuit characteristics, shown in Fig. 37, has the following rating:

164 kv-a	Y-connected armature
3600 rpm	2200 volts
2 poles	60 cycles

The full-load current per terminal:

$$I = \frac{164{,}000}{\sqrt{3} \times 2200} \quad \text{or} \quad 43.0 \text{ amperes}$$

Rated volts to neutral:

$$V_n = \frac{2200}{\sqrt{3}} \quad \text{or} \quad 1270 \text{ volts}$$

The maximum voltage on the open-circuit curve is 3200. At this voltage the field current is 31 amperes, and this same excitation produces a short-circuit current of 70 amperes. These values will be used to calculate X_s.

$$\frac{3200}{\sqrt{3}} = 1846 \text{ volts per phase}$$

$$Z_s = \frac{1846}{70} \quad \text{or} \quad 26.37 \text{ ohms}$$

The effective resistance per phase is 0.60 ohm at 75 C. This will be neglected in determining X_s.

Hence:

$$X_s \approx 26.37 \text{ ohms}$$

To calculate the regulation at unity pf, lay off as reference vector the phase value of V_t (equaling V_n). Solve for E_0:

$$E_0 = V_t + IR_e + jIX_s \quad \text{(See Fig. 38.)}$$

$$= 1270 + 43 \times 0.60 + j43 \times 26.37$$

$$E_0 = \sqrt{1295.8^2 + 1133^2} \quad \text{or} \quad 1721 \text{ volts}$$

$$\text{Regulation} = \frac{E_0 - V_t}{V_t} 100$$

$$= \frac{1721 - 1270}{1270} 100$$

$$= 35.5 \text{ per cent at unity pf}$$

The regulation at 0.80 pf, lagging, is calculated below. Refer to Fig. 38 for the vector diagram. The IR_e and IX_s drops are rotated with the current. The no-load voltage then becomes

$$E_0 = V_t + I(R_e + jX_s)(\cos\theta - j\sin\theta)$$

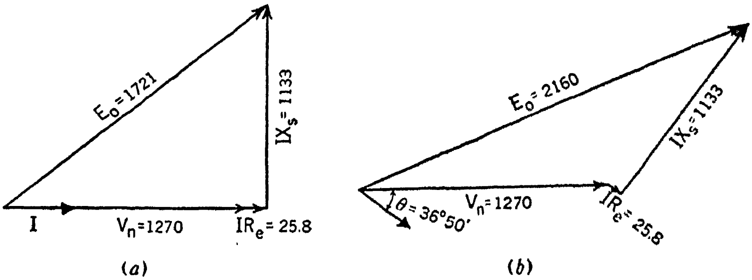


FIG. 38. (a) Synchronous impedance diagram for a unity pf load; (b) 80 per cent pf, lagging.

The operator $\cos\theta - j\sin\theta$ rotates the current vector clockwise through θ degrees. Of course the voltage-drop triangle rotates similarly.[1] Then

$$E_0 = 1270 + 43(0.60 + j26.37)(0.8 - j0.6)$$

$$= 1270 + 699 + j892.5$$

$$E_0 = 2160 \text{ volts}$$

$$\text{Regulation} = \frac{2160 - 1270}{1270} 100$$

$$= 70.1 \text{ per cent at } 0.80 \text{ pf, lagging}$$

44. Errors in the Synchronous-impedance Method. Of the various methods for calculating the regulation, this one yields results most likely to be inexact. Regulation so obtained is higher than actual tests show, and hence this is called the *pessimistic method*. The results are, however, more likely to be "on the safe side." However, this method is theoretically accurate for round-rotor or non-salient-pole machines with distributed field windings when saturation is not considered.

It assumes that the synchronous impedance or reactance is a constant. Actually if other values of excitation had been used on Fig. 37 rather than OF, the synchronous impedance would have been different. From these curves, X_s is obviously not a constant; and one of the large sources of error lies in the degree of saturation. At low saturation of the magnetic circuit, the effect of armature reaction is greater than at increased

[1] Complex quantities can be avoided in the solution of these problems by the use of accurate diagrams from which the values can be scaled.

saturation. It is impracticable to obtain a measure of synchronous impedance by this method with normal saturation as the armature-sustained, short-circuit current would then be several times normal (roughly, two to four times rated).

On alternators of both the salient and non-salient types a change in pf changes the demagnetizing effect of the armature reaction. As pointed out previously, armature reaction opposes or assists the field mmf, depending upon whether the pf is lagging or leading. At or near unity pf, armature reaction has little effect on the field strength. Examination of the diagram of Fig. 36 shows that the pf of the alternator during short circuit is much lower than normal, and hence the effect of armature reaction is larger than at loads of, say, 80 per cent lagging pf.

Because of the influence of pf and saturation upon synchronous reactance and armature reaction, one approach to the method of duly considering these effects is to divide the current into *active* and *reactive* components. These components are referred not to the terminal voltage but to the internally generated value. The two effects of armature reaction in demagnetizing and cross-magnetizing are then considered separately with the appropriate current components. Thus synchronous reactance will be divided into *direct* (X_d) and *quadrature* (X_q) synchronous reactances for consideration in a later chapter. The method just described makes use of what amounts to the direct component.

45. Old American Institute of Electrical Engineers Method. As explained above, the calculation of synchronous impedance by the short-circuit characteristic is open to error due to low saturation and other factors. The error caused by low saturation is avoided by a method formerly recommended by the American Institute of Electrical Engineers before its recent adoption of the American Standards method. Two test runs are necessary: the open-circuit saturation curve and the full-load saturation curve at zero pf. At zero pf load, the Potier vector diagram takes on the form shown in Fig. 39, except that the distance *ab* is added to IX_l to replace the effect of the armature reaction mmf, M_a. Compare this with Fig. 34. In this diagram:

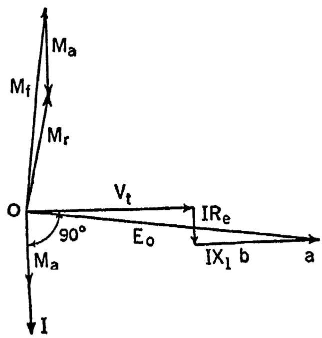


FIG. 39. Vector diagram of an alternator at zero pf, lagging.

M_f = the mmf of the field in ampere turns per pole
M_a = the mmf of armature reaction in ampere turns per pole

M_r = the resultant mmf in ampere turns per pole after the armature-reaction effect is considered

V_t = the terminal voltage in volts per phase

E_0 = the voltage generated by the useful field flux

I = the rated load current per terminal in amperes

X_l = leakage reactance of armature winding in ohms per phase

$IX_s = IX_l + ab$ (where X_s is the synchronous reactance)

Consider the voltage vectors on Fig. 39. At zero pf, the difference between the terminal voltage V_t and the internal generated voltage E_0

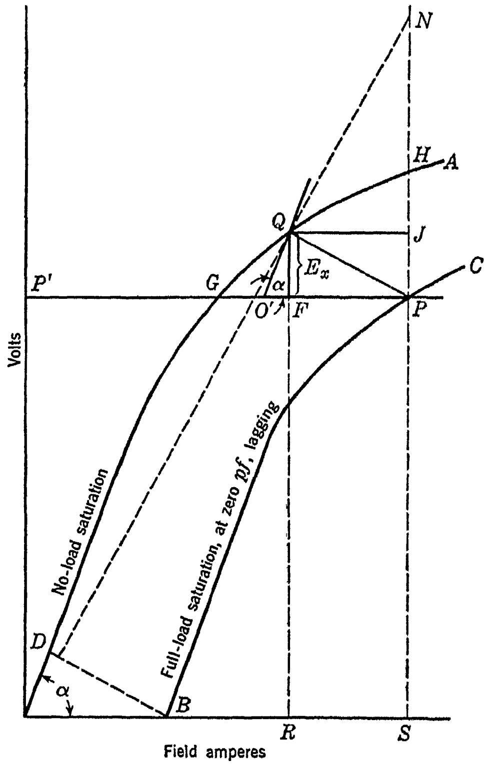


FIG. 40. Curves for determining Potier triangle and Potier reactance. For lagging pf, OB represents the field current necessary to overcome the demagnetizing effect of armature reaction. HJ represents the reduction in emf due to armature mmf. Field-pole flux leakage under load tends to lower slightly the no-load saturation curve.

represents the IX_s drop per phase. Refer now to Fig. 40, noting that SP is the terminal voltage per phase at zero pf and rated amperes. This same excitation, by this reasoning, would produce an open-circuit

voltage of SH, corresponding to E_0 on the vector diagram. Hence, if these curves are plotted in phase values,

$$\frac{PH}{I} = X_s$$

This is the saturated synchronous reactance by the old A.I.E.E. method.

46. Potier Triangle; Potier Reactance. Although the method just described has made use of the no-load saturation and the zero pf (full-load current) curve for determining synchronous reactance, more detailed use can be made of these curves as will be shown below.

Consider that the initial field excitation is OS, corresponding to M_f on Fig. 39. The demagnetizing effect of armature reaction in ampere turns (or amperes, if armature-reaction ampere turns are divided by effective turns) is RS or FP. Since it is an mmf it must be subtracted horizontally as shown. The net excitation is then OR, corresponding to M_r on Fig. 39.

This excitation would produce a voltage RQ on the no-load saturation curve, but directly subtracting from this is the effect of the leakage-reactance drop. On Fig. 40 this drop is shown as E_x or QF. It results in a terminal voltage of RF or OP', equal to the point on the no-load saturation curve from which the analysis started.

Because each of these effects has been considered in its proper relation, i.e., as mmf and emf drops, respectively, and the process results in a final voltage equal to that assumed in going through the reactions, the conditions must be as described.

Note the drop E_x. This is called the Potier reactance drop when obtained from this construction. It is not quite equal to the leakage-reactance drop because of the change in field leakage with load. It will be considered as approximately equal to IX_l, but is more nearly so in non-salient-pole machines; otherwise $E_x > IX_l$.

From the above explanations it can be seen that the internal voltage RQ must correspond to the value of Ob (line not drawn) on Fig. 39.

To obtain the approximate leakage reactance:

$$QF = E_x$$

$$\frac{E_x}{I} \approx X_l \qquad [25]$$

Construction Method. Up to this point we have been considering the triangle QFP which is really a part of the triangle $QO'P$, known as the Potier triangle. Given the no-load saturation and the lagging zero pf

curves, it can be constructed in the following manner. The curves can be drawn in terminal or phase values and plotted against either field amperes or field ampere turns per pole, so long as consistency is observed in the calculations.

Identify the points *BOD* and the angle α on the lower part of Fig. 40.

Draw the line $P'P$ through the curves, at a height above the origin corresponding to rated voltage.

Move the line *OB* up to $O'P$. Draw the line $O'Q$ through O' parallel to the bottom end of the no-load saturation curve. This locates the point *Q*.

Draw in the lines *QF* and *QP*.

47. Summary. Re-examining Fig. 39, it will be seen that the difference in internal and terminal voltages at zero pf, interpreted from the open-circuit and zero pf curves of Fig. 40, result in a means of calculating X_s.

Considering the mmf vectors of Fig. 39 and using the mmf and emf effects in their correct relations on the construction of Fig. 40 result in an expression for leakage-reactance drop (E_x) and an approximate measure of the armature-reaction mmf, *FP*. This latter value may not be accurate on salient-pole machines because of the non-sinusoidal nature of armature reaction. Furthermore, for such machines in which the field-pole leakage influences the Potier reactance, the entire triangle and the values obtained therefrom may be subjected to fairly large errors.

48. Load-characteristic Curves. The zero pf saturation curve under consideration is only one special case of load-characteristic curves. Others represent such conditions as zero pf at other than rated load or saturation curves at pf's other than zero.

In the experimental determination of the zero pf curve it is not usually necessary to obtain a pf of less than, say, 20 per cent. At such a load the demagnetizing effect is 98 per cent of that obtained for zero pf, or a difference of 2 per cent. At 30 per cent pf, the difference is 4.6 per cent.

Load characteristics at various pf's are shown in Fig. 41. Note that they all pass through the same lower point, corresponding to zero terminal volts at full-load current. This is obtained only by short-circuiting the armature terminals, and hence no external load exists; the pf is fixed by internal relationships.

If the no-load saturation and the zero pf curves are given, to obtain a load characteristic at any other pf, the following method is suggested:

(*a*) Calculate the armature-resistance drop per phase, which may or may not be used, depending upon its magnitude with reference to the synchronous-reactance drop *PA* (Fig. 41).

(*b*) Refer to Fig. 42*a*. Using *O* as the reference point, draw the IR_e drop, making the desired pf angle with the horizontal.

(*c*) Draw the synchronous reactance drop $IX_s = AP$, thereby locating the point *A*.

(*d*) Draw *AS* from *A* to the base line; *AS* is read from Fig. 41. This fixes the value of terminal voltage (*SO*) which would result for the given pf.

Then *SP′* in Fig. 41 equals *SO* in Fig. 42*a* and would locate the point *P′* on, say, the 0.8 pf curve of Fig. 41. Or, *SP″* (Fig. 41) is equal to *SO* in Fig. 42*b* and would locate the point *P″* on the unity pf curve.

For any other excitation, IX_s would correspond to such a value as *LB*, and the E_0 to use on Fig. 42 would be *LR*.

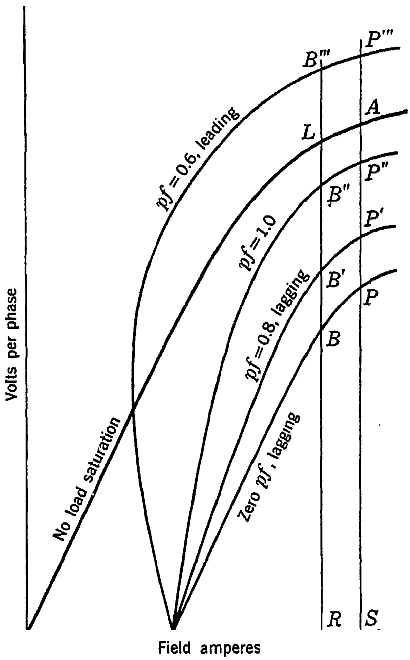


FIG. 41. Load-characteristic curves.

By this process, which simply consists in calculating regulation diagrams in reverse, the several load-characteristic curves are obtained.

If the no-load saturation and zero pf curves are given, to obtain a zero pf curve at any other current, the following procedure is suggested.

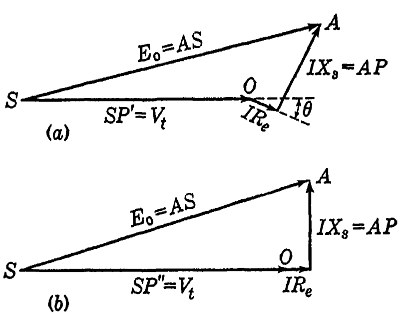


FIG. 42. Construction diagrams for determining points on load-characteristic curves.

The altitude of the Potier triangle and the base *FP* (Fig. 40) are both proportional to the current. Hence for half current, neglecting saturation effects on the two components, the Potier triangle reduces to half size.

If a triangle of a size proportional to the change in current is moved along the no-load saturation curve, so that the point *Q* remains on this curve and the base remains horizontal, the new point *P* will trace the zero pf curve for the required load current.

49. The Magnetomotive-force Method. In determining the regulation by the synchronous-impedance method, the demagnetizing effect of armature reaction and the consequent lowering of voltage were taken into account as a voltage drop. This voltage, plus the drops caused by leakage reactance and the effective resistance of the armature, was subtracted vectorially from the generated emf to give the terminal voltage. In the mmf method for determining regulation, the demagnetizing effect of armature reaction is subtracted from the field mmf as such, and in addition the effect of leakage reactance is converted roughly into an equivalent demagnetizing force which would produce the same effect of lowering the terminal voltage. Results obtained by these calculations are uniformly high, i.e., they usually show better regulation than actual tests, and as a result this is called the *optimistic method*.

50. Calculation Procedure for the Magnetomotive-force Method. To obtain the regulation by this method, the test curves shown in Fig. 37 are required. The vector diagram is shown in Fig. 43.

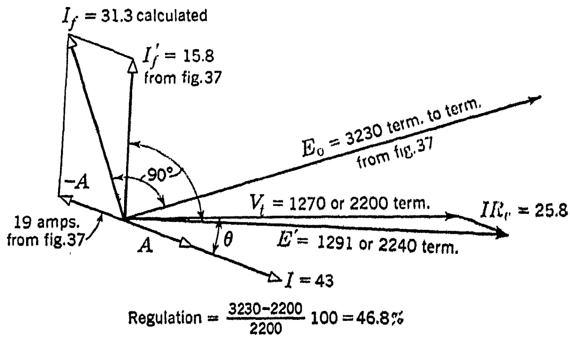


FIG. 43. Vector diagram of an alternator by the mmf method.

Procedure. Terminal voltage per phase (V_t) is used as the reference vector. Lay off the armature IR_e drop in phase with the current, at the pf angle for which the regulation is desired. Determine the field current I'_f required to produce the voltage E', using the open-circuit curve.

It is assumed that on short circuit all the excitation is opposed by the mmf of armature reaction and armature reactance. Hence A represents the mmf (or field current) required to produce rated current on short circuit.

The excitation I_f required to produce a terminal voltage V_t is then the vector sum of I'_f and $(-A)$. With no load on the alternator, the excitation I_f would produce a voltage E_0 as read from the open-circuit curve. The regulation is then calculated as shown with Fig. 43.

51. American Standards Association Method. The theory developed up to this point will now be used to determine the regulation of a synchronous generator by a method embodying some elements of the Potier triangle values and the mmf analysis. These details follow the method

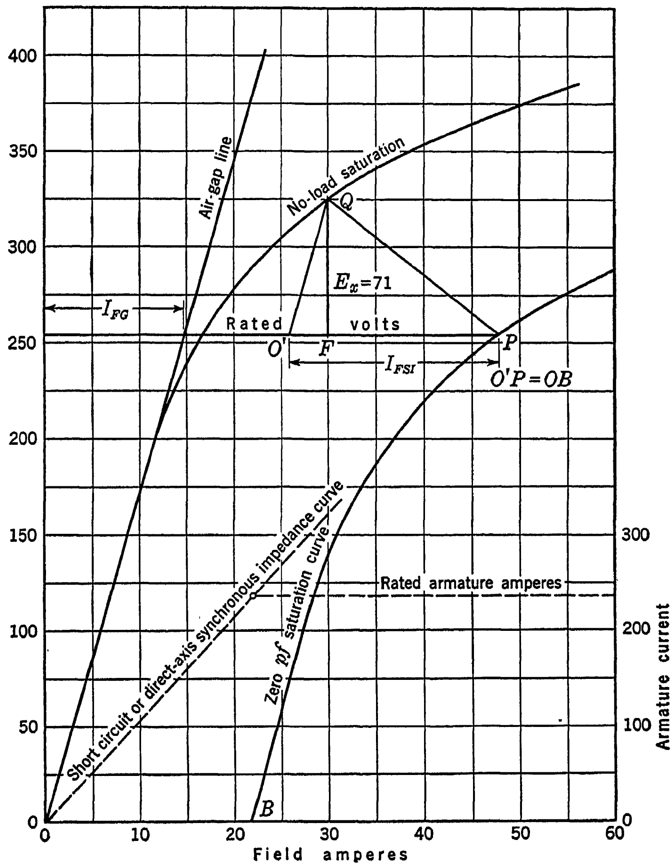


FIG. 44.

to be found in the American Standards for Rotating Electrical Machinery (Jan. 6, 1936) and in the Proposed Test Code for Synchronous Machines (A.I.E.E. Standards, January, 1937).

The test curves required are shown in Fig. 40, or more accurately in Fig. 44. Chiefly, these consist of the open-circuit or no-load saturation curve, the zero pf curve and the short-circuit current line, designated in the Standards as the *direct-axis, synchronous-impedance* curve. To these are added (1) the air-gap line, by drawing a straight line through the

origin, tangent to the lower part of the no-load saturation curve; (2) the Potier triangle, following the procedure previously described; and (3) several other identified excitation values which will be discussed.

These curves can be plotted in terminal volts or volts to neutral so long as a consistent treatment is followed, but we will assume that voltage to neutral will be used.

To obtain the regulation or the field current for the alternator at any pf, the construction of Fig. 45 will be followed.

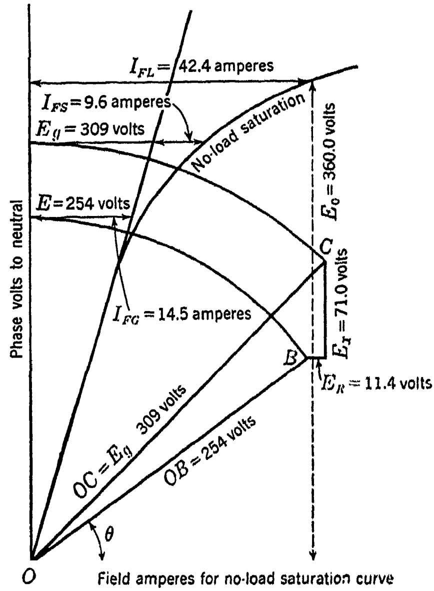


FIG. 45.

(*a*) Lay off rated volts to neutral *OB* at an assumed pf angle θ, with the horizontal base line.

(*b*) Calculate the armature-resistance drop per phase (E_R) and lay it off as shown, parallel with the base.

(*c*) To this add the Potier reactance drop E_x as read from Fig. 44. Then draw *OC*. This is the internally generated voltage E_g in Fig. 34, assuming that E_x equals IX_l.

(*d*) Project *OB* and *OC* by arcs to the vertical line, equaling the voltages E and E_g, respectively. Then project these intercepts horizontally to the no-load saturation curve which has been drawn as shown.

(*e*) We are now ready for the final process. Refer to Fig. 46. Draw I_{FG}, corresponding to the field current required to obtain no-load rated volts per phase if no saturation has taken place. (See Fig. 44.)

(*f*) On short circuit the field amperes necessary to produce rated armature current is shown as I_{FSI}, which agrees with the abscissa of the zero pf curve at the base. This field current, I_{FSI}, is added vectorially to the value I_{FG} at the pf angle θ with the vertical, as shown in Fig. 46. The use of this value and its position will be made clear by the comparison with the mmf analysis, shown in Fig. 43.

(*g*) If no saturation occurred, the required field current to produce a voltage which would yield rated full-load terminal volts would be OS on Fig. 46. Actually, saturation requires that a mmf corresponding to I_{FS}

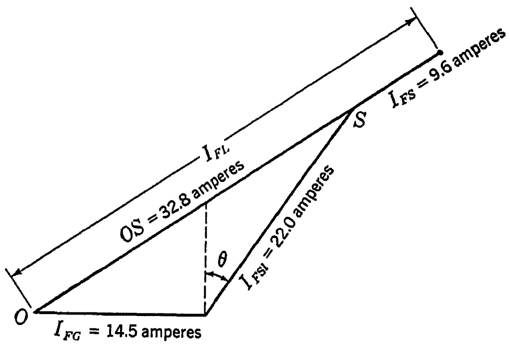


FIG. 46.

(Fig. 45) is needed to force the flux through the iron of the circuit, at the degree of saturation at which the generator is operating. Hence I_{FS} is added to OS, yielding a total designated I_{FL}.

(*h*) The transfer of I_{FL} to Fig. 45 or 44 enables the voltage to be read, corresponding to this excitation. Hence the no-load voltage (E_0) is that corresponding to an excitation I_{FL} and the regulation becomes

$$\frac{E_0 - V}{E_0} 100$$

52. Calculation of Regulation; American Standards Method. The curves of Fig. 44 refer to an alternator rated at 180 kv-a, 440 volts, 300 rpm, 60 cycles, three phases, Y connected. R_e per phase is 0.048 ohm at 75 C.

The voltage to neutral is 254 and the rated line current is 236 amperes per terminal.

Following the construction method of Article 46, the Potier triangle was drawn as shown. Note that the curves were drawn using volts to neutral.

$$E_x \text{ scaled } = 71 \text{ volts}$$

$$\text{Potier reactance} \approx X_l = \tfrac{71}{236} \quad \text{or} \quad 0.301 \text{ ohm}$$

To calculate the regulation at 0.80 pf lagging, refer to Fig. 45. At the pf angle, having a cosine of 0.80, $OB = 254$ volts.

$$E_R = 0.048 \times 236 \quad \text{or} \quad 11.4 \text{ volts}$$

$$E_x = 71 \text{ volts}$$

$$OC = \text{internal volts, scaled as } 309$$

Project points B and C to the voltage axis and read

$$I_{FG} = 14.5 \text{ amperes}$$

$$I_{FS} = 9.6 \text{ amperes}$$

$$I_{FSI} = 22.0 \text{ amperes}$$

Following the construction shown in Fig. 46, lay off I_{FG} and I_{FSI}, and scale $OS = 32.8$ amperes.

$$I_{FL} = 32.8 + 9.6 \quad \text{or} \quad 42.4 \text{ amperes}$$

An excitation of 42.4 amperes produces a voltage of 360 per phase on the no-load saturation curve, with unchanged speed and excitation. This corresponds to 622 volts, terminal to terminal.

$$\text{Percentage regulation} = \frac{622 - 440}{440} 100 \quad \text{or} \quad 41.7\%$$

The full-load field current of 42.4 amperes, for an 0.80 pf load is an indication of the required exciter capacity for this synchronous generator.

Calculations for other pf's would follow the same procedure shown here. The term *regulation* implies full-load current, but, if voltage change or excitation for other loads are required, the procedure need not be modified beyond an appropriate change in E_R and E_x. As mentioned previously, both base and altitude of the Potier triangle are proportional to current, and hence for smaller currents the triangles are similar.[2]

[2] Sterling Beckwith, "Approximating Potier Reactance," *Elec. Eng.*, July, 1937.

CHAPTER VII

CALCULATION OF ARMATURE REACTION AND REACTANCE

53. Chapter Outline.

Calculation of Armature Resistance.
Calculation of Armature Reaction.
  Usual Method.
  Step-curve Method.
  Fourier's Series.
  Examples.
Calculation of Leakage Reactance.
  Example.

54. Introduction. When the regulation has been determined from test data, its accuracy can frequently be checked through the determination of various machine constants. Such determinations are based on physical measurements on the machine and other items such as winding data on both armature and field. Methods and underlying theory are given in this chapter.

55. Calculation of Resistance. The resistance of the armature circuit per phase can be calculated from the conductor length, conductor area, number of conductors in series, and the number of parallel paths through the winding. The resistance at 75 C is the value specified by A.I.E.E. standards.

$$R = (1 + 0.004t°_c) \times 9.7\frac{l_c N_a}{a \times n} \qquad [28]$$

where $t°_c$ = the temperature in degrees Centigrade
l_c = mean length of one turn in feet
N_a = series turns per phase
a = area of one conductor in circular mils (1 sq in. = 1,273,000 cir mils)
n = number of parallel paths per phase

This value found for R will be increased by the factors making up effective resistance. In general, R could be multiplied by 1.6 to

give an effective value for 60-cycle machines, and 1.2 for 25-cycle machines.[1]

56. Armature Reaction. In dealing with armature-reaction effect by the use of synchronous reactance or in terms of field current having an equivalent demagnetizing effect, one should not lose sight of the fact that fundamentally it is a mmf, built up by the current in the armature-winding turns. Methods for obtaining a quantitative measure of this mmf will be discussed here.

The mmf of armature reaction rotates at synchronous speed inasmuch as it bears a fixed position with reference to the synchronously rotating field structure; a fixed position, i.e., except for shifts brought about by pf change from direct demagnetization at zero pf lagging, to cross-magnetization at unity pf and cumulative magnetization at zero pf lead.

In dealing with synchronous and induction motors, we will find that the connection of a polyphase supply to a properly distributed polyphase winding, placed about the stator periphery, results in a synchronously rotating mmf. This provides the motive power, in a sense, for such motors. The currents in the windings of these motors come from the supply source. The windings of a-c generators are essentially identical with those of synchronous and induction machines. The current flowing through the windings represents the load current supplied by the generator to its load. Yet these currents, regardless of their origin, flowing through the polyphase windings in their proper phase relationships, produce exactly the same rotating mmf's which would give motor action. In the motor it becomes the rotating field mmf; in the alternator we call it armature reaction.

The principle is emphasized at this point as it indicates the nature of our problem, to evaluate the resultant mmf of various phase windings distributed in space about the stator periphery and carrying currents differing in time phase.

The measure of armature reaction will be obtained in terms of crest ampere turns per pole; this will be designated A.

[1] For a discussion of transposed armature conductors and their effect see:

I. H. Summers, *Trans. A.I.E.E.*, Vol. 46, p. 101, May, 1927.
C. M. Laffoon and J. F. Calvert, *J.A.I.E.E.*, Vol. 46, p. 573, June, 1927.
S. L. Henderson, *Elec. J.*, Vol. 23, p. 348, July, 1926.

For a discussion of the *skin effect* in armature conductors, see:

A. B. Field, *Trans. A.I.E.E.*, Vol. 24, p. 761, 1905.
R. E. Gilman, *Trans. A.I.E.E.*, Vol. 39, p. 997, 1920.
W. V. Lyon, *Trans. A.I.E.E.*, Vol. 40, p. 1361, 1921.

$$A = \text{effective } NI \text{ per pole}$$

$$\text{Crest } NI \text{ per pole} = K\frac{Z}{2P}I \qquad [29]$$

I = effective current per armature conductor
Z = total armature conductors
P = the number of poles
K = a constant to be determined

Because of the combination of displaced mmf's of various phase windings, we can expect that K will not be unity, as that would indicate that

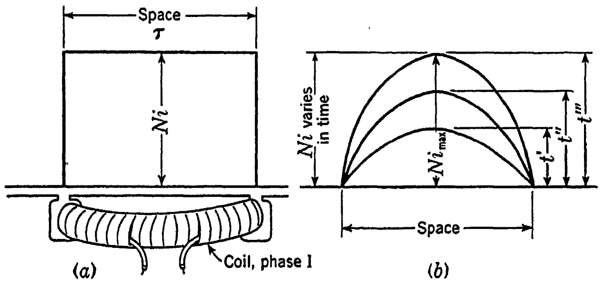


FIG. 47. Mmf of a single coil as a blocked wave in space and as a sine wave in space and time.

all ampere turns on the distributed polyphase winding were equally effective. Our problem is to determine K.

We will consider first the case of a two-phase alternator having concentrated windings of full pitch. The mmf of one coil is shown in Fig. 47*a*. The mmf is assumed to be the same at all points on the interior of the coil. Hence the *space* distribution of mmf is a blocked figure as shown. We wish to deal with this mmf as a space vector, and hence we will assume that the distribution of mmf over the area enclosed by the coil is a sine wave in space. Although not a true representation, this assumption is not usually so far in error as it would appear in dealing with this concentrated coil. The assumed space distribution is shown in Fig. 47*b*.

Because an alternating current flows through this coil, the magnitude of the current changes from instant to instant, and hence the instantaneous changes in mmf follow a sine wave in time. The mmf value at different instants is shown as t', t'', t''', etc.

Suppose we consider next the concentrated winding of the second phase on this alternator.

The coil under one pole will be assumed to yield (for convenience in

vector representation) a sine wave of mmf in *space* distribution, and, as it really does, a sine wave of mmf in *time*. But the winding is 90 electrical degrees removed in space, and the current is 90 electrical degrees in time, from the first coil considered. Hence from instant to instant we have an action as shown in Fig. 48. Coils I and II represent the concentrated windings under a pole, carrying two-phase currents. At the instant

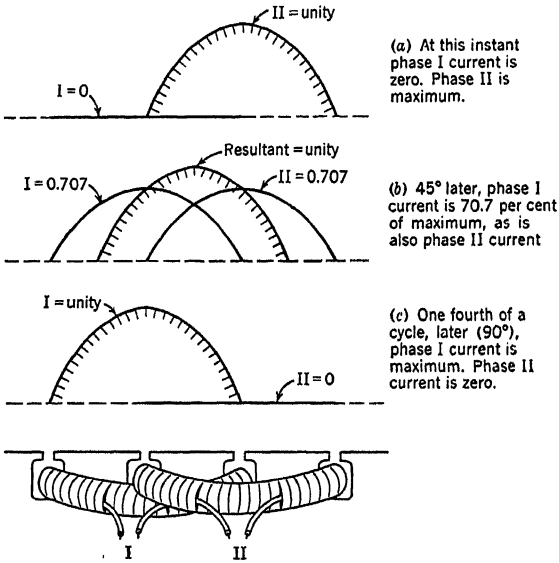


FIG. 48. Armature reaction in a two-phase alternator.

shown in *a*, the current of phase I is zero. Phase II current is a maximum, and the crest value of mmf, built up by this coil, is $I_m N$ ampere turns per pole, or $\sqrt{2}NI$, in terms of effective amperes and the number of turns N in one coil.

At an instant, 45 electrical degrees later, phase I current is 70.7 per cent of maximum (sin 45° = 0.707), as is phase II current. The resultant mmf of the two coils is the sum of the two waves, point by point. Its maximum is still the same as that shown in *a*, and hence the crest value is still $\sqrt{2}NI$. Note the condition shown in *c*. Again the wave has a crest value of $\sqrt{2}NI$. For convenience only 1 loop (1 pole) is shown. We have indicated a shift over one quarter of the cycle; during that shift, the magnitude of the mmf has not changed, but its position has shifted 90 electrical degrees around the air gap, exactly equal to the cyclic change in electrical degrees. This illustrates, and verifies, the statement that armature reaction is a synchronously revolving mmf

with a crest value of $\sqrt{2}NI$ in terms of the number of turns per coil. Since there are 2 coils building up mmf in this case, the crest value would be $0.707N'I$, if N' represented the turns per pole, both phases. Continuing a step further, with Z equal to *total* winding conductors, the turns per pole for both phases must be $Z/2P$ and hence the crest value of mmf will be

$$A = 0.707 \frac{Z}{2P} I$$

I is the effective current per conductor, or if Z represents the total *series* conductors of both phases, I would be the effective current per terminal.

These conclusions are based on an investigation of several points in time. If other points were investigated or if a three-phase winding were used for this analysis, it would be found that the constant would be the same.

The single-phase case will be investigated later.

A full-pitch concentrated winding has been assumed in this analysis. Just as short pitch reduces the induced voltage, it reduces the mmf built up by a coil, as does also the distribution of the winding. Hence a distributed, short-pitch winding produces an mmf which is k_pk_d times the mmf produced in a concentrated full-pitch coil. Our final formula for effective mmf of armature reaction per pole is

$$A = 0.707 \frac{Z}{2P} \cdot Ik_pk_d \qquad [30]$$

Unfortunately this is not the complete picture of the phenomenon. It is the result of assuming that the more or less blocked figure of mmf in space can be replaced by a sinusoidal distribution in space. Other viewpoints are commonly accepted and put to use, and we will develop them. For want of a better title, the above analysis will be called the *usual method.*

57. The "Step-curve" Method. This method of evaluating armature reaction is more exact than the preceding analysis and is much used by designers. The results vary only slightly from the first method, except in alternators having only a few slots per pole. The method will be explained by application to a machine with the following constants:

Number of phases, 3.
Number of slots per pole, 6.
Number of conductors per slot, 2.
Double-layer, full-pitch winding.

Assume that the machine is loaded until the armature current is 100 amperes per terminal at unity pf. Refer to Fig. 49. At the instant shown, the voltage and current of phase II are at their maximum values. Phases I and III are at one-half of their maximum values. The maximum value of the current in phase II is 141 amperes, and since one turn is assumed per coil, the values of currents shown in a will also be the same as those for ampere turns of armature reaction. These ampere turn values are added algebraically and shown graphically. This step curve

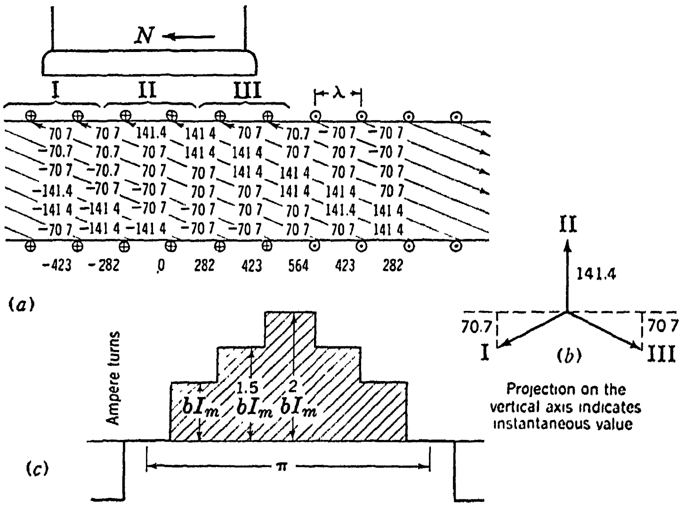


FIG. 49.

shows the distribution of armature reaction mmf around the air gap. The various ordinates represent ampere turns surrounding each tooth. The crest value of this wave is 564 ampere turns, or in more general terms,

$$2bI_m = 564$$

where b = the conductors per slot. In this case $b = 2$.

Next consider the instant at which the field has moved 90°. The instantaneous values of the currents in the armature windings are as shown in Fig. 50a. The ampere turns surrounding each tooth are again added algebraically and the step curve of Fig. 50c obtained as the distribution of armature reaction.

At this point one of several possibilities is to be selected as a means of evaluating these results.

(a) The average armature reaction ampere turns per pole can be taken as the average of the two crest values.

(*b*) The average armature reaction ampere turns per pole can be taken as the average of the two areas under the curves.

(*c*) The armature reaction ampere turns per pole can be calculated as the crest value of a sine wave with an area equal to the average of the above two areas.

Methods *a* and *c* are most commonly used, though *b* is good also.

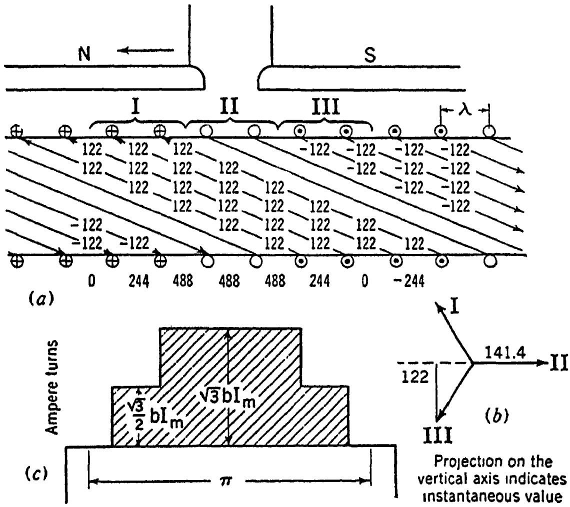


FIG. 50.

58. Formulas for Ampere Turns Given by Step Curves. To avoid the labor of plotting the step curves each time a result is wanted, formulas to fit each of the three viewpoints will next be given.

Method a. The crest value of the curve shown in Fig. 49*c* is $2bI_m$; that of the curve shown in Fig. 50*c* is $\sqrt{3}bI_m$. The average of these two is

$$\frac{2+\sqrt{3}}{2}bI_m = \frac{2+\sqrt{3}}{2}b\sqrt{2}I \qquad [31]$$

If the total conductors on the armature are Z, then Z/P are the conductors per pole. Since there are 6 slots per pole in this example,

$$\frac{Z}{6P} = b \text{ (or the conductors per slot)}$$

The above expression becomes

$$\frac{2+\sqrt{3}}{2}\cdot\frac{Z}{6P}\cdot\sqrt{2}I = 0.88\,\frac{Z}{2P}\,I \qquad [32]$$

Hence the step-curve method of determining the armature reaction ampere turns by average of the crests of the two waves gives a constant of 0.88 as compared with 0.707 for the usual method.

Method c. The areas of the two curves of Figs. 49c and 50c must be determined in order to use the third method of evaluating armature reaction. Let λ equal the slot pitch. Then the area under the curve of Fig. 49c is:

$$\begin{array}{r} 1.0bI_m\lambda \\ 1.5bI_m\lambda \\ 2.0bI_m\lambda \\ 1.5bI_m\lambda \\ 1.0bI_m\lambda \\ \hline 7.0bI_m\lambda \end{array}$$

In a similar manner, the area under the curve of Fig. 50c is found to be $6.93bI_m\lambda$. The average of these areas is equal to $6.965bI_m\lambda$. If NI_{aver} represents the average ampere turns under 1 pole, then the total area under such a curve (with 6 slots per pole) will be $NI_{\text{aver}} \times 6\lambda$. Then

$$NI_{\text{aver}} \times 6\lambda = 6.965bI_m\lambda$$

$$NI_{\text{aver}} = 1.16bI_m$$

$$= 0.547 \frac{Z}{2P} \cdot I$$

With the mmf of armature reaction represented as a sine wave having an area equal to the average area obtained above, the crest value of the wave would be

$$\frac{\pi}{2} NI_{\text{aver}} = 0.861 \frac{Z}{2P} I \qquad [33]$$

If the curves are plotted, the step-curve method will correct for both pitch and distribution factors, and no further correction need be made. The constants determined above, however, hold only for the given number of slots per pole, etc. Table V shows constants calculated for various combinations of slots and pitches using method *c*. The use of this table eliminates the labor of constructing the curve for every new condition.

Regardless of the actual distribution of the winding, it is satisfactory to assume concentrated full pitch. The constants in Table V can then be corrected by multiplying by the distribution and pitch factors. This gives good results.

TABLE V

CONSTANTS FOR DETERMINING ARMATURE REACTION AMPERE TURNS BY THE STEP-CURVE METHOD. CORRECTED FOR VARIOUS PITCHES AND SLOTS PER POLE

Crest NI per pole $= KbI$

b = conductors per slot

I = effective current per conductor

Three-phase

Slots per pole	Pitch	K	Slots per pole	Pitch	K
3	1	1.382	15	1	6.450
6	1	2.580		$\frac{14}{15}$	6.443
	$\frac{5}{6}$	2.582		$\frac{13}{15}$	6.370
	$\frac{4}{6}$	2.236		$\frac{12}{15}$	6.165
	$\frac{3}{6}$	1.890		$\frac{11}{15}$	5.920
9	1	3.870		$\frac{10}{15}$	5.620
	$\frac{8}{9}$	3.844		$\frac{9}{15}$	5.230
	$\frac{7}{9}$	3.680		$\frac{8}{15}$	4.895
	$\frac{6}{9}$	3.380			
	$\frac{5}{9}$	2.978			
12	1	5.285			
	$\frac{11}{12}$	5.170			
	$\frac{10}{12}$	5.000			
	$\frac{9}{12}$	4.825			
	$\frac{8}{12}$	4.478			
	$\frac{7}{12}$	4.140			
	$\frac{6}{12}$	3.660			

Two-phase—Full-pitch

Slots per Pole	K	Slots per Pole	K
2	0.949	8	3.160
4	1.579	10	3.220
6	2.437	12	4.740

59. Method Based on Fourier's Series. By the use of Fourier's theorem it is possible to express any single-valued periodic function by a series of sine waves. Consider the block of mmf in Fig. 51 (built up by

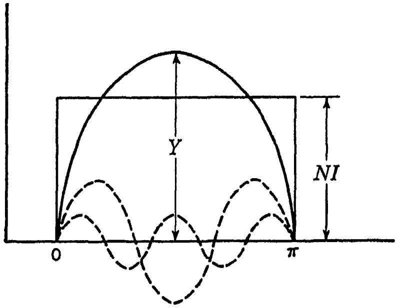


FIG. 51.

a single coil) having a value of y constant between $x = 0$ and $x = \pi$. Thus we can write

$$y = NI, \quad 0 < x < \pi$$

The coefficient of the mth harmonic in a Fourier's series =

$$A_m = \frac{2}{\pi}\int_0^{\pi} f(x) \sin mx dx$$

Here $f(x) = y = NI$; so

$$A_m = \frac{2}{\pi}\int_0^{\pi} NI \sin mx dx = \frac{2NI}{\pi}\left(-\frac{1}{m}\cos mx\right)_0^{\pi}$$

When m is odd,

$$A_m = \frac{2NI}{\pi m}(1 + 1) \quad \text{or} \quad \frac{4}{\pi m} NI$$

When m is even,

$$A_m = \frac{2NI}{\pi m}(-1 + 1) \quad \text{or} \quad 0$$

Then for the complete series

$$y = NI = \frac{4NI}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x \cdots\right) \qquad [34]$$

That is, by replacing a rectangular figure of width 0 to π by its Fourier's series, the fundamental will have a peak value of $4/\pi$ times the altitude of the rectangle.[2] Hence the equation for ampere turns of

[2] Byerly, "Fourier's Series and Spherical Harmonics."

armature reaction, neglecting all the harmonics and considering only the fundamentals, would be

$$A_{\text{crest}} = \sqrt{2}\,\frac{4}{\pi}\,NI \qquad [35]$$

for one coil in terms of turns N and effective amperes.

In dealing with a two-phase machine, the ampere turns at the crest would become

$$\sqrt{2}\,\frac{4}{\pi}\,NI = \sqrt{2}\,\frac{4}{\pi}\,\frac{\frac{1}{2}Z}{2P}\,I$$

$$= 0.9\,\frac{Z}{2P}\,I \qquad [36]$$

For a three-phase machine the result is the same.

These equations must be corrected for pitch and distribution by multiplying by $k_p k_d$. The general form of the results is, of course, similar to that derived by the usual method and the step-curve method. The difference is in the coefficients resulting from the different viewpoints.

60. Armature Reaction in a Single-phase Alternator. To develop the theory for armature reaction in a single-phase machine, a full-pitch

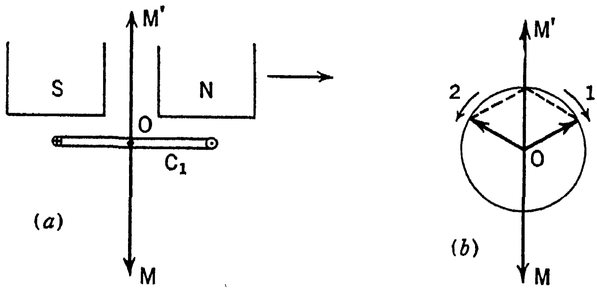


FIG. 52. The oscillating vector of armature reaction is separated in b into two oppositely rotating vectors.

winding of only 1 coil will be used. This is shown in Fig. 52. The armature coil is assumed to be connected to a unity pf load, and at the instant shown both armature voltage and current are at their maximum values. The mmf of the armature is shown as the vector M. As the field rotates this mmf will vary from M through O to M'. That is, armature reaction will oscillate along the MM' axis and will be fixed in space with respect to the armature. Such an alternating vector, which revolves as it alternates sinusoidally at the same frequency, can be replaced by two vectors of one-half its magnitude and rotating in opposite directions at the same frequency as the original. Oscillating vector M is the equivalent of the

two rotating vectors shown in Fig. 52b. The resultant of 1 and 2 will lie along MM' at all times. With respect to the stationary armature, both of these vectors are rotating oppositely at synchronous speed. With respect to the moving field, one vector is stationary and the other rotates at twice synchronous speed.

If Z is the total number of armature conductors, that part of armature reaction which is fixed with respect to the field is:

$$\text{Armature reaction per pole} = \frac{1}{2} \cdot \frac{\frac{1}{2}Z}{P} \cdot \sqrt{2}I$$

$$= 0.707 \frac{Z}{2P} I \qquad [37]$$

Or, by the Fourier's series viewpoint this would be

$$A = 0.9 \frac{Z}{2P} I \qquad [38]$$

In either case the factors $k_p k_d$ would be used as required.

The component which rotates in the opposite direction from the field, or at twice synchronous speed with respect to it, has the same value and

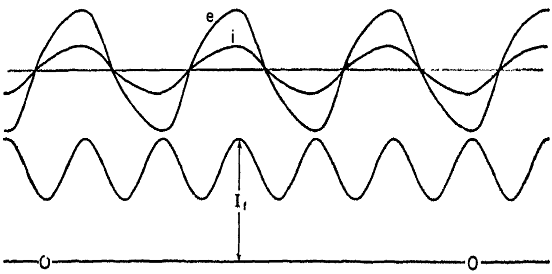


FIG. 53. Curves drawn from an oscillogram representing unity pf load on a single-phase alternator. The backward-sweeping flux of armature reaction generates the second harmonic in the d-c field. (Field current I_f was supplied by a storage battery.)

causes a pulsating flux across the faces of the field poles and a pulsation of the main pole flux. This component of double frequency (with respect to the field) results in a third harmonic voltage being generated in the armature winding, and causes a second harmonic of current in the d-c field winding which is superimposed on the direct current used for excitation. (See Fig. 53.) It can be eliminated very largely by the use of a squirrel cage or other damper winding placed in the pole shoes. It is much less pronounced in machines with solid poles than in machines with laminated poles.

61. The Single-phase Method Extended to Polyphase. The method just shown for single-phase machines will be extended to the three-phase alternator. Consider first the forward rotating components of the armature reaction mmf's. The components will add, with due regard to their space and time relationships. The backward components are of double frequency and when referred to a point on the armature, the three components are 240 degrees and 480 degrees, respectively, behind the first component. Their vector sum is zero, and hence the double-frequency components of armature reaction balance out, so long as the load current is balanced. Unbalancing results in incomplete neutralization, and the appearance of a second harmonic in the field current.

Mathematical expressions for the components will be derived below.

Let N' be the number of armature turns per phase
i = instantaneous value of phase current
θ' = the angle of lag of the currents behind the excitation voltage. Note that this is not the angle between the current and the terminal voltage, but the voltage which would be produced by the field excitation. Sinusoidal variation of flux, in space and time, will be assumed.

The currents in the three phases will be, respectively,

$$i_1 = I_m \sin (wt - \theta') \tag{39}$$

$$i_2 = I_m \sin (wt - 120^\circ - \theta') \tag{40}$$

$$i_3 = I_m \sin (wt - 240^\circ - \theta') \tag{41}$$

Next consider a point a in the air gap, removed from the center of the poles (zero flux) by an electrical angle α. The mmf at that point, at an instant t, due to all three phases, will be

$$N'i_1 \sin (\alpha + wt) + Ni_2 \sin (\alpha + wt - 120^\circ) + Ni_3 \sin (\alpha + wt - 240^\circ) \tag{42}$$

But since the currents i_1, i_2, and i_3 vary in time, their actual values must be used in order to generalize, and hence the mmf at point a will be

$$N'I_m \sin (wt - \theta') \sin (\alpha + wt) + N'I_m \sin (wt - 120^\circ - \theta') \sin (\alpha + wt - 120^\circ) + N'I_m \sin (wt - 240^\circ - \theta') \sin (\alpha + wt - 240^\circ) \tag{43}$$

Using the trigonometric expansion for the product of two sines, this reduces to

$$\begin{aligned}0.5N'I_m[&\cos(\alpha + \theta') - \cos(2wt + \alpha - \theta') \\ &+ \cos(\alpha + \theta') - \cos(2wt + \alpha - \theta' - 240°) \\ &+ \cos(\alpha + \theta') - \cos(2wt + \alpha - \theta' - 480°)]\end{aligned} \quad [44]$$

Note the terms involving $2wt$. They represent the backward rotating components. Their vector sum is zero, and our equation can hence be reduced to

$$1.5N'I_m \cos(\alpha + \theta') \quad [45]$$

Let us convert these ampere turns into effective amperes and conductors on all three phases,

$$1.5N'I_m = 1.5\frac{N}{3}\sqrt{2}I \quad \text{or} \quad 0.707NI$$

$$= 0.707\frac{Z}{2P}I \quad [46]$$

Hence, the mmf of armature reaction at a point a is

$$0.707\frac{Z}{2P}I\cos(\alpha + \theta') \text{ ampere turns per pole} \quad [47]$$

This is independent of time and varies only according to the point selected. The maximum value is removed from a point midway between the poles, equal to the angle between the current and the internal voltage.

The actual magnitude should be corrected by pitch and distribution factors, and, if the Fourier method is used, the coefficient will change from 0.707 to 0.9.

62. Examples of Armature Reaction Calculation. The machine in which the effect of armature reaction will be calculated is rated as follows:

180 kv-a	24 poles	300 rpm
three-phase	60-cycle	440 volts

Test curves for this machine are shown in Fig. 44.

Design Data:

Armature: 144 slots, 2-layer winding, coil pitch $\frac{5}{6}$. Each coil has 5 turns of 4 straps, each 0.175 by 0.080 in. Armature is Y connected with 2 circuits. Mean length of turn, 34 in. Width of slot, 0.504 in. Depth of slot, 2.50 in. Net axial length of iron, 4.5 in.

Field: Each pole is wound with $128\frac{1}{2}$ turns. Pole and air-gap dimensions are shown in Fig. 54.

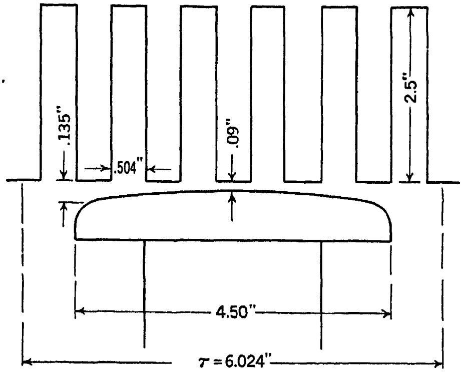


FIG. 54.

'olution:

$$\text{Rated } I \text{ per phase} = \frac{180{,}000}{\sqrt{3} \times 440}$$

$$= 236.0 \text{ amperes}$$

Number of coils = 144 (same as number of slots in a two-layer winding)

Total turns = 144×5 or 720

Total conductors = 1440

Total conductors in series = 720 (two-circuit winding)

The four conductors of each turn will be considered as laminations of one conductor.

Current per conductor = 118 amperes

(*a*) *Armature Reaction by the Usual Method.*

$$0.707 \frac{ZI}{2P} = 0.707 \frac{1440 \times 118}{2 \times 24}$$

$$= 2500 \text{ ampere-turns per pole}$$

This must be corrected for pitch and distribution.

Coil pitch $= \frac{5}{6}$

$k_p = \sin\left(\frac{5}{6} \times 90°\right)$

$= 0.966$ (or this can be obtained from Table I)

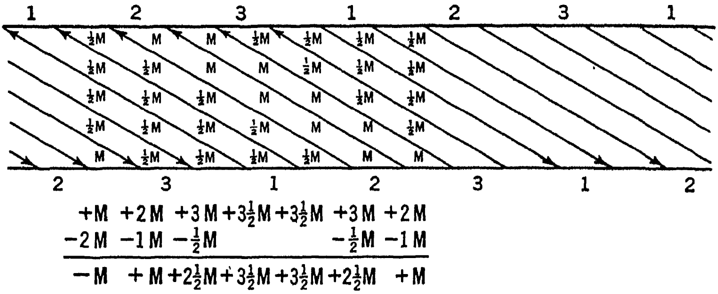
$k_d = 0.966$ (from Table II)

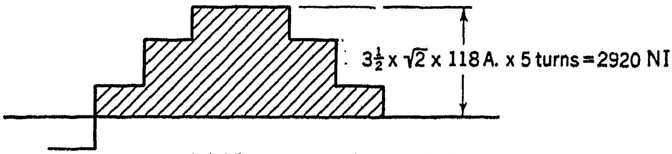
$2500 k_p k_d = 2336$ ampere turns per pole

(*b*) *The Step-curve Method.*

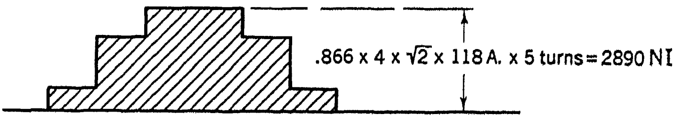
In Fig. 55 we have the data for step curves of armature reaction. There are 118 amperes per coil at full load (due to the two-circuit winding). Method *a* assumes that the current of the second phase is at its maximum. As this is a three-phase armature, I_1 and I_3 will then be one-half of their respective maxima. The winding is

shown with $\frac{5}{6}$ pitch, and the direction of the currents (shown as M and $\frac{1}{2}M$ fo simplicity) determines the magnitude of the stepped wave.





(*a*) The current of phase 2 is max.
1 and 3 at ½ max.



(*b*) At this instant the current of phase 3 is zero
phases 1 and 2 are 866 max.

FIG. 55.

The crest value of the first wave is 2920 ampere turns; that of the second is 2890. The average is 2905 ampere turns, as determined by method *a*, Article 58.

(*c*) *Fourier's Series.*

$$0.9\frac{ZI}{2P} = 0.9\frac{1440 \times 118}{2 \times 24}$$

$$= 3190 \text{ ampere turns per pole}$$

Corrected for pitch and distribution:

$$3190k_pk_d = 2975 \text{ ampere turns per pole}$$

The following table compares the values of armature reaction ampere turns per pole as determined by the various methods.

METHOD	NI PER POLE
Usual method	2336
Step-curve method	2905
Fourier's series	2975

63. Leakage Reactance. The components making up the leakage reactance have already been discussed qualitatively. The usual procedure is to calculate the reactance resulting from each component flux, and to obtain the sum of these individual reactances. That is, each is calculated as though no other flux component was present. Actually, the fluxes have no real existence as they show up as distortions or harmonics in the mutual or air-gap flux.

Extensive literature has appeared on this subject. However, some confusion of terms and components still exists. The subject cannot be discussed here in detail, but the fundamentals whereby leakage reactance can be calculated will be presented briefly. This is a subject which can appear very confusing to the novice. Enough material will be presented to clarify some of this mystery; then it must be remembered that all the advanced work built around this subject is merely modification, or further improvement, of these fundamentals.

Slot-leakage reactance and end-coil reactance are the only two components which will be calculated. In addition, tooth-tip reactance is sometimes included, but it will not be presented here.

64. Slot Leakage. When a conductor or series of conductors is embedded in a slot (Fig. 56), current flow builds up a magnetic field as

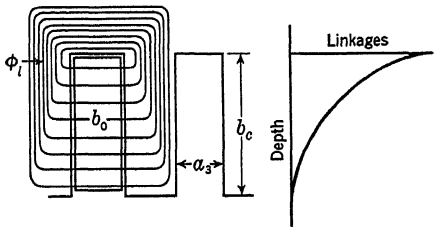


FIG. 56.

shown. Alternations of the current cause the building up and collapse of this flux about the conductors, thereby inducing in the winding a slot-leakage reactance voltage.

The reluctance of the flux path will be considered for the air or copper only, neglecting the iron as being of negligibly small reluctance.

Let ϕ_l = leakage flux lines per ampere per conductor per inch of slot length

P = the number of poles

b_0 = the conductors in series per slot

S_1 = the number of stator slots

L = the gross length of the core in inches

q_s = slots per pole containing phase conductors in series

The leakage flux per slot per ampere is then $\phi_l L b_0$. If each flux line linked every conductor in the slot, the linkages per slot per ampere would be $\phi_l L b_0^2$. Since each phase has q_s slots per pole containing phase conductors in series, the reactance of the winding from the slot leakage flux is then:

$$X_{\text{slot}} = 2\pi f \phi_l L b_0^2 P q_s \, 10^{-8} \text{ ohm} \tag{48}$$

Each flux line does *not* link each conductor (or element of one conductor) in the slot. This fact is considered by using a "slot constant" which expresses the ratio of effective linkages to actual linkages.

Slot Constants. Referring again to Fig. 56, it will be assumed that the slot is full of conductors (or contains one large conductor) that carry uniformly distributed current. The latter assumption is not quite correct in practice. The flux distribution from top to bottom of the slot is linear, but the interlinkages will be parabolic. Since the average height of a parabola is one-third of its maximum value, the same result is obtained as though the effective area of the flux path were

$$\tfrac{1}{3} \times b_c \times \text{(axial length of the slot)}$$

A more rigorous analysis can be made for the above, holding for sloping slot sides or any other shape than straight slots, by using elements of slot depth and the summation by calculus.

Permeance of air is proportional to area divided by length of path; and to use inch units, we then would have as permeance of the slot-leakage flux path

$$\lambda_s = 0.4\pi \times 2.54 \times \frac{\frac{1}{3}b_c}{a_3}$$

$$= 3.19 \frac{b_c}{3a_3} \text{ per inch of axial length} \tag{49}$$

The constant 0.4π, inherent with the magnetic circuit for the system of units used, will be absorbed with the permeance rather than with the ampere turns.

The slot constant K_s will then be defined as the value $b_c/3a_3$, and it holds only for this slot shape.

The previous expression for slot-leakage reactance (equation 48) involved flux lines. We now know that they are equal to $3.19K_s$ for each inch of axial slot length. But as used in engineering offices for many years, this theoretical value of 3.19 is increased to 4.2 simply because it has been shown to result consistently in greater accuracy. Furthermore, if S_1 equals the total stator slots and there are m phases, the slots per phase are S_1/m. Also $b_0 P q_s$ must represent the total series conductors

per phase, which will be represented by the symbol C. Then for a core length of L inches,

$$X_s = 2\pi f C^2 m \, 10^{-8} \left(\frac{4.2LK_s}{S_1}\right) \text{ ohm per phase} \qquad [50]$$

The above equation assumes that all conductors are carrying currents in time phase. On double-layer windings, the fact that one coil side is in the top and one in the bottom of the slots would modify the individual effects, but the average would be as shown. For fractional-pitch windings, if the pitch is such as to result in a phase displacement of θ degrees between sides 1 and 2 of a coil, then coil side 2 will be in a slot with another coil side whose conductors are carrying current θ degrees ahead of the current in 2.

The out-of-phase components of the voltages induced by the leakage fluxes will be canceled out, but the in-phase components will be reduced. This results in less reactance drop with apparently less reactance. Such effect is considered by a correction for the pitch, designated k_s. Values can be found in Alger's paper on this subject, as well as in the early work of Adams on induction-motor leakage reactance. In their simplest form, they are fundamentally the same. Values of k_s for pitches of 0.5 to 1.0 can be expressed as $(0.625p + 0.375)$, where p is the pitch as a decimal.

65. End-connection Leakage. The significance of the end-connection leakage has been discussed briefly in Article 30. The flux, built up around the coil ends, alternates in such a way as to give the ends an inductance and hence cause a reactance drop. Because this is largely an air path, the flux per ampere is low, but what helps to complicate the calculations is the fact that the adjacent conductors may belong to various phases, may be surrounded by conducting material used as braces for the end material, or may be formed to lie close to or far from parts of the magnetic circuit. More elaborate analyses have been made, but we will point out a simple empirical relationship shown by Behrend for induction-motor, coil-end leakage. The permeance of the air path by his work is shown to be $\frac{\tau}{2P}$. Then by similarity to the slot-leakage formula,

$$X_{\text{end}} = 2\pi f C_m^2 \, 10^{-8} \frac{\tau}{2P} K_e \qquad [51]$$

where τ = the pole pitch in inches

K_e = a construction constant, influenced by the placement of the coils and various other factors mentioned above. For salient-pole machines, an average value of 1.21 will be used.

The above equation must be corrected for pitch, and for pitches of 0.5 to 1.0, a correction factor $k_e = (1.25p - 0.25)$ should be used.

66. Example of Calculation; Armature Leakage Reactance. To illustrate the method, we will use data from the same machine for which armature reaction was just calculated.

$$
\begin{aligned}
\text{Series conductors per phase} &= 240 \\
\text{Slot width, } a_3 &= 0.504 \text{ in.} \\
\text{Slot depth, } b_c &= 2.5 \text{ in.} \\
\text{Pole pitch, } \tau &= 6.024 \text{ in.} \\
\text{Axial length of stacking, } L &= 4.5 \text{ in.} \\
\text{Number of stator slots, } S_1 &= 144 \\
\text{Phases, } m &= 3 \\
\text{Poles, } P &= 24 \\
\text{Pitch, } p &= \tfrac{5}{6} \text{ or } 0.833 \\
\text{Slot constant, } K_s &= b_c/3a_3 = 1.65
\end{aligned}
$$

The armature winding is made up of 10 conductors, each of 4 parallel straps per pole, connected for 2 parallel paths.

From equation 50 for full pitch,

$$X_{\text{slot}} = 2\pi f \times 240^2 \times 3 \times 10^{-8} \left(\frac{4.2 \times 4.5 \times 1.65}{144}\right) \quad \text{or} \quad 0.140$$

$$k_s = (0.625p + 0.375) \quad \text{or} \quad 0.895$$

$$X_{\text{slot}} = 0.140 \times 0.895 \quad \text{or} \quad 0.1255 \text{ ohm for } \tfrac{5}{6} \text{ pitch}$$

For the end-connection leakage, equation 51,

$$X_{\text{end}} = 2\pi f \times 240^2 \times 3 \times 10^{-8} \left(\frac{6.024 \times 1.21 \times 0.79}{2 \times 24}\right) \quad \text{or} \quad 0.079 \text{ ohm}$$

The value 0.79, used above, is the correction for pitch:

$$k_e = (1.25 \times \tfrac{5}{6} - 0.25) \quad \text{or} \quad 0.79$$

The total leakage reactance then is 0.2045 ohm. This is smaller than the Potier reactance calculated for this same machine as 0.318 ohm in Article 52. Potier reactance is usually larger than leakage reactance in salient-pole machines although the results are doubtless influenced by neglecting tooth-tip reactance in these calculations.

CHAPTER VIII

TWO-REACTION ANALYSES

67. Chapter Outline.

Blondel Two-reaction Analysis.
  Application and Example.
Direct- and Quadrature-axis Synchronous Reactances.
Vector Diagrams.
Determination of Values.

68. Blondel Two-reaction Method. Because Potier's general diagram inherently assumes that the field and armature fluxes are sinusoidally distributed in the air gap, there is a tendency for this method to give results that are somewhat higher than those of actual test. Sinusoidal flux distribution is nearly true for fields with distributed iron and copper, but is not true for salient-pole machines.

In order to remove the criticism of results for salient-pole machines, Professor André Blondel[1] published in 1904 the following method. It can be found in several different forms, differing mostly in details for finding the coefficients. The method is applicable to either design or experimental data for the determination of regulation.

In recent years this method has been greatly extended. An outline of the more modern approach of the two-reaction theory will be presented here, also.

69. Division of Armature Magnetomotive Force into Sine and Cosine Components. It has been explained that at zero pf, lagging, the armature-current distribution is such that the armature mmf directly opposes the mmf of the main field. At unity pf load the armature mmf crowds the flux forward in the direction of rotation. This causes the trailing pole tips to be strengthened and the leading tips to be weakened. Saturation in the pole tips and armature teeth acts to prevent the flux density at the trailing pole tips from rising as much as it falls at the leading pole tips. The net effect is a reduction in pole flux, even though arma-

[1] A. Blondel, *Transactions of the St. Louis Electrical Congress*, 1904.

A. Blondel, *Comptes rendus*, Vol. 159, p. 604, 1914.

Arnold and LaCour, "Die Synchronen Wechselstrommaschinen," Chapters II and III.

V. Karapetoff, "Essays on Synchronous Machinery," *Gen. Elec. Rev.*, 1911.

ture ampere turns add and subtract in equal amounts. This effect, while not entirely negligible, is usually neglected. It is present in varying degrees at all pf's above zero.

When an alternator is operating at normal power factors of 60 to 90 per cent, lagging, such as are common to industrial loads, a combination of both these effects takes place. Figure 57 shows this in greater detail. The part of the armature ampere turns which weakens the main flux is shown in diagonal sectioning, and that part which strengthens it is

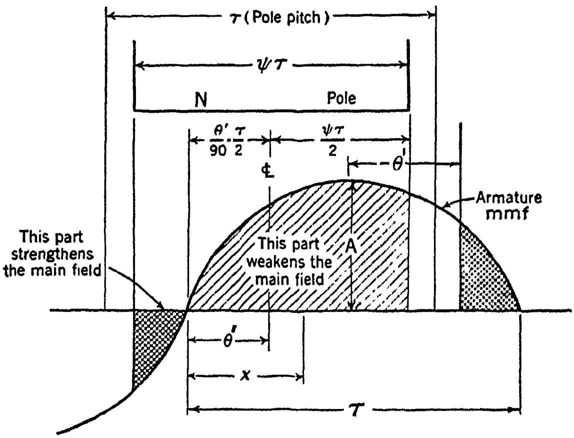


FIG. 57.

cross-hatched. The resulting demagnetizing ampere turns can be found by subtracting the latter area from the former and by dividing by the circumferential width, $\psi\tau$, of the pole shoe. To allow for fringing, this length should be increased by twice the radial thickness of the air gap. This correction is small and is usually neglected.

Note how this figure agrees with the theory developed in Article 61. In that presentation it was shown that the magnitude of the armature mmf varied as the cosine of $(\alpha + \theta')$ wherein α was the angular distance between the midpoint of the poles and any point on the air gap under consideration. For a maximum, α would equal $-\theta'$. This is clearly indicated on Fig. 57.

To evaluate the area of the large part of the wave:

$$A_1 = A \int_0^{\frac{\psi\tau}{2} + \frac{\theta'}{90}\frac{\tau}{2}} \sin\frac{x}{\tau}\pi dx \qquad [52]$$

$$= A\frac{\tau}{\pi}\left[1 - \cos\frac{\pi}{2}\left(\psi + \frac{\theta'}{90}\right)\right] \qquad [53]$$

(an examination of the figure should explain the limits of this integration),

in which τ = pole pitch in inches
ψ = fraction of the pole pitch covered by the pole
θ' = the pf angle between the current and the induced emf in electrical degrees

The area of the small part is

$$A_2 = A \int_0^{-\left(\frac{\psi\tau}{2} - \frac{\theta'}{90}\frac{\tau}{2}\right)} \sin \frac{x}{\tau} \pi dx \tag{54}$$

$$= A \frac{\tau}{\pi}\left[1 - \cos \frac{\pi}{2}\left(\psi - \frac{\theta'}{90}\right)\right] \tag{55}$$

The difference in area:

$$A_1 - A_2 = A \frac{2\tau}{\pi} \cdot \sin \frac{\psi\pi}{2} \cdot \sin\left(\frac{\pi}{2}\frac{\theta'}{90}\right) \tag{56}$$

The average demagnetizing ampere turns:

$$\frac{A_1 - A_2}{\psi\tau} = A\left(\frac{2 \sin \frac{\pi\psi}{2}}{\psi\pi}\right) \cdot \sin \theta' = Ak \sin \theta' \tag{57}$$

This shows that the demagnetizing ampere turns for a given machine are proportional to $\sin \theta'$. Accordingly, we may conclude that the armature reaction ampere turns consist of two parts: $A \sin \theta'$, which is demagnetizing; and $A \cos \theta'$, which is cross-magnetizing. Do not confuse $A \sin \theta'$ and $A \cos \theta'$ with the respective areas below and above the axis of Fig. 57. $A \sin \theta'$ is proportional to the difference in these two areas or the net demagnetizing ampere turns. This analysis neglects the effects of change in space distribution of the gap flux with load.

To apply this analysis to the calculation of alternator regulation, the demagnetizing and cross-magnetizing components must be evaluated.[2] They will vary with the ratio of pole arc to pole pitch (ψ) and with the angle θ'. In addition, A, the crest value of armature reaction ampere turns per pole, must be used, and any of the methods derived in the preceding figures may be applied. The Fourier series analysis will be used, wherein:

$$A = NI \text{ of armature reaction per pole} = 0.9 \frac{ZI}{2P} k_d k_p$$

[2] See Karapetoff, "Experimental Electrical Engineering," Vol. II, Third Edition, Chapter XLVII.

Hence the average demagnetizing ampere turns per pole (abbreviated A_D)

$$A_D = 0.9\frac{ZI}{2P}k_dk_p \cdot \frac{2\sin\frac{\pi\psi}{2}}{\psi\pi} \cdot \sin\theta'$$

$$= \frac{ZI}{P}k_dk_p\left(0.9 \cdot \frac{1}{2} \cdot \frac{2\sin\frac{\pi\psi}{2}}{\psi\pi}\right)\sin\theta' \qquad [58]$$

The value in the parenthesis will be called D and is given for various values of ψ in Table VI.

TABLE VI

ψ =	0.6	0.7	0.75	0.8	1.0
D =	0.386	0.365	0.353	0.341	0.286

70. The Cross-component. It is obvious that any sine wave can be separated into two components in quadrature, yielding a resultant equal to the original. In fact, the analysis just followed showed that the wave

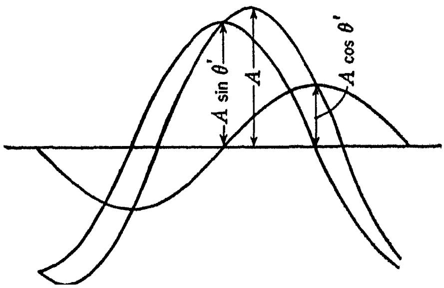


FIG. 58. The two components of armature-reaction ampere turns.

of armature mmf broke down into components proportional to $\sin\theta'$ and $\cos\theta'$. Such components are shown in Fig. 58, and applying the idea specifically to the problem at hand, we have just evaluated the demagnetizing component, expressed by equation 58. We will next consider the cross-magnetizing component in detail.

The cross-magnetizing component neither increases nor decreases the field excitation as a whole, but increases the mmf acting over one-half of each pole face and decreases by an equal amount the mmf acting over the other half of each pole face. This is shown in Fig. 59a. The shaded

portion shows approximately that part of the cross mmf which is really effective.

In Fig. 59*b* we see the mmf of the cross-component, and a curve of the flux which it would produce, considering the greatly increased reluctance of the air path between the salient poles. This flux is considered as being made up of a Fourier's series. All components except the fundamental will be neglected here. The flux path will include one-half of the air gap, the pole shoe, back across the other half of the gap, and through the armature to the starting point. (See Fig. 31.) This cross-flux is constant for any given armature current and pf, and is fixed in position with respect to the poles. It will therefore revolve with them and produce an emf, E_c, in the armature winding.

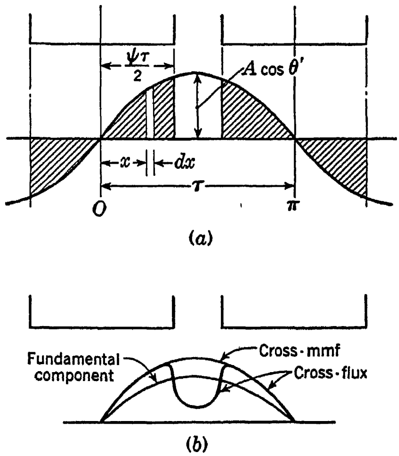


FIG. 59. The cross-component of armature reaction.

71. Evaluating E_c. The expression for the wave of cross-mmf as a Fourier's series, with all but the fundamental neglected, involves the integration of the wave of Fig. 59 between the limits as shown. Crest value of the fundamental:

$$(A \cos \theta') \frac{2}{\pi}\left(\int_0^{\psi\pi/2} \sin^2 x \cdot dx + \int_{\pi - (\psi\pi/2)}^{\pi} \sin^2 x \cdot dx\right)$$

$$= A \cos \theta' \left(\psi - \frac{1}{\pi} \sin \psi\pi\right) \quad [59]$$

The average value of this mmf is

$$A_c = \frac{2}{\pi} A \cos \theta' \left(\psi - \frac{1}{\pi} \sin \pi\psi\right) \text{ ampere turns per pole} \quad [60]$$

By substituting the value of A:

$$A_c = \frac{ZI}{P} k_d k_p \left[\frac{0.9}{\pi}\left(\psi - \frac{1}{\pi} \sin \psi\pi\right)\right] \cos \theta' \quad [61]$$

The value of the expression in brackets will be designated C and is shown in Table VII.

TABLE VII

ψ =	0.6	0.7	0.75	0.8	1.0
C =	0.085	0.126	0.150	0.175	0.286

72. The Vector Diagram. The diagram of Fig. 60 can be built up by noting the following relationships. When the generator delivers a current of I amperes, at a terminal voltage V_t and at a pf angle θ, there will be set up an armature-resistance drop IR_e and a leakage-reactance drop IX_l in phase, and in quadrature, respectively, with the current. These latter values add vectorially to the terminal voltage to give the emf E_a, which is generated in the armature by the air-gap flux. The generated emf can be split into the two components, in quadrature with each other, of which E_f is due to the main-pole flux and E_c is due to the cross-flux

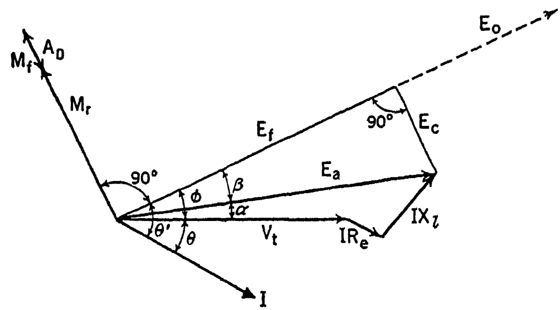


FIG. 60.

set up in the interpolar space and pole edges by the cross-component of armature reaction.

To generate the voltage E_f, requires a resultant mmf along the axis of the field poles which is designated M_r. The initial mmf of the field winding alone is M_f, but the direct component of armature reaction A_D, is subtracted therefrom to yield M_r. This subtraction, of course, presumes a lagging current; for leading current the so-called demagnetizing component would add to that of the field winding.

Note that the excitation M_f is presumed to result in a no-load voltage of E_0 as read from the saturation curve, and hence the regulation would be:

$$\frac{E_0 - V_t}{V_t} 100 \text{ per cent} \qquad [62]$$

Several ideas should be grasped from the above explanation.

The cross-component of armature reaction sets up a flux cut by the armature conductors and so generates the voltage E_c, which may be treated as a voltage drop.

The demagnetizing component of armature reaction is an mmf subtracting arithmetically from the field-winding mmf.

Unfortunately the calculation cannot be carried through in the order as given above. A_C and A_D depend upon the angle θ' between the voltage E_f and the armature current. The angle θ' is derived in part from the relative values of E_c and E_f, and hence roundabout methods must be used to complete the diagram. This will be explained below.

The terminal voltage per phase of a generator is designated V_t, and the regulation is desired at a load of I amperes, lagging θ degrees behind V_t. The effective resistance of the armature and the leakage reactance are known.

V_t, I, IR_e, and IX_l can be laid off on the vector diagram and E_a can be calculated.

$\theta' = \theta + \phi$

v = volts per ampere turn per pole generated on the lower part of the saturation curve

Then it is assumed that

$$E_c = vA_c \tag{63}$$

$$= v \cdot \frac{ZI}{P} \cdot k_dk_pC \cos(\theta + \phi) \tag{64}$$

But

$$E_c = E_a \sin \beta \text{ (from Fig. 60)}$$

Hence

$$E_c = E_a \sin \beta = v \cdot \frac{ZI}{P} \cdot k_dk_pC \cos(\theta + \phi) \tag{65}$$

By putting $\phi = \beta + \alpha$, this can be changed to

$$\tan \beta = \frac{v \dfrac{ZI}{P} k_dk_pC \cos(\theta + \alpha)}{E_a + v \dfrac{ZI}{P} k_dk_pC \sin(\theta + \alpha)} \tag{66}$$

Since

$$E_a = V_t + I(R_e + jX_l)(\cos \theta - j \sin \theta) \tag{67}$$

the angle α can be found. All the values of equation 66 can be substituted and the angle β calculated. Then

$$E_c = E_a \sin \beta \tag{68}$$

73. Example of Regulation Calculated by the Two-reaction Method. The machine data used here will be the same as that of Articles 52 and 62 for which armature reaction was calculated.

180 kv-a   24 poles   440 volts   300 rpm

Armature: 144 slots, 2-layer winding, coil pitch $\frac{5}{6}$. Each coil has 5 turns of 4 conductors. Armature is connected 2–Y (2 circuits), etc. (See Fig. 44 also.)

$$E_{\text{per phase}} = \frac{440}{\sqrt{3}} \quad \text{or} \quad 254 \text{ volts}$$

$$I_{\text{per terminal}} = \frac{180{,}000}{\sqrt{3}E} \quad \text{or} \quad 236 \text{ amperes}$$

$$R_e \text{ per phase at 75 C} = 0.03 \times 1.6 \quad \text{or} \quad 0.048 \text{ ohm}$$

$$X_l \text{ per phase} = 0.301 \text{ ohm} \quad \text{(See Article 52.)}$$

$$E_a = V_t + I(R + jX)(\cos\theta - j\sin\theta)$$

The regulation will be calculated at 80 per cent pf, lagging.
Hence, from equation 67,

$$E_a = 254 + 236(0.048 + j0.301)(0.8 - j0.6)$$

$$E_a = 305.72 + j49.96$$

$$= 309.0 \text{ volts}$$

$$\sin\alpha = \frac{49.96}{309.0} \quad \text{or} \quad 0.1615$$

$$\alpha = 9°\ 18'$$

If a pf of 0.80 is the assumed value,

$$\theta = 36°\ 52'$$

$$\cos(\theta + \alpha) = 0.6926$$

$$\sin(\theta + \alpha) = 0.7214$$

$$\frac{ZI}{P} = \frac{1440 \times \frac{236}{2}}{24} \text{ (from Article 62)}$$

$$= 7080$$

$$k_d = 0.966 \quad \text{(from Article 62)}$$

$$k_p = 0.966 \quad \text{(from Article 62)}$$

$$\psi = \frac{4.5}{6.024} \quad \text{say, 75 per cent (from Fig. 54)}$$

$$C = 0.150 \quad \text{(from Table VII)}$$

To obtain v: Figure 44 gives the open-circuit saturation curve for this machine. Referring to this curve, 90 volts per phase requires 5 field amperes. Each pole has 128.5 turns or 642.5 ampere turns.

$$\text{Volts per ampere turn} = \frac{90}{642.5} \quad \text{or} \quad 0.140$$

$$v\frac{ZI}{P} k_d k_p C \cos (\theta + \alpha) = 0.140 \times 7080 \times 0.966 \times 0.966 \times 0.150 \times 0.7214 \quad \text{or} \quad 100$$

$$v\frac{ZI}{P} k_d k_p C \sin (\theta + \alpha) = 96.0$$

$$E_a = 309 \text{ volts}$$

$$\tan \beta = \frac{100}{309 + 96.0} \quad \text{(from equation 66)}$$

$$\tan \beta = 0.2475$$

$$\beta = 13° \, 54'$$

$$\sin \beta = 0.2402$$

$$\cos \beta = 0.9707$$

$$\begin{aligned} E_c &= E_a \sin \beta \\ &= 309 \times 0.2402 \\ &= 74.2 \text{ volts} \end{aligned}$$

$$\begin{aligned} E_f &= E_a \cos \beta \\ &= 309 \times 0.9707 \\ &= 300 \text{ volts} \end{aligned}$$

To generate 300 volts requires a field current of 24 amperes on Fig. 44. This is

$$24 \times 128.5 = 3090 \text{ ampere turns per pole}$$

It is represented on Fig. 60 by M_r.

The demagnetizing component of armature reaction must be added directly to this magnetization to obtain M_f.

The demagnetizing component from equation 58 is

$$A_D = \frac{ZI}{P} k_d k_p D \sin \theta'$$

$$D = 0.353 \quad \text{(from Table VI)}$$

$$\begin{aligned} \theta' &= \theta + \alpha + \beta \\ &= 36° \, 52' + 9° \, 18' + 13° \, 54' \\ &= 60° \, 4' \end{aligned}$$

$$\sin \theta' = 0.866$$

$$A_D = 7080 \times 0.966 \times 0.966 \times 0.353 \times 0.866 \quad \text{or} \quad 2020$$

$$\begin{aligned} M_f &= M_r + A_D \\ &= 3090 + 2020 \quad \text{or} \quad 5110 \text{ ampere turns per pole} \end{aligned}$$

$$\frac{5110}{128.5} = 39.8 \text{ amperes, field current}$$

From the saturation curve this corresponds to a phase voltage of 352. The regulation is then

$$\frac{352 - 254}{254} 100 = 38.5 \text{ per cent}$$

This same machine was used for the example of Article 52; the regulation determined at an 80 per cent lagging pf was 41.7 per cent by the American Standards method. This result compares favorably with that given above.

74. Modified Vector Diagram. The Blondel two-reaction theory, as just presented, is closely allied to the original form of this development. It has been extended by the Doherty and Nickle [3] theory which is by far the most complete and accurate analysis yet available for alternator calculations. Because of its importance, it will be presented here in outline form, stressing the general method of approach.[4] The same concepts will also be applied to synchronous motors.

[3] R. E. Doherty and C. A. Nickle, "Synchronous Machines I and II," *Trans. A.I.E.E.*, Vol. 45, pp. 912–914, 1926.

R. E. Doherty and C. A. Nickle, "Synchronous Machines III, Torque Angle Characteristics under Transient Conditions," *Trans. A.I.E.E.*, Vol. 46, pp. 1–18, 1927.

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S. B. Crary, "Two-reaction Theory of Synchronous Machines," *Elec. Eng.*, pp. 27–31, January, 1937.

B. R. Prentice, "Fundamental Concepts of Synchronous Machine Reactances," *Trans. A.I.E.E.*, Vol. 56, 1937.

B. L. Robertson, T. A. Rogers, and C. F. Dalziel, "The Saturated Synchronous Machine," *Elec. Eng.*, July, 1937.

[4] Because of its detailed nature and bulk, the specialized exposition will not be given in this text.

In deriving the separate components of armature reaction for the Blondel theory, it was found that the directly demagnetizing portion was a function of $A \sin \theta'$ and the cross-magnetizing component depended upon $A \cos \theta'$. Since armature-reaction ampere turns A naturally involve armature current, the first step in this process is the association of the current with the terms $\sin \theta'$ and $\cos \theta'$. The vector diagram of Fig. 60

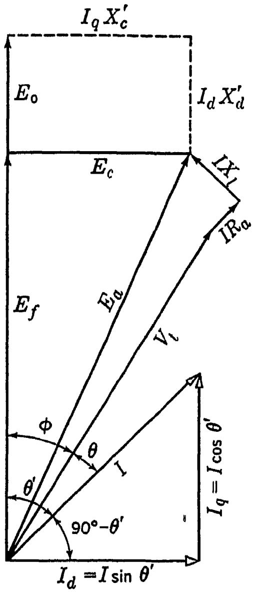


FIG. 61. Transition diagram between the conventional Blondel analysis and the modern two-reaction theory.

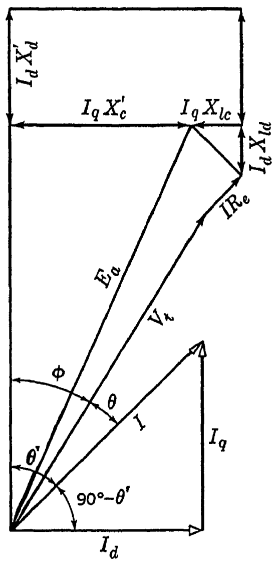


FIG. 62. Second step in the transition. The leakage-reactance drop has been separated into components.

is then redrawn as shown in Fig. 61. The directly demagnetizing effect resulted in a change in voltage represented by a difference in the length of E_f and E_0. (This difference will depend upon saturation, but for the present this will be neglected.) Since it is a function of $\sin \theta'$ and is in quadrature with the component of armature current represented by $I \sin \theta'$ it follows that the change in voltage from E_0 to E_f might be represented by a fictitious reactance drop.

$$E_0 - E_f \propto I_dX'_d$$

The term X'_d is as yet undefined although it will be designated in general as a direct component of reactance, and similarly the current with which it is associated is the direct component of current. This current will be so designated, even though it is in quadrature with the position of the no-load voltage.

In a similar manner we find that the cross-magnetizing component E_c will be considered a fictitious reactance drop since it is proportional to $\cos\theta'$ and in quadrature with the current component $I_q = I \cos\theta'$.

$$E_c \propto I_q X'_c$$

X'_c is a cross-reactance, as yet undefined.

The idea of associating components of a drop with suitable components of current will be carried a step further as shown in Fig. 62. Here the leakage-reactance drop will be separated into components associated with suitable current components so that vectorially

$$IX_l = I_d X_{ld} + I_q X_{lc}$$

The slots containing conductors carrying I_d may occupy a different position, with reference to the poles, from those carrying I_q (presuming that each current component has a separate existence). In such a case it is possible that certain parts of the leakage flux may cause a difference in the leakage reactance of the two components. In calculating the reactance from design data, this fact can be considered. Otherwise the usual method of calculating the leakage reactance would result in usable values.

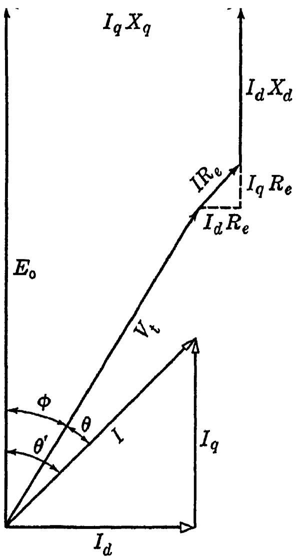


FIG. 63. Final step in the transition, resulting in the two-reaction diagram built up with direct-axis synchronous reactance ($X_d = X'_d + X_{ld}$) and quadrature-axis synchronous reactance ($X_q = X'_c + X_{lc}$). The resistance drop can also be divided into quadrature components, or in many cases this can be neglected entirely. Its relative size is exaggerated here.

As shown in Fig. 61, the effect of the reactance X'_c merely replaces the effect of the voltage E_c, but by grouping the reactances associated with the current component I_q we obtain a new reactance

$$X'_c + X_{lc} = X_q \qquad [69]$$

Similarly, associating the reactive drops, using the direct component of current, we obtain a new reactance

$$X'_d + X_{ld} = X_d \qquad [70]$$

By making use of these combined terms we have a comparatively simple diagram as shown in Fig. 63.

If necessary, the resistance drop may be divided into components, although for finding the regulation this factor may frequently be dropped

entirely. Now if X_d and X_q are known, the vector diagram can be solved for the determination of regulation.

By trigonometric considerations,

$$\tan \theta' = \frac{V \sin \theta + IX_q}{V \cos \theta + IR_e} \qquad [71]$$

also

$$\phi = \theta' - \theta$$

and

$$\tan \phi = \frac{I(X_q \cos \theta - R_e \sin \theta)}{V_t + I(R_e \cos \theta + X_q \sin \theta)}$$

$$E_0 = V_t \cos \phi + IX_d \sin \theta' + IR_e \cos \theta' \qquad [72]$$

75. Direct- and Quadrature-axis Synchronous Reactances. The replacing of the mmf effect of the direct component of armature reaction by a fictitious reactance X_d, which contains a leakage-reactance component, is, of course, parallel to the old concept of synchronous reactance. Because of this similarity of the two reactances with which we will deal, they are called *direct-axis synchronous reactance* and *quadrature-axis synchronous reactance*, respectively. As shown on the vector diagrams, they must always be used with their respective components of current.

The two reactances arise from fluxes which have widely different paths. Neglecting, for the moment, the leakage fluxes, the direct component of mmf acts on the main magnetic circuit of the machine. The quadrature component has a magnetic circuit largely through the air gaps and interpolar space. For this reason the quadrature-axis synchronous reactance is smaller than the direct component and is less affected by saturation. In a non-salient-pole machine X_d is nearly equal to X_q and would be exactly so were it not for the slight differences in the two magnetic circuits on which they operate. Such differences may be brought about by the slots on the rotor in which the field winding is placed.

76. Determination of X_d *and* X_q.

(*a*) *Direct Axis.*

(1) The direct-axis synchronous reactance can be calculated from the no-load-saturation curve and the short-circuit-current curve as shown in dealing with the original concept of synchronous reactance. Note that the short-circuit armature current versus the field current has already been labeled *direct-axis synchronous reactance* in Fig. 44. Dividing the rated phase voltage by the short-circuit current at the same excitation gives X_d. To obtain the *unsaturated* value of X_d, the voltage on the *air-*

gap line should be used. For the same field excitation, the short-circuit armature current is obtained so that

$$X_d = \frac{V_n \text{ on air-gap line}}{I_{sc}} \qquad [73]$$

(2) The Proposed Test Code (A.I.E.E.) makes use of the term *per unit values* in dealing with reactances. Suppose an alternator has a leakage reactance so that the drop it causes is 5 per cent of the rated terminal voltage when rated current flows. In this case the magnitude of the leakage reactance could be expressed as 5% X. Instead of using 100 as the base, which is inherent with the use of percentages, *unity* is taken as the base in per unit values, thereby moving the decimal point two places. Hence in the above case the per unit leakage reactance is 0.05.

In this code, the per unit direct-axis synchronous reactance (at rated kilovolt-amperes) is defined as the ratio of the field current at rated armature current on sustained symmetrical short circuit to the field current at normal, open-circuit voltage on the air-gap line.

Example (Refer to Fig. 44).

$$X_d \text{ (per unit)} = \frac{I_{FSI}}{I_{FG}} \qquad [74]$$

$$= \frac{22}{14.5} \quad \text{or} \quad 1.516$$

To evaluate this, we must consider that the machine under consideration had a rated current of 236 amperes and a voltage per leg of 254. This yields "base ohms" of

$$\frac{\text{Rated volts}}{\text{Rated current}} = \frac{254}{236} \quad \text{or} \quad 1.07 \text{ ohms per leg}$$

Then since

$$\frac{\text{Actual ohms}}{\text{Base ohms}} = \text{per unit values}$$

$$(\text{Per unit values}) \times (\text{base ohms}) = \text{actual values in ohms}$$

$$1.516 \times 1.07 = 1.624 \text{ ohms, } X_d$$

or

$$\% X_d = 151.6$$

By the first method given,

$$X_d = \frac{V_n \text{ air-gap line}}{I \text{ short circuit}}$$

Refer again to Fig. 44. The short-circuit current corresponding to the same excitation as rated voltage on the air-gap line is 155 amperes.

$$X_d = \frac{254}{155} \quad \text{or} \quad 1.64 \text{ ohms}$$

This is a fair check with the previous method.

(*b*) *Direct- and Quadrature-Axes.*

(1) As a test method of obtaining X_d, the alternator under investigation should be driven at slightly less than synchronous speed with its field circuit open. Balanced, reduced voltage is applied to the armature terminals. Applied armature volts, armature current, and the voltage

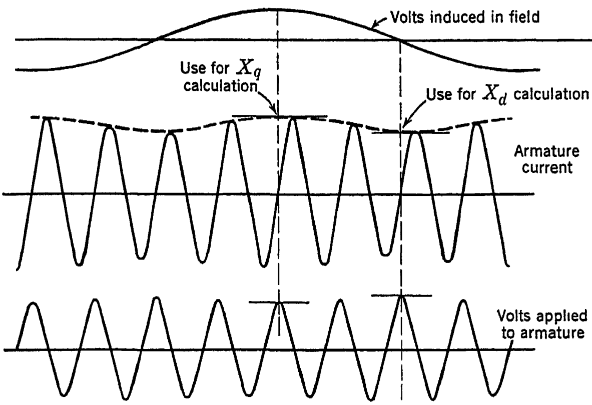


FIG. 64.

induced in the field are read. Variation in reading will occur as shown in Fig. 64. At the instant when the voltage across the field is zero,

$$X_d = \frac{\text{armature volts per phase}}{\text{armature current}} \qquad [75]$$

The above method follows the procedure outlined in the Proposed Test Code. The reasoning is explained below.

As the field structure rotates through the gap, at slightly greater or lesser speed than synchronous, it is exposed to the rotating mmf of armature reaction. The physical poles and the armature-reaction mmf are alternately in phase and out, the change occurring at slip frequency. When the axis of the poles and the axis of the armature-reaction mmf wave coincide, the armature mmf acts through what is ordinarily the field magnetic circuit. The voltage applied to the armature is then equal to the drop caused by the direct component of armature reaction and leakage reactance. The entire armature current is in the position of $I \sin \theta'$, being completely wattless except for the effect of armature resistance. Continued rotation brings the armature mmf in quadrature with the field poles. Under this condition the applied voltage is equal to the leakage-reactance drop plus the equivalent voltage drop of the cross-magnetizing field E_c. It follows then that the fluctuation in current as the field

slips in and out of step is a measure of the two components of synchronous reactance, containing as they do, by definition, component effects of leakage reactance as well.

Because the applied voltage may vary slightly with change in current and the swing of the ammeter may be influenced by inertia, an oscillograph record is recommended for reading the values as shown. The voltage induced in the field is a measure of the rate of change of flux through the field circuit. The flux is a maximum and the rate of change is zero at the instant the mmf's coincide. This point serves as an indication for taking the readings described in the Proposed Test Code method. It corresponds to the instant of minimum current.

From the oscillograph record,

$$X_d = \frac{\text{maximum voltage}}{\text{minimum current}} \qquad [76]$$

$$X_q = \frac{\text{minimum voltage}}{\text{maximum current}} \qquad [77]$$

Both are unsaturated values, and both should be converted into terms of reactance per phase.

The above test method is the only one which will be presented here for determining quadrature-axis synchronous reactance. Complex analytical methods are available. Of course the procedure shown for obtaining the value of E_c in the Blondel analysis can be used in conjunction with the proper component of leakage-reactance drop to yield a value of X_q.

77. Conclusion. The extension of the two-reaction theory just presented typifies the approach to modern synchronous machine theory but does not go into the details of the Doherty and Nickle analysis. The use of direct and quadrature synchronous-reactance components results in a simpler vector diagram and in a more direct solution from constants obtained through test or design data. The Doherty and Nickle analysis, with the related papers by other writers, have presented the most complete and rigorous method of analytical treatment that has yet appeared for either steady state or transient conditions.

CHAPTER IX

STANDARDS, LOSSES, AND HEAT RUNS

78. Chapter Outline.

Generator Standards.
Generator Efficiency.
  Direct Measurement.
  Loading-back.
  Loss Measurement.
Losses.
  Friction.
  Core Loss.
  Field Loss.
  Armature Copper Loss.
  Load or Stray Power Losses.
Determination of Losses.
  Rated Motor Method.
  Retardation Method.
Heat Runs.

79. Standards. Many of the usual types of electrical machines are built according to specifications set up by various standardizing groups. The more prominent of these are the American Institute of Electrical Engineers (A.I.E.E.), the American Standards Association (A.S.A.), and the National Electrical Manufacturers Association (N.E.M.A.). Their standards include figures for minimum permissible performance and for test and calculation methods as quoted in the preceding chapters. In addition, other items are standardized, only a few of which will be listed below.

Ratings. Standard kilowatt ratings for 60-, 50- and 25-cycle 0.8 power factor lagging, synchronous generators, except waterwheel driven, shall be: 1, 2, 3, 5, 7.5, 10, 15, etc., up to 8000.

Various speeds for which each kv-a rating should be constructed are also listed. For instance, 125 kv-a or 100 kw are recommended for speeds of 720, 600, 514, 450, 400, 360, 327, 300, 277, 257, 240, or 225 rpm.

Voltages. Standard voltages shall be 120, 240, 480, 600. 2400, 2500, 4160 or 4330, 6900, 11,500, 13,800, and 23 000 volts.

Frequencies. Standard frequencies shall be 25, 50, and 60 cycles per second.

Excitation Voltage. The standard excitation voltage for field windings shall be 125 volts direct current.

Power Factor. The standard power factor for synchronous generators shall be either unity or 0.8 lagging.

In this regard it might be pointed out that the actual pf at which a generator operates depends upon load conditions. It can be controlled only by such practices as providing more or less leading current through synchronous motors or static condensers to neutralize the usual lagging currents of most loads. Extremely low pf operation, lagging, requires increased exciter capacity, heavier field currents, and appropriate field-winding design. Rather than attempt to list as standard equipment a series of a-c generators with field windings and exciters for various pf operation, or an extreme design with reserve capacity for low pf which would not be required on a high pf load, standardization on 0.80 pf, lagging, or unity pf is a logical step.

Voltage Regulation. The voltage regulation for synchronous generators for ratings larger than 200 kw at any speed and for ratings smaller than 200 kw at speeds 450 rpm and lower shall not exceed the following:

Full-load Regulation in Per Cent

Power factor	50 C generator	40 C generator
0.8	40	34
0.9	35	30
1.0	25	20

Normal Efficiency. N.E.M.A. tables are available, showing the efficiency to be expected of all standard ratings of engine-type synchronous generators. They cover many pages, but a few values will be given here to indicate a typical range.

50 C rise   2300 volts   400 rpm   3 phase   0.8 pf

Field rheostat losses are excluded. Full-load efficiencies are given below in percentages.

Kw =	25	50	100	200	500	1000	1500
Eff =	85.2	88.3	90.6	92.3	94.1	95.2	95.8

The above material is adapted from the N.E.M.A. Motor and Generator Standards 38–49 and 34–24. They do not apply to turbo-alternators, new standards for these types now (1941) being under consideration. It should not be assumed from such standardization that machines of

"off-standard" construction cannot be built for special requirements. To quote:

An Adopted Standard of the National Electrical Manufacturers Association defines a practice of construction to the observance of which in the interest of the public all members of the Association should adhere, and in no event should a member of the Association represent as standard any product falling below such standard.

It is distinctly understood that adopted standards relate only to products commercially standardized and subject to repetitive manufacture, and do not apply to products built to meet the special requirements of individual customers.

80. Efficiency of Alternators.[1] The efficiency of a-c generators can be determined through one of three methods:

1. Direct Measurement of Output and Input. The mechanical input cannot always be measured accurately. In general, this method is impracticable.

2. Circulating Power or Loading-back Method. This involves two machines of the same rating, connected electrically and mechanically. Data can be taken from such tests over the entire rating of the machines with an expenditure of energy equal to the losses in both. However, for large machines on commercial tests such a setup is difficult and expensive.

3. Conventional Efficiency from Measurement of Losses. This involves a determination of the losses, which are very similar to those found in d-c machines. However, some of the losses in a-c generators are not determinable with any degree of accuracy, and more or less correct assumptions must be made regarding them.

81. Losses. The losses in a-c generators are:

(*a*) Bearing friction.
(*b*) Windage.
(*c*) Brush friction at slip rings.
(*d*) Eddy current and hysteresis loss at no load.
(*e*) Field and field rheostat losses.
(*f*) Armature copper loss.
(*g*) Load or stray power losses.

82. Friction Losses. The friction loss in the bearings varies with a number of factors, including the lubrication, temperature of the bearing, and the load. Load variation produces a more noticeable change in bearing friction on belt-driven machines, but in spite of this it is usually assumed to be a constant value for any one speed.

$$\text{Bearing friction loss} = 0.81\, D \cdot L \left(\frac{V}{100}\right)^{3/2} \text{watts} \qquad [78]$$

[1] P. L. Alger, "A Comparison of the Efficiencies of Synchronous Machines as Determined by Various Methods," *Gen. Elec. Rev.*, Vol. 29, p. 765, November, 1926.

wherein D = bearing diameter, in inches
L = bearing length, in inches
V = peripheral velocity of the shaft, in feet per minute

The Proposed Test Code for Synchronous Machines (A.I.E.E., January, 1937) recommends a method of separating friction and core losses by running the generator as an idle synchronous motor. Kilowatts input, applied volts, and field current should be read. The run should be made at an excitation resulting in unity (or above 0.95) pf.

The voltage is lowered in steps to obtain data for the curve shown in Fig. 65. If the voltage could be extended to zero, its intercept would

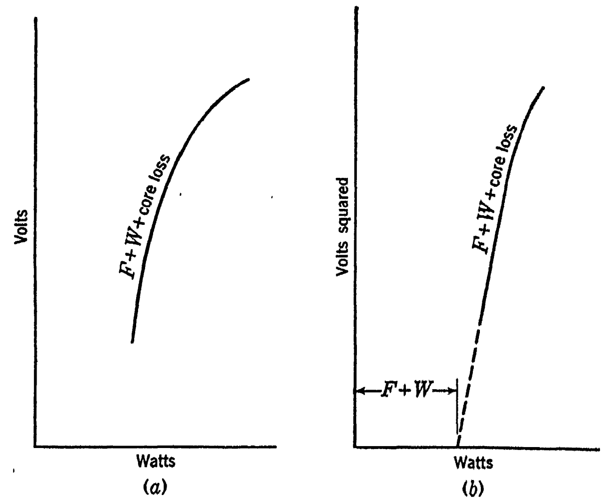


FIG. 65. In b the original data of friction, windage, and core loss versus volts have been replotted against volts squared. Extension of this line to zero voltage gives the value of friction and windage losses.

indicate friction and windage since core loss is zero at no excitation. This range is impossible, but on the assumption that core loss at low densities varies as the voltage squared, a re-plot of these losses versus volts squared results in a straight line at the lower portion of the curve. Extension of this line gives the intercept, indicating friction and windage losses.

On large machines the input during such a run includes copper losses in the armature which are increasingly important as the capacity of the machine reduces. For greater accuracy, the copper losses must be subtracted from the no-load inputs before the curves of Fig. 65 are plotted.

83. Eddy-current and Hysteresis Losses.[2] The losses in the armature teeth and core, caused by the field flux, make up the majority of the so-called *core loss.* In addition, other losses arise from the same source.

Eddy-current Losses in the Pole Faces Caused by the Armature Slots. The difference in reluctance between the slots and the teeth produces space ripples in the flux wave and gives rise to these losses. If the air gap were large compared to the width of the slot, the effect would be a minimum; conversely, wide slots and small air gap increase the losses. When the field flux distribution is not sinusoidal the tooth eddy-current loss becomes different from that for a sine distribution. This difference is usually neglected.

Eddy-current Losses in the Armature Conductors (in Addition to Those Caused by the Armature Leakage Flux). Owing to the change in flux intensity over the face of the pole (more or less sinusoidal), the voltages produced by the flux will be different in the two sides of the conductor. The tufting of the flux near the outer ends of the teeth also causes a difference in voltage between the top and bottom of the conductor. These differences in voltages between the conductor edges would produce eddy currents unless the conductor were laminated horizontally and vertically. This implies, then, that, in cases where the effect is noticeably large, armature coils should be built up of turns in parallel rather than of large copper strap.[3]

All the above losses are assumed to be independent of the load, but vary with the excitation; if the excitation is to be varied with load and pf change to maintain constant voltage, then these losses change also.

84. Field and Field Rheostat Losses. The copper losses in the field circuit are obtained by adding the I^2R loss of the field winding and the I^2R loss of the field rheostat or, more simply, by multiplying the excitation voltage by the field current. This loss is constant for any given load and pf, but varies with these two. The maintenance of rated voltage at low (lagging) pf requires a comparatively large field current and gives the maximum excitation loss.

85. Armature Copper Loss. The losses in the armature windings of an a-c generator are I^2Rm, where R is the ohmic or d-c resistance per phase and m is the number of phases. The Standards of the A.I.E.E. recommend that the resistance should be that at 75 C. Whether the

[2] S. L. Henderson and C. R. Soderberg, "Recent Improvements in Turbine Generators," *Trans. A.I.E.E.*, Vol. 47, No. 2, p. 549.

[3] For a discussion of laminated conductors see:
A. B. Field, *Trans. A.I.E.E.*, Vol. 24, p. 761, 1905.
H. W. Taylor, *Elec. Rev.*, p. 102, June 23, 1920.

ohmic or the effective resistance should be used depends upon the treatment of the stray power losses.

86. Load or Stray Power Losses. These losses are classed as indeterminable, although an approximate value is often used. They include the eddy-current losses in the armature conductors and the change in the hysteresis and eddy-current losses in the armature teeth and armature core with load. Load losses are caused by the armature leakage flux and by the change in the main flux distribution only. Although they do not vary with the first power of the armature current, such a relationship is often assumed. If a value is obtained for these losses from test data, this value is added to the other losses. In such a case the armature I^2R loss is calculated from the ohmic resistance of the armature. If load losses are otherwise neglected, the resistance of the armature may be increased to include them. This is done by using the effective resistance.

87. Efficiency from Losses. The conventional efficiency of an a-c generator can be calculated from the following formula:

Efficiency in percentage

$$\frac{\text{output watts}}{\text{output watts} + W_f + W_i + W_r + I^2Rm + W_l} 100 \qquad [79]$$

wherein W_f = friction loss, in watts, including windage
W_i = core or eddy-current and hysteresis losses at the excitation required to give a terminal voltage equal to the calculated internal voltage
W_r = field and field rheostat losses
I = armature current per phase
R = resistance of the armature in ohms at 75 C
m = number of phases
W_l = load loss in watts

The preceding losses must correspond to the values of output assumed. Thus, if the efficiency of a three-phase alternator were to be calculated at half-rated current and 80 per cent pf, lagging, the formula would become:

Efficiency in percentage

$$= \frac{\sqrt{3}EI_1\,0.80}{\sqrt{3}EI_1\,0.80 + W_f + W'_i + W'_r + I_1{}^2Rm + W'_l} 100 \qquad [80]$$

wherein W'_i = core or eddy-current and hysteresis losses at the excitation necessary to give a terminal voltage equal to the

calculated internal voltage. This varies on account of change in magnitude and direction of I_1.

W'_r = field and field rheostat losses determined from the value of I_f necessary to produce rated voltage at half-rated current and 80 per cent pf

W'_l = one-half of the full-load value, unless data are available over a range of current

I_1 = one-half of rated value of armature current

88. The Determination of Losses. Several methods have been found to be practicable for the determination of losses.

(*a*) The rated motor method.
(*b*) The retardation or deceleration method.
(*c*) Calculation from design data.

89. The Rated Motor Method. The generator to be tested is driven at rated speed by a motor of known efficiency. The motor need be large enough to supply only the losses of the generator. If possible, motor and generator should be direct coupled; if belted, correction is necessary to account for belt loss.

(*a*) When the generator is run at rated speed without excitation, the motor output (calculated from motor input and known efficiency) is a measure of the bearing, brush, and windage losses of the generator.

(*b*) If the generator field is excited, the additional motor output is a measure of the core losses. Usually a series of values are taken to obtain core loss versus field current, or core loss versus E_0, over a wide range.

(*c*) The armature winding of the generator is short-circuited through ammeters of low resistance, and the generator is run at rated speed with reduced excitation. Adjustment of field current will give the recommended short-circuit amperes of 25 to 150 per cent as recommended by the Proposed Test Code previously quoted. The friction losses of *a* and the calculated I^2R losses of the armature are subtracted from the generator input. Calculated copper losses should be figured at the temperature of test. The remainder is the stray power loss.

The above values determined by the rated motor tests are:

Friction loss.
Core loss.
Stray power loss.

The excitation loss must be determined by calculating the field current necessary for the assumed load conditions.

The I^2R loss in the armature is determined by measurement of the armature resistance.

90. Example of Calculation of Efficiency.[4] Required to find the efficiency of the following generator from the data obtained by the rated motor method:

500 kv-a	three-phase
2300 volts	60 cycles
360 rpm	125.6 amperes per terminal

Excitation voltage 110 volts

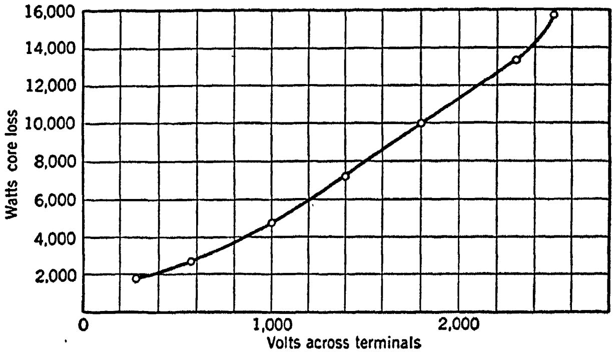


FIG. 66. Core loss.

When run at rated speed by a d-c motor, the friction and windage loss was 5050 watts.

The core-loss data are shown in Fig. 66.

Short-circuiting the armature and varying the excitation yield a stray power loss curve as shown in Fig. 67.

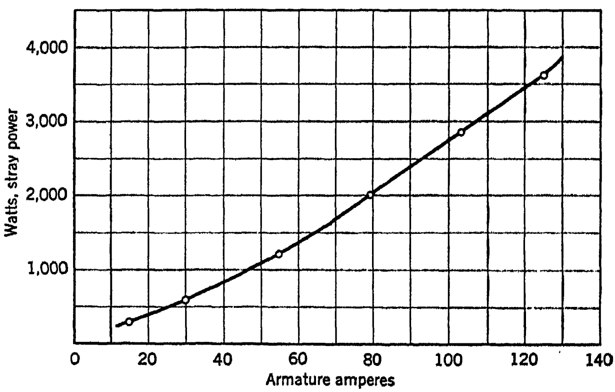


FIG. 67. Stray power loss.

[4] Barclay and Smith, "The Determination of the Efficiency of the Turbo-Alternator," *J.I.E.E.*, Vol. 57, p. 293, 1919.

The resistance of the armature across terminals was found to be:

$$R_{1-2} = 0.231 \text{ ohm}$$
$$R_{2-3} = 0.231 \text{ ohm}$$
$$R_{3-1} = 0.229 \text{ ohm}$$

These measurements were made by direct current and have been corrected to 75 C.

Open-circuit and zero pf characteristics of the generator are shown on Fig. 68.

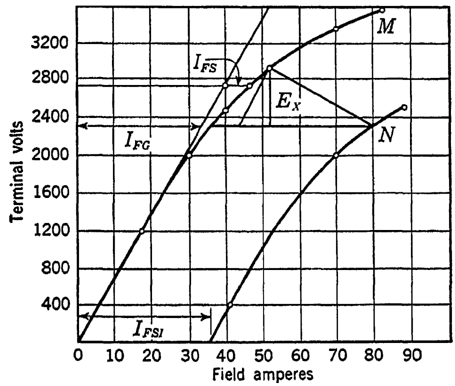


FIG. 68.

The efficiency of this generator is to be determined at rated kilovolt-ampere load and 80 per cent pf, lagging.

$$E_{\text{per phase}} = \frac{2300}{\sqrt{3}} \text{ or } 1329 \text{ volts}$$

$$E_{\text{internal per phase}} = 1329 + IR(\cos\theta - j\sin\theta)$$

$$R_{\text{average per phase}} = (0.231 + 0.231 + 0.229) \div 6$$
$$= 0.115 \text{ ohm}$$

$$\therefore E_{\text{internal per phase}} = 1329 + 125.6 \times 0.115(0.8 - j0.6)$$
$$= 1341 \text{ volts}$$

$$E_{\text{internal between lines}} = 1341 \times \sqrt{3}$$
$$= 2324 \text{ volts}$$

Core loss corresponding to this voltage = 13,900 watts. (See Fig. 66.)

Stray power loss at rated armature current = 3600 watts. (See Fig. 67.)

Determination of required excitation:

As shown in Article 51, the American Standards Association method will be used to determine the required excitation. (See Fig. 68.)

$$E_X = 610 \text{ volts between terminals or } 352 \text{ volts per leg}$$

$$E_R = 125.6 \times 0.115 \text{ or } 14.47 \text{ volts per leg}$$

$$E_g = \frac{2300}{\sqrt{3}} + (14.47 + j352)(0.8 - j0.6)$$

$$= 1575 \text{ volts per leg or } 2730 \text{ volts between terminals}$$

$$I_{FS} = 7.5 \text{ amperes}$$

$$I_{FSI} = 35.5 \text{ amperes}$$

$$I_{FG} = 33.5 \text{ amperes}$$

Using a construction similar to that of Fig. 46, $OS = 61.8$, and hence $I_{FL} = OS + I_{FS}$ or 69.3 amperes.

The power required for excitation is

$$69.3 \times 110 \quad \text{or} \quad 7630 \text{ watts}$$

We will obtain the effective resistance, more as a means of illustrating the method than for its accuracy.

$$I^2R \text{ in three phases} = 3 \times 125.6^2 \times 0.115$$

$$= 5442 \text{ watts}$$

The total loss attributed to armature current is the I^2R loss plus the stray power loss. This is:

$$5442 + 3600 = 9042 \text{ watts}$$

$$\therefore 3I^2R_{\text{effective}} = 9042 \text{ watts}$$

$$R_{\text{effective}} = \frac{9042}{3I^2}$$

$$= 0.1912 \text{ ohm}$$

$$\text{Ratio of } \frac{R_{\text{effective}}}{R_{\text{ohmic}}} \doteq 1.662$$

This compares favorably with the ratio 1.6 mentioned for 60-cycle machines.

Summary of losses:

Friction and windage	5,050 watts
Core loss	13,900 watts
Stray power loss	3,600 watts
Power for excitation	7,630 watts
Armature copper loss	5,442 watts
Total of all losses	35,622 watts

$$\text{Output} = \sqrt{3} \times 2300 \times 125.6 \times 0.80$$
$$= 400{,}000 \text{ watts}$$

$$\text{Efficiency} = \frac{400{,}000}{400{,}000 + 35{,}622}\,100 = 91.8 \text{ per cent}$$

91. The Retardation Method. When a motor or generator is running and its driving power is shut off, the machine decreases in speed and finally comes to rest. The interval of time required to bring this about depends directly upon the kinetic energy represented by the rotating mass and inversely upon the friction or other losses which dissipate that kinetic energy. Such a curve of speed versus time is shown in Fig. 69. Its shape depends upon the variation of the friction losses with speed.

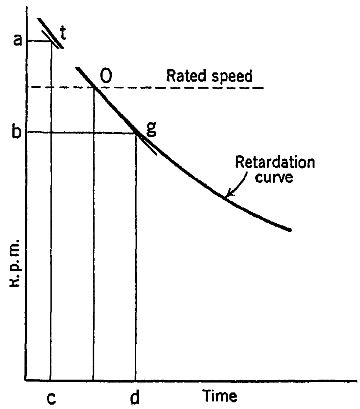


Fig. 69.

The following paragraphs will outline a method whereby such data can be used in determining the losses of an alternator. Two difficulties are encountered in applying this method: first, in determining accurately the speed-time curve; and second, in determining the "polar mass moment of inertia" of the machine. The latter quantity can be eliminated by indirect methods as will be shown.

This retardation method of loss measurement is particularly applicable to large machines which cannot readily be driven by a rated motor and in which the kinetic energy of the rotating parts is so great as to maintain speed for several minutes.[5]

92. Theory.[6] The energy stored in a rotating body at an angular velocity ω is $\frac{1}{2}J\omega^2$ where J equals the polar mass moment of inertia of the

[5] J. Allen Johnson, "The Retardation Method of Loss Determination As Applied to the Large Niagara Falls Generators," *Trans. A.I.E.E.*, Vol. 45, p. 747, 1926.

[6] Methods of calculation and test procedure are given in the Proposed Test Code for Synchronous Machines, *A.I.E.E.*, January, 1937.

rotating parts. Any decrease in the kinetic energy of the rotating parts must equal the work done in overcoming the losses during any interval of time. Let the loss for the velocity ω equal W. Hence, for the time dt:

$$Wdt = d(\tfrac{1}{2}J\omega^2)$$

When differentiated:

$$Wdt = J\omega d\omega$$

$$W = J\omega \frac{d\omega}{dt} \qquad [81]$$

The expression $d\omega/dt$ is the rate of change of angular velocity and is therefore acceleration. Or, since this analysis is to be applied to retardation, or negative acceleration, equation 81 should be written

$$W = -J\omega\alpha \qquad [82]$$

With ω expressed as radians per second and α as the radians per second per second, the moment of inertia will be in cgs units. Then by multiplying the second term by 10^{-7} the power is expressed in watts.

The value of α can be determined by the construction shown in Fig. 69. A tangent tg is drawn to this curve at rated speed. Select points t and g and draw lines at, bg, tc, and gd. Then the retardation is

$$\alpha = \frac{ba}{cd}$$

This must be interpreted in terms of radians per second per second.

93. Example. A 5 kv-a, 220-volt, three-phase, 1800-rpm alternator is driven at a speed of 2000 rpm and allowed to come to rest. Its driving motor is direct coupled, and the retardation curve a of Fig. 70 represents the friction losses in both motor and generator along with the core losses in the driving motor. (Its field was excited throughout the run.)

$$WR^2 \text{ (total)} = 22.8 \text{ lb-ft}^2$$

$$= 9{,}550{,}000 \text{ gram-cm}^2$$

For 1800 rpm:

$$\omega = 60\pi \text{ radians per second}$$

Select points to obtain the slope

$$\alpha = \frac{680}{12.3 \times 60} \text{ radians per second per second}$$

(Numbers refer to points selected on the curve.)

$$= 5.81 \text{ radians per second per second}$$

$$P_{\text{watts}} = 9{,}550{,}000 \times 60\pi \times 5.81 \times 10^{-7} \quad \text{or} \quad 1046 \text{ watts}$$

The solution of this example requires a knowledge of the moment of inertia. If, instead, the above losses can be determined by some other method, substitution in the equation will permit the moment of inertia to be determined. Additional retardation curves can be determined under the following conditions. Excite the alternator field until rated voltage is obtained at no load and rated speed. Then raise the speed and cut off the driving power, reading speed at regular intervals as the generator slows down. Such a curve is shown in *b* Fig. 70. By substituting the previously determined value for moment of inertia in

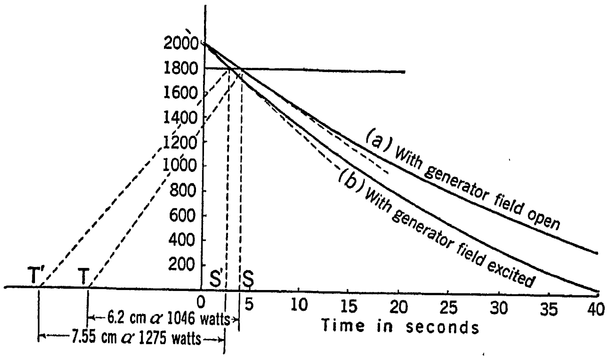


FIG. 70.

equation 81, along with the new value of α at 1800 rpm, the value of watts (W_1) now represents friction losses in both machines, core loss in the driving motor, and core loss in the alternator at that particular excitation. Then $W_1 - W$ equals the alternator core loss. Other runs can be made at different excitations to determine the corresponding losses.

If the armature is short-circuited and the field excited so that rated current flows at rated speed, a retardation curve can be used to determine the stray power losses. From the total loss (W_2) subtract the friction and calculated I^2R losses of the armature. (The core loss is negligible at this low excitation.) The difference is armature stray power loss.

94. Second Method for Using Retardation Curves. Equation 81 can also be written:

$$W = -Kn\frac{dn}{dt} \qquad [83]$$

where W = watts loss

n = rpm

K = a constant which absorbs the moment of inertia and the ratios of radians per second to revolutions per minute, etc.

This leads to another method of determining the losses from retardation curves. Refer to Fig. 71. The vertical line OS is dropped from the curve at the speed for which the losses are to be determined. Draw OT

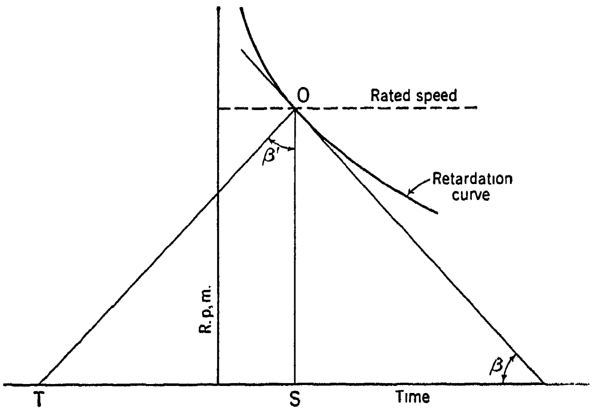


FIG. 71.

perpendicular to the tangent through O. The slope of the curve at the point O is dn/dt. Then

$$\tan \beta = \frac{dn}{dt}$$

and

$$\tan \beta' = \frac{TS}{OS}$$

Since

$$OS = \text{the speed}$$

$$TS = n\frac{dn}{dt}$$

Now since the power loss at the speed n equals

$$-Kn\frac{dn}{dt}$$

the length TS is proportional to the loss. Figure 70 shows such construction for curves obtained on the previously mentioned machine.

TS = 6.2 cm, and as determined by direct measurement the losses under the conditions of this run were 1046 watts

$T'S'$ = 7.55 cm, and since the scale has been determined this represents 1275 watts

The core loss in the alternator at this excitation is then

$$1275 - 1046 = 229 \text{ watts}$$

95. To Eliminate the Moment of Inertia. In the example given here the polar mass moment of inertia was known. In other cases it is not necessary to know this value if the losses corresponding to one of the conditions under which a retardation run was made can be calculated or measured by another method. All other losses can then be evaluated.

If the losses can be varied by a known amount the other losses can be determined by proportion.

If the moment of inertia can be varied by adding a flywheel of known moment, the moment of inertia of the entire setup can be determined.

96. Obtaining the Curves. Considerable difficulty is experienced, especially with small machines, in reading the speed at short intervals of time. It is especially important that accurate readings be obtained at and near rated speed.

(*a*) A special tachometer can be used which reads speed at known intervals of time.

(*b*) An electric tachometer can be used.

(*c*) The differential voltmeter method can be used. The curves shown here were replotted from oscillograph records made by such a method. The armature circuit of the driving motor was opened so that the set would slow down and stop, and a voltmeter and an element of the oscillograph were connected in parallel across the two opened leads. As the motor and generator slowed down, the voltmeter measured the difference between line voltage and the motor counter emf. This is proportional to the drop in speed; and one speed, say, the initial value, is all that is necessary to determine the scale.

97. Heat Runs. The difficulty and expense of providing power to large alternators by which they can be loaded to determine their temperature rise have caused the evolution of a number of methods of artificially reproducing load conditions. The temperatures of the windings, bearings, and of various other parts are then determined by means of attached thermometers, or embedded thermocouples or resistances.[7] Some of the methods of making these runs are described briefly below.

1. Hobart-Punga Method. This method makes use of alternate runs with a short-circuited and open-circuited armature. On open circuit the sources of heat are field loss, core loss, and friction and windage. The core loss is exaggerated on the open-circuit run by over-excitation.

On short circuit the sources of heat are copper loss in the field and armature, core loss, friction, and windage. The I^2R loss is exaggerated by permitting a short-circuit current to flow, larger than normal. During this period the core loss is below normal, owing to decreased excitation.

[7] See Standards of the A.I.E.E., 7–150 to 7–175.

The success of this method lies in the relative lengths of the alternate periods of open and short circuit. Mathematical formulas have been worked out [8] whereby the proper time and magnitude can be determined so that over a given test the various losses occur in the proper proportion. Owing to the difficulty of adjustment to provide proper armature copper and core losses, it is not always possible to produce normal field copper loss. In such event, the temperature rise of the field copper can readily be calculated.[9]

2. Zero Power Factor Method. This method is very satisfactory if a source of power is available equal in kilovolt-amperes to the rating of the alternator under test. It requires a power expenditure equal only to the losses in the machine, however. The alternator is run as an over-excited synchronous motor, operating at rated speed and frequency, with normal armature current.[10] Because of the over-excitation, core loss and especially the copper loss of the field winding will be excessive. By running alternately over- and under-excited, it is possible to correct the error if the relative periods of each are correct. All these methods require at least approximate knowledge of the various losses.

3. Goldsmith Method. This method employs direct current for heating the armature. The alternator is run at full-load excitation to give the correct field copper and core loss. The armature is heated by direct current, the source of which must be shielded from the high alternating voltage generated in the armature. One method of accomplishing this result is to reconnect the Y-wound armature into a delta with a corner open. The alternating voltage is practically zero across the open corner, so that direct current can be introduced. This method has the objection of giving rise to extra losses from the direct current, chiefly in the pole faces.[11]

[8] *Elec. World,* April 22, 1905.
Hobart, *Trans. A.I.E.E.*, Part II, p. 1278, 1916.

[9] Miles Walker, "Specifications and Design of Dynamo-electric Machinery," Chapter X, Longmans, Green and Co.
Humburg, "Distribution of Temperature in Field-coils of Rectangular Section." *Elek. und Maschinenbau,* Vol. 27, 1909.

[10] This method is recommended by the Proposed A.I.E.E. Test Code.

[11] For another method of making a heat run developed by Mordey and modified by Behrend, see:
Mordey, *J.I.E.E.*, Vol. 22, 1893.
Behrend, *Elec. World,* Vol. 42, Oct. 31 and Nov. 14, 1903.

CHAPTER X

UNBALANCED LOADS. VOLTAGE REGULATORS

98. Chapter Outline.

Effect of Unbalanced Loads.
Voltage Regulators.

99. Effect of Unbalanced Loads. In large central stations supplying load to many consumers the chances of obtaining unbalanced loads are much more remote than in small power plants with relatively few connected units of load. Unbalanced loads are especially likely to occur if much of the generator output is used on single-phase consuming devices such as lamps. Hence the problem of unbalance of load on the three phases of a generator is especially applicable to smaller units.

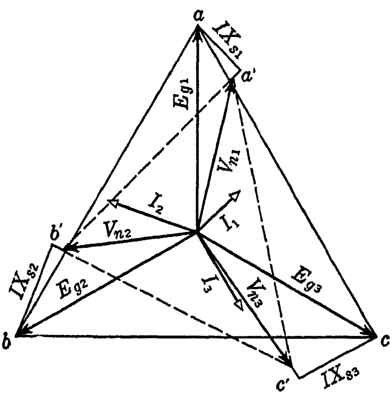


FIG. 72. Vector diagram of an alternator with unbalanced load.

As the load varies on an alternator, the automatic voltage regulator actuates the exciter output to that value necessary to maintain the correct voltage to which the regulator is adjusted. Suppose that a regulator is connected to a 2300-volt, three-phase alternator. The regulator is actuated from a potential transformer connected across the terminals of phase A. An increased load on this one phase results in the regulator's increasing the excitation I_f, thus increasing the generated voltage, from which the increased IZ_s drop is subtracted, resulting in constant terminal voltage being maintained on this phase. But an increased excitation increases the generated voltage on all three phases. Since the loads on the other phases did not increase, IZ_s is not increased and the terminal voltages are high on the lightly loaded phases.

In alternators with only 5 or 10 per cent regulation, even large unbalance in load would not cause an excessive difference in the terminal

voltage of the three phases. In modern alternators with a regulation of, say, 40 per cent, a slight unbalance in load can cause considerable difference in phase voltages. This is illustrated in Fig. 72.

It has been pointed out in the discussion of armature reaction that a single-phase alternator builds up two components of armature reaction: one fixed with reference to the field; the other rotating in the opposite direction at twice synchronous speed with respect to the field poles.

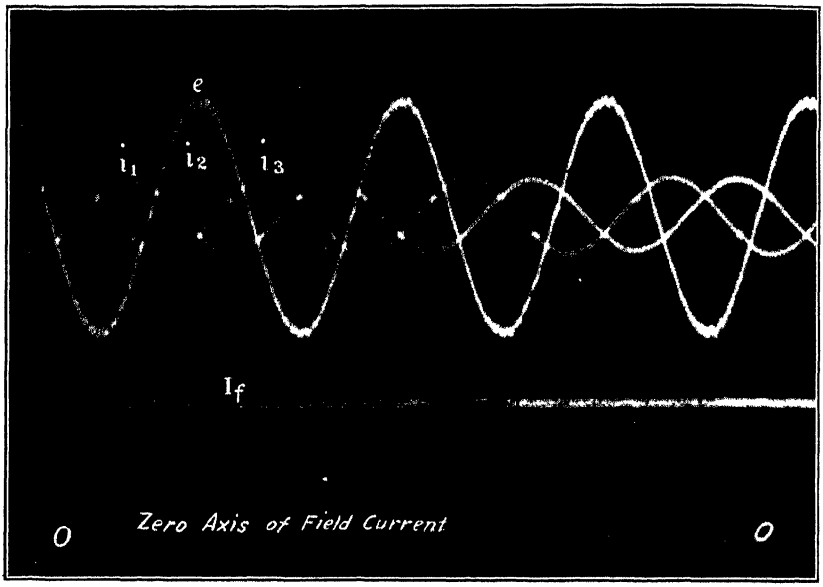


FIG. 73. Oscillograph record of a three-phase alternator, showing one of the line voltages, the three balanced line currents, and the d-c field amperes.

This variable component results in a double-frequency pulsation of the main flux and in increased iron losses. It also introduces a double-frequency current in the field circuit. Excessive heating often occurs in the field structure. If damper windings are added to the rotor to minimize armature reaction, violent vibrations may occur in the frame and foundation under certain conditions.[1]

In a polyphase alternator, carrying a balanced load, the variable components of armature reaction from the different phases are balanced out by each other, and the component of the reaction which is rotating at synchronous speed with respect to the poles is all that remains. This is

[1] B. G. Lamme, "Dampers on Large Single-Phase Generators," *Electrical Engineering Papers*, Westinghouse Electric & Mfg. Co., E. Pittsburgh, p. 139, 1919.

why we are able to say that armature reaction in polyphase machines is fixed with reference to the poles.

Now, considering a polyphase alternator with unbalanced load, the variable components of armature reaction do not cancel, and a double-frequency flux sweeps across the pole faces (Fig. 74.) The resulting increase in losses may cause dangerous heating of the entire pole core and

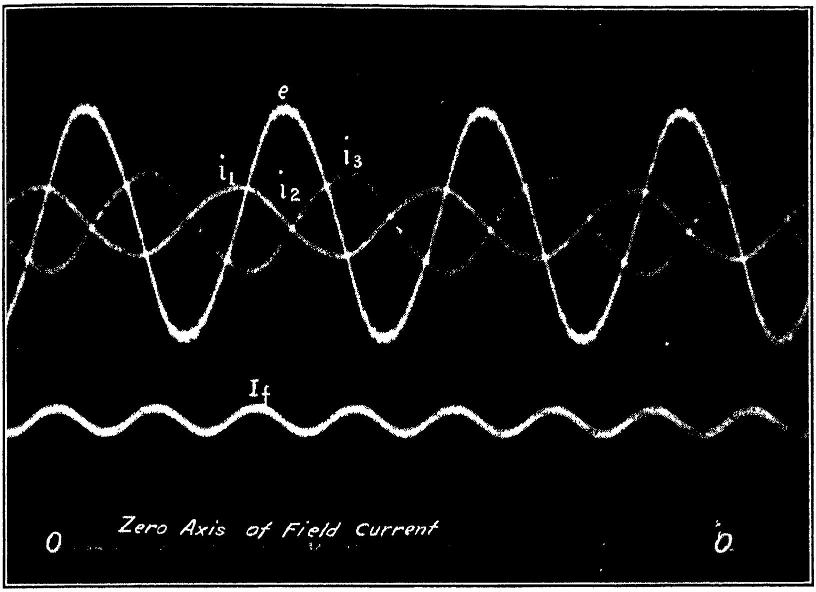


FIG. 74. Oscillograph record of a three-phase alternator, showing one of the line voltages, three unbalanced line currents, and the second harmonic current in the d-c excitation.

winding. This is not an infrequent cause of the "burning out" of field windings.

100. Voltage Regulators.[2] No attempt will be made here to describe the theory and construction of machine voltage regulators beyond the brief analysis of operation given below.

The type which has probably found the greatest application in America was developed by A. A. Tirrill. On the ordinary setup a d-c generator, known as the *exciter*, supplies the direct current needed to excite the field of the alternator. A field rheostat in series with the

[2] J. G. Tarboux, "Electrical Power Equipment," Chapter V, p. 131, McGraw-Hill Book Co.

S. Q. Hayes, "Switching Equipment for Power Control," Second Edition, Chapter XVI, p. 247, McGraw-Hill Book Co.

exciter and field needs occasional adjustment, but is ordinarily fixed at some value. Control can be obtained by adjusting the field rheostat of the exciter, which in turn varies the exciter voltage and the alternator field current. Next, imagine an exciter field rheostat set at such value of resistance that the exciter voltage is too low to maintain the proper field current to give rated alternator voltage. If this exciter field rheostat is short-circuited, the field current of the exciter, its voltage, and the

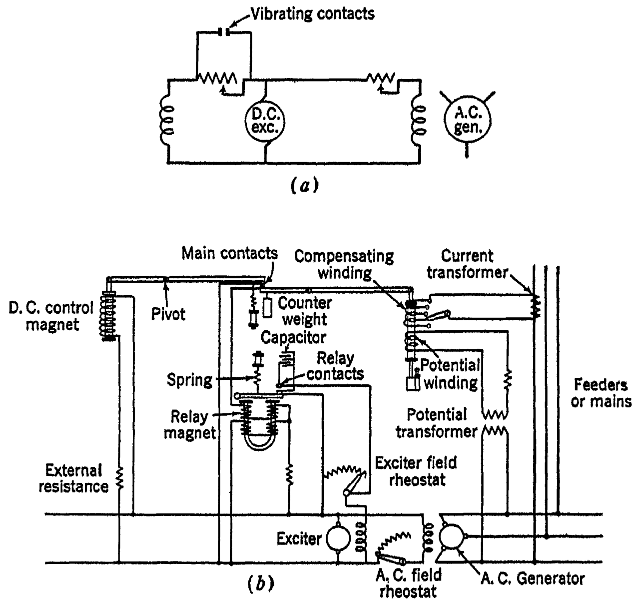
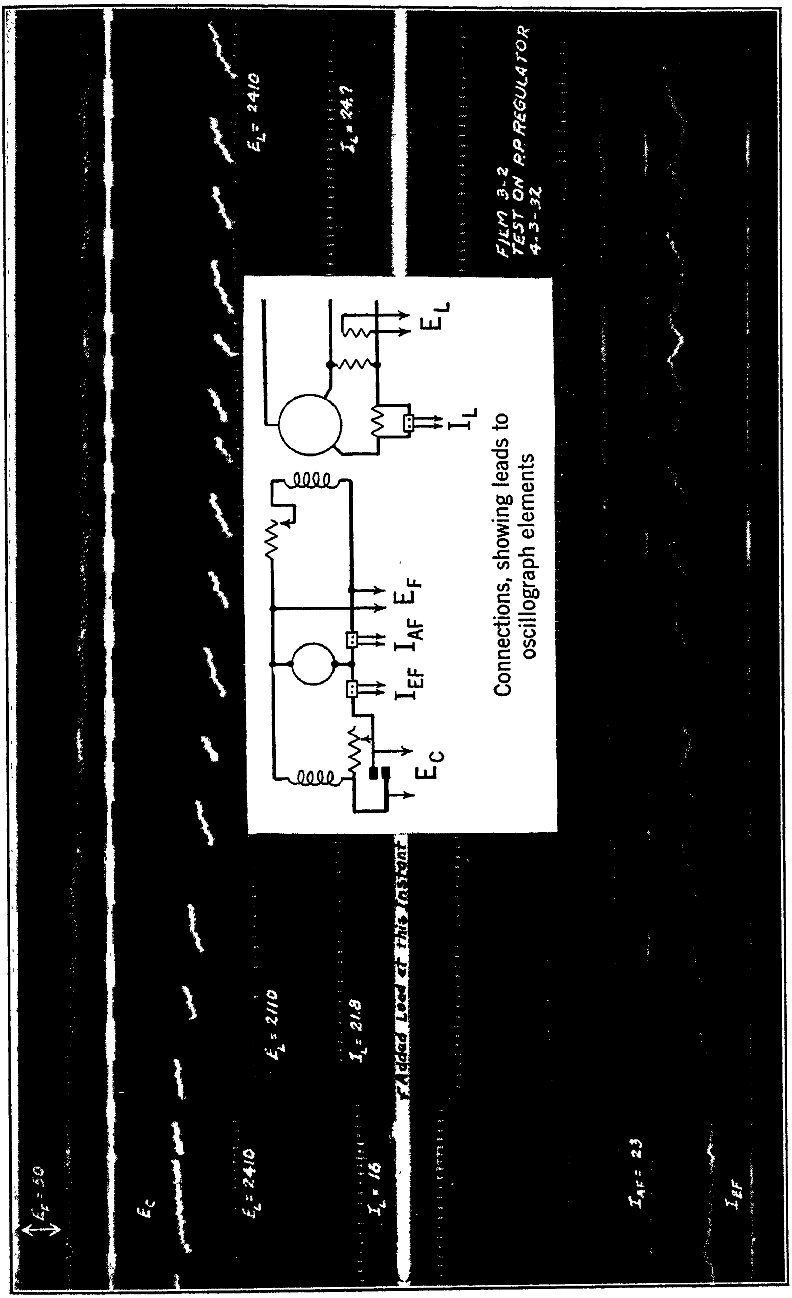


FIG. 75. (*a*) Machine circuits upon which the automatic regulator operates. (*b*) Schematic diagram of connections for the vibrating type of regulator.

field current of the alternator and its voltage, all rise above the normal values. By making the short circuit around the field rheostat alternately open and close, the field can be kept in alternate states of growth and decrease, with an average value which results in rated alternator voltage at all loads. This idea is illustrated in Fig. 75. The schematic diagram of connections with the contact actuating relays is shown also. The actual apparatus possesses a number of refinements not shown here.

A second type of regulator employs what is known as *rheostatic control.* Its operation duplicates that of an attendant by adjusting the field rheostat of the alternator. A reduced alternator voltage actuates a motor drive, swinging the field rheostat arm so as to reduce its resistance. This increased field current brings the alternator voltage back



E_F = 50
E_C
E_L = 2410
I_L = 16
E_L = 2110
I_L = 21.8
E_L = 2410
I_L = 24.7
FILM 3-2
TEST ON P.P. REGULATOR
4-3-32
I_AF = 23
I_EF
E_L
I_L
E_F
I_AF
I_EF
E_C
Connections, showing leads to oscillograph elements

FIG. 76.

to normal, at which point the rheostat arm remains stationary until further load deviation.

In recent years considerable development work has been done on electronic regulators.

101. Oscillogram of Vibrating-type Regulator Operation. An oscillograph record of the vibrating-type regulator is shown in Fig. 76. This regulator was operating on a three-phase, 60-cycle, 2400-volt, 186-kv-a alternator. The exciter was rated at 11 kw, 125 volts. At a load of 16 amperes (I_L) the terminal voltage was 2410 (E_L). At this small load the exciter voltage (E_F) required to supply the alternator field current (I_{AF}) is comparatively low, and the contacts shorting the exciter field rheostat remain open for longer intervals of time. Note that E_C is the voltage across contacts. This is used merely to indicate whether the contacts are open or closed.

An induction motor is thrown on the 2400-volt lines at the instant shown. Alternator regulation is such that the terminal voltage immediately drops to 2110 volts between lines. The load current is then 21.8 amperes.

When the regulator contacts are closed, the exciter field current, I_{EF} (about 1 ampere), increases, increasing the exciter voltage E_F, which in turn forces an increased current I_{AF} through the alternator field circuit and raises the voltage. When the contacts are open, the exciter field current reduces; repetition of the cycle results in the saw-toothed effect shown for I_{EF}. (This type of regulator does not maintain its contacts closed until the voltage is restored to normal.[3]) A net rise in voltage is accomplished by a change in the relative times during which the contacts are open or closed. That change is clearly indicated here. The exciter response is determined largely by the inductance of its field circuit.[4] In this case approximately 2 seconds are required to restore the voltage to normal after load increase.

102. Bibliography on Electronic Regulators.

"An Electronic Regulator for an Alternator," by C. C. Whipple and W. E. Jacobsen. *Elec. Eng.*, June, 1935.

"An Electronic Voltage Regulator," by P. H. Craig and E. F. Sanford. *Elec. Eng.*, February, 1935.

"An Electronic Regulator for A-c Generators," by F. H. Gulliksen. *Elec. Eng.*, June, 1934.

[3] C. A. Nickle and R. M. Carothers, "Automatic Voltage Regulators," *Trans. A.I.E.E.*, Vol. 47, p. 957, July, 1928.

[4] C. A. Boddie and F. L. Moon, "The Application of D. C. Generators to Exciter Service," *Trans. A.I.E.E.*, Vol. 39, Part II, p. 1595, 1920.

TRANSFORMERS

CHAPTER XI

TRANSFORMER CONSTRUCTION

103. Chapter Outline.

Transformer Construction.
- Types of Core.
- Core Losses.
- Windings and Insulation.
- Cooling Methods.
- Transformer Oil.
- Bushings.

Standards.

104. The Transformer. The transformer consists of an iron core about which are wound two sets of windings. Power is supplied to one set of windings, called the *primary*, which builds up a magnetic flux through the iron. This flux generates a counter emf in the primary and limits the current which can be drawn from the supply. The same flux system generates an emf in the second winding, or secondary, to which the load is connected. This secondary emf causes current to be supplied to the load. Hence power is transferred electromagnetically from the source to the secondary winding and load.

The ability of this comparatively simple machine to transform power input at one voltage to a different voltage on the output side makes it of tremendous importance to the electrical industry. The power lost in such transformation is practically insignificant.

The ratio of the primary and secondary voltages depends upon the ratio of turns on the respective windings. Step-up transformers have more turns on the secondary than on the primary; step-down transformers have the reverse ratio.

Some idea of the effectiveness at which these transformations take place can be gained from Table VIII.

105. Shell and Core Types. The iron laminations through which the flux is built up can be constructed in different ways with respect to the

TABLE VIII

TYPICAL CHARACTERISTICS *

(60-cycle, 2300- to 230/115-volt Transformers)

Kv-a	Iron Loss, Watts	Copper Loss, Watts	Percentage Efficiency (Load)				Percentage Regulation	
			$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{4}$	Pf = 1	Pf = 0.8
1	21	29	91.6	94.1	95.2	95.2	3.1	3.2
5	44	99	95.1	97.2	97.4	97.2	2.1	2.8
10	74	168	96.7	97.7	97.8	97.6	1.8	2.7
15	103	222	96.9	97.9	98.0	97.8	1.7	2.6
25	137	367	97.4	98.1	98.1	98.0	1.6	2.6
50	258	607	97.6	98.3	98.4	98.3	1.3	2.6
75	460	825	97.3	98.2	98.3	98.3	1.2	2.5
100	615	1120	97.3	98.2	98.3	98.3	1.3	2.8
200	1130	2400	97.4	98.2	98.3	98.2	1.3	3.1

* Two classes of transformers are usually obtainable for distribution systems: (1) high efficiency; (2) reduced efficiency. The former uses high-grade silicon steel to reduce the iron losses. This type has a greater first cost. The more economical type for a certain setup is determined from the cost of power lost, period of service, and relative first costs. Consequently each type is justifiable economically for its own field.

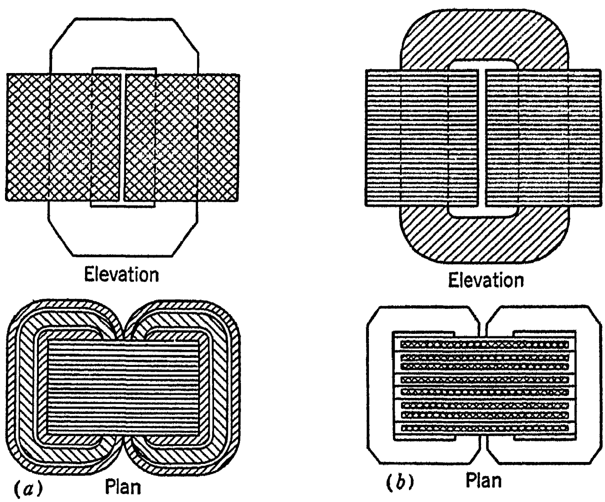


FIG. 77. (a) The simple rectangular core type of construction. (b) The shell type.

winding. Figure 77*a* shows the core type in which the iron forms a core linked by all the magnetic flux set up by the emf impressed on the primary. The shell type of 77*b* consists of two or more paths through which the flux divides. In a sense, the iron forms a shell about the windings. Other arrangements are shown in Fig. 78.

These constructions can be designed of such proportions as to give similar characteristics.

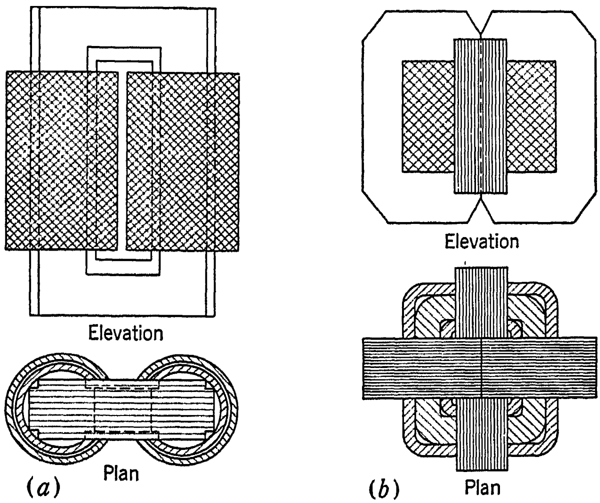


FIG. 78. (*a*) The cruciform core permits circular coils to be used. (*b*) Distributed shell construction.

106. Transformer Cores. The magnetic core is built up of laminations of high-grade silicon or other sheet steel, usually insulated from one another by varnish, though frequently the surface coating of oxide and possibly occasional layers of paper are relied upon to keep down eddy losses. The usual thickness of laminations for 60 cycles is 0.014 inch, corresponding to 29 gauge. Thicknesses of 0.020 inch are common for 25-cycle transformers; other thicknesses are much used also. For the influence of thickness of laminations upon eddy-current losses see Article 141.

Two losses occur in the iron core, owing to the varying flux. These are eddy-current and hysteresis losses. The silicon content of the iron and the nature of the annealing are very important in determining the hysteresis loss. Up to 4 per cent silicon is used in the best grade of annealed sheets. (See Article 141.) This makes the material hard to punch and very brittle.

In recent years the wound-core transformer has been used as shown in Fig. 79*a*. It is wound of one strip of sheet steel, presenting an economy over the usual lamination construction. The whole magnetic path is active, with only one effective air gap of large area and short length as shown in Fig. 79*b*. A reduction in core loss and an increase in permeability is claimed through the flux running with the grain direction.[1]

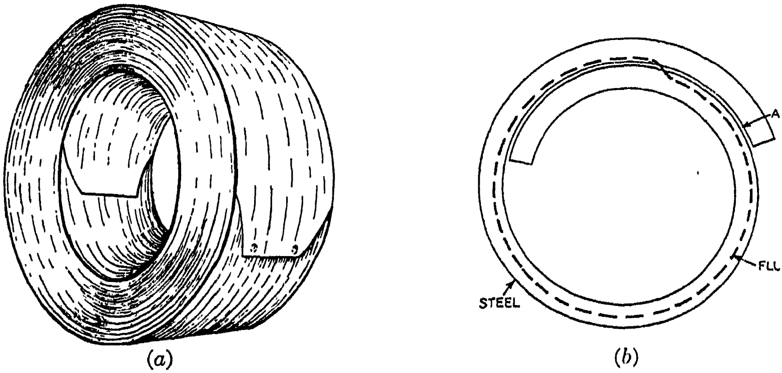


FIG. 79. A wound core is first prepared as shown in *a* from a precut, cold-rolled strip of low silicon content. This is rewound through the prepared coils which form the electric circuit. Flux path is shown in *b*.

107. Windings and Insulation. The windings of transformers are built up of copper wire or strap. Heavy current capacity requires conductors of large cross-section. To reduce the eddy-current losses within the conductors, several small wires or paralleled straps are preferable to one large strap. This gives rise to unequal reactance of the components of the conductor and can be eliminated only by transposing the conductors.

Two types of coil construction are commonly used. These are (1) concentric, and (2) "pancake" or interleaved. In each, spacers are provided between adjacent coils to permit ventilation or to aid in the dissipation of heat.

Double cotton, single cotton with an underlayer of enamel, or synthetic enamel insulation is most commonly used as conductor covering, particularly for wire windings. In addition, strips of insulating paper are placed between layers, and the completed coil is taped and impregnated with insulating compound.

The end turns on the coils of a high-voltage transformer require special insulation. This is necessary because of the distributed capacity of

[1] E. D. Treanor, "A Wound-core Distribution Transformer," *Elec. Eng.*, November, 1938.

the transformer windings which exposes the end-turn insulation to great voltage strains when the transformer is switched on and when surges

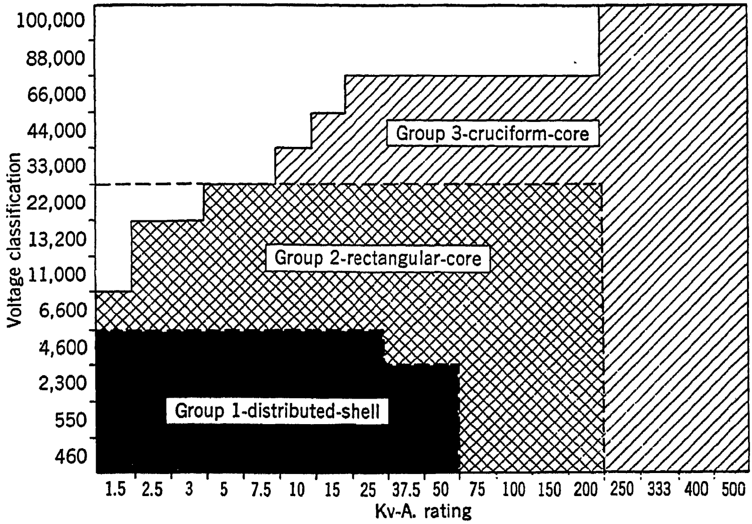


FIG. 80. Sizes and construction types of single-phase transformers. (*Westinghouse Elec. and Mfg. Co.*)

occur on the line. A simple circuit, representing this effect, is shown in Fig. 81. The capacity effects between turns and between high and low potential windings are omitted for simplicity.

Circuit *ab* represents the primary coils connected to the line through switch *S*. The capacity between each turn and core is represented by the condensers. When switch *S* is closed, if the instantaneous direction of flow is from *a* to *S*, condenser 1 will be charged to line potential. The inductance of the winding prevents the instantaneous flow of current directly through the turns, and condenser 2 will not be charged until an instant later. During this instant, full line potential will exist across this end turn, and the insulation will be subjected to great strain. A high voltage will be induced in an adjacent secondary turn.

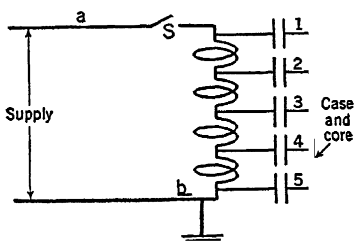


FIG. 81. A transformer winding, showing distributed capacity between turns and the transformer case and core.

As this phenomenon may exist at either end of the winding, extra

insulation must be provided on about 10 per cent of the turns to protect them during this transient condition.

Some idea of the space taken up by insulation around the conductors and coils can be gained from what is known as the copper space factor. This is the ratio of net copper cross-sectional area to area of the window in the transformer core. It varies from 50 per cent in low-voltage transformers to less than 10 per cent for voltages of 66,000.

108. Cooling Methods. All the losses in a transformer are dissipated as heat from the windings and core. The losses are small, but, owing to the compactness of design, unsafe temperatures will be reached if special means are not provided for cooling.

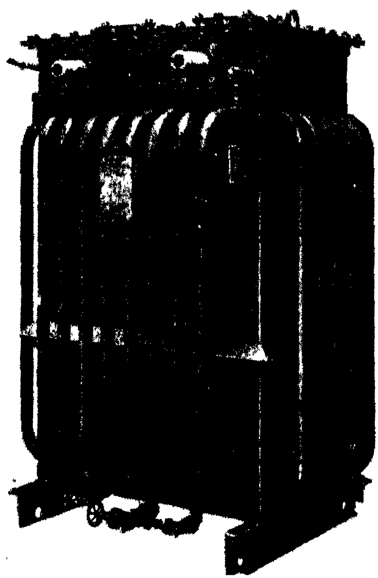


FIG. 82. Transformer with cooling radiators. (*Westinghouse Elec. and Mfg. Co.*)

In most transformer designs the watts loss per pound in the iron and the watts loss per pound in the copper have rather narrow limits. This means that the losses to be dissipated vary, roughly, as the volume of the material, i.e., as l^3, l representing one dimension of the transformer. On the other hand, the cooling surface of a transformer varies as l^2. As transformers increase in size the ratio of heat generation volume to surface for dissipation (l^3/l^2) becomes very large.

For small transformers a smooth case readily dissipates the heat; but, as can be seen from the above ratio, the problem of getting rid of the heat in large transformers is more difficult. This will explain the progressive design with increasing transformer size, of smooth tanks, fluted tanks, tubular construction, radiator construction, and finally in the largest sizes, the necessity of artificial cooling.

Natural Radiation. Very small transformers for metering and power uses are cooled by natural radiation and convection of heat from their surfaces.

Oil-immersed, Self-cooled. The transformer is immersed in a tank filled with oil. Heated oil rises through the circulating ducts of the winding and cools on its downward path against the sides of the tank. The oil

gives up its heat to the sides of the tank, from which it is then radiated to the air. Large capacities require corrugations on the surface of the tank or radiating jackets to increase the surface area. Figure 82 illustrates one of these latter designs.

Oil-immersed, Water-cooled. Instead of depending entirely upon the conduction of the heat from the oil to the outside surface of the tank, some of the heat can be dissipated from the oil by coiled tubes in the top of the transformer tank. Circulating water is forced through these coils. Occasionally the oil is circulated and cooled outside of the transformer.

Air Blast. Instead of immersing the transformer in oil, the heat is dissipated by a blast of air forced through special ventilating ducts in the core and between sections of the winding. This method of cooling requires a supply of clean air, and fans and special construction to assure its correct distribution. Its advantage lies in reduced fire and explosive risks. This type of ventilation is confined to systems of 25,000 volts or less.

109a. Transformer Oil. The selection of oil used for cooling and insulating transformers is of great importance. Its desirable characteristics include: high dielectric strength; freedom from moisture and particles in suspension; absence of alkalies, acids, and sulphur; low viscosity; and low sludging tendencies.[2]

Chemical impurities attack the materials comprising the transformer or tank; sludge and sediment increase the viscosity and tend to clog oil ducts. Moisture and minute suspended particles seriously decrease the dielectric strength.[3]

Great care is taken in filling transformers to preclude the presence of moisture in the tank through condensation or other causes. In operation, water may contaminate the oil as a result of the breathing of the transformer through leaky covers, or of leaky cooling coils.

In recent years widespread use has been made of a new synthetic dielectric. This is a non-inflammable derivative of organic hydrocarbons, in which chlorine has been substituted for hydrogen in the hydrocarbon molecule. The dielectric constant is unusually high, being about equal to that of cellulose. The breakdown strength is greater than that of mineral oil. This dielectric is marketed under the trade name of Pyranol.[4]

109b. Transformer Bushings. One of the important problems in transformer design is that of getting the leads from the external circuit

[2] See Standards of the A.I.E.E. and bulletins of the American Society for Testing Materials.

[3] Dean Harvey, "Electrical Insulating Oils," *Elec. J.*, February and March, 1928.

[4] F. M. Clark, "The Dielectric Strength of Non-Inflammable Synthetic Insulating Oils," *Elec. Eng.*, June, 1937.

into the transformer case. On lower voltages this has been accomplished by using bushings of porcelain around the conductor. But as voltages increased, it was found necessary to increase the bushing sizes to large proportions. In modern transformers the problem is met by the use of large porcelain or composition bushings up to about 33,000 volts; above that voltage the condenser and oil-filled types are used.

110. Ratings and Standards. As an aid to the purchaser of transformers, the N.E.M.A. has standardized on transformer ratings and types. The reader is referred to their bulletin on standards as well as to those of the A.S.A. and the N.E.L.A. for more complete details. These standards apply to power and distribution transformers, covering standard ratings, percentage taps to be provided, etc.

Voltage Ratings. Standard values for transformers have been set up as follows:

Distribution types: *440*; 550; *2300*; *4000*; 4600; 6600; 11,000; *13,200*; 22,000; *33,000*; 44,000; *66,000.*

Power types: *2300*; *4000*; 4600; 6600; 11,000; *13,200*; 22,000; 33,000; 44,000; *66,000*; 110,000; *132,000*; 154,000; *220,000*; *330,000.* Values in italics are recommended, suggesting the trend for future practice.

CHAPTER XII

ELECTROMOTIVE-FORCE EQUATION. VECTOR DIAGRAMS

111. Chapter Outline.

The Equation of Electromotive Force.
Ratios of Transformation.
  Turns.
  Volts.
  Amperes.
Flux: Mutual and Leakage.
Vector Diagrams.
  No-load.
  Load.
Equivalent Resistance and Reactance.

112. Electromotive-force Equation. When a coil of wire is connected to a source of alternating current, the reactions built up in the coil must be equal and opposite to the applied voltage. If the resistance of the coil is small, practically all the applied voltage is used up in overcoming the self-induced emf generated by the alternating flux cutting through the turns of the coil. This reaction is

$$e_{\text{self-induced}} = -N\frac{d\phi}{dt}10^{-8}\text{ volt} \qquad [84]$$

where N is the number of turns on the coil.

If the flux is assumed to follow the sine law, its instantaneous value is

$$\phi = \phi_{\max} \sin 2\pi ft$$

and from equation 84,

$$e_{si} = -N\frac{d(\phi_{\max}\sin 2\pi ft)}{dt}10^{-8}$$

$$= -N\phi_{\max}\cos 2\pi ft \times 2\pi f \times 10^{-8}$$

To obtain the effective value of this expression, divide by $\sqrt{2}$ and take the maximum value of the term cos $2\pi ft$; thus:

$$E = \frac{2\pi}{\sqrt{2}}fN\phi_{\max}10^{-8} \qquad [85]$$

The cosine term has no significance except to derive instantaneous values. It is not needed to obtain the effective value, but it shows that since the flux wave was assumed to be a sine function and the voltage wave appeared as a cosine function, there exists a 90° relationships between ϕ and E. This will appear on the vector diagrams.

From equation 85 the expression for effective values becomes:

$$E = 4.44 f N \phi_{\max} 10^{-8} \tag{86}$$

or

$$E = 4(\text{form factor}) f N \phi_{\max} 10^{-8} \tag{87}$$

The flux is kept as its maximum value in this equation since this is the most convenient way to designate it. It will be noted that the final emf equation is the same (except for the distribution and pitch factors) as that for an alternator. So long as we assume a sine wave of flux cutting through the coils it does not matter whether the cutting is accomplished by actually moving a constant field of sine distribution in space or by having the flux increase and decrease according to the sine function in time.

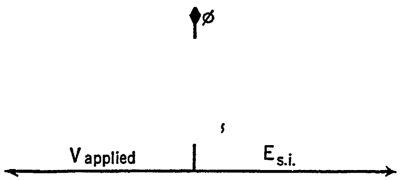


FIG. 83. Elementary vector diagram of an ideal reactance coil.

The vector diagram for a perfect reactance coil with no appreciable resistance is shown in Fig. 83. The flux will be treated as a vector and is 90° behind the applied voltage, or in phase with the magnetizing current I_ϕ. The self-induced voltage is equal and opposite to the applied voltage, and the current through the coil lags the applied voltage by 90°.

Let us next consider a coil on an iron core, the winding resistance being taken into account. Since the effective applied voltage must at all times equal the vector sum of the effective reactions through the circuit,

$$V_{\text{applied}} = I_n r + (-E_{si})$$

The minus sign indicates that the self-induced voltage has been rotated through 180° in order to put it on the same side of the diagram with the applied voltage, inasmuch as the self-induced voltage is a counter emf which must be overcome.

In Fig. 83 the current and the applied voltage are 90° out of phase, the pf is zero, and no power is consumed by this ideal coil, although it alternately absorbs and gives up power. Actually, with an iron core in the transformer the varying flux sets up eddy-current and hysteresis losses in the iron, hence the current must be enough in phase with the applied voltage to give a power loss equal to the iron losses. Refer to

Fig. 84. The current taken by this iron-cored reactance is I_n. It can be considered as being made up of two components: I_ϕ is the magnetizing component, i.e., it is the current necessary in the coil to build up ϕ_{max} through the core; ϕ_{max} in turn is fixed as the flux necessary to equation 87 in order that the self-induced voltage will be equal, or nearly equal, to the applied voltage. Hence, assuming that the reluctance is constant over the cycle,

$$\phi = \frac{0.4\pi N I_\phi}{\text{reluctance of the magnetic circuit}} \qquad [88]$$

$$\text{Reluctance} = \frac{\text{length of magnetic path in centimeters}}{\text{area in square centimeters} \times \text{permeability}} \qquad [89]$$

where I_ϕ is the magnetizing current needed to build up ϕ lines. Hence any factor which tends to increase the reluctance of the flux path will also increase the required magnetizing current.

The other component of the no-load current is I_{h+e}. This component will be considered as in phase with $-E_{si}$ and when multiplied by that voltage gives the watts iron loss.

Since the $I_n r$ drop is always in phase with the current, the position of $I_n r$ is determined by the relative values of I_ϕ and I_{h+e}. The power factor of the reactance coil is cos θ.

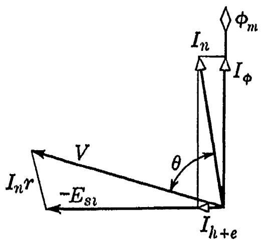


FIG. 84. Vector diagram of reactance coil with iron losses and resistance drop.

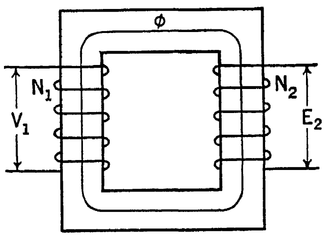


FIG. 85.

113. The Elementary Transformer. Reactance coils such as have been described are valuable for voltage reduction, for the protection of electrical circuits, and for increased stability of electrical systems. However, we are interested in them here only as a first step in the understanding of transformer action.

To an iron-cored reactance, a second winding is added in Fig. 85. The first winding N_1 is connected to the source; the secondary N_2 is connected to the load. These windings are shown on separate legs. Actually, to keep down magnetic leakage, they are wound on the same leg.

Examination of the formula for induced voltage in N_1 shows that, for a given winding connected to a constant-voltage source, a definite self-induced voltage must be built up, approximately equal to the applied voltage.

$$E_{si} = 4.44 f N \phi_{max} 10^{-8} \tag{90}$$

Assuming that the same flux cuts all the windings on the core, then a voltage equal to E_{si} would be built up in any other coil on this iron core, having the same number of turns. That is, a definite number of volts will be built up per turn, by the flux. The ratio of voltages becomes

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \tag{91}$$

where E_1 and E_2 are the voltages induced in the primary and secondary windings, respectively. This ratio of transformation is denoted as a, and in the pages which follow it will be used as greater than unity. (Thus 2, rather than $\frac{1}{2}$.)

114. Current Ratios. When an impedance is connected across the secondary leads a current flows through this load from N_2. Neglecting the small losses of the transformer itself, the input equals the output,

$$E_1 I_1 \approx E_2 I_2$$

or

$$V_1 I_1 \approx V_t I_2$$

and

$$\frac{V_1}{V_t} \approx a \tag{92}$$

more exactly,

$$\frac{E_1}{E_2} = a$$

(V_1 and V_t refer to the terminal values of primary and secondary voltage, respectively, distinguishing from E_1 and E_2 which imply induced voltages.)

If, in addition, the exciting current is neglected, then

$$\frac{I_2}{I_1} = a \tag{93}$$

The primary and secondary currents vary in inverse ratio to the voltages or turns. This relationship, however, depends upon another reaction as well.

When a secondary current flows, the turns of the secondary build

up an mmf, N_2I_2. This force is in such a direction as to oppose that of the primary N_1I_1. Since the flux ϕ_m is fixed as the value necessary to build up the required counter emf it must remain constant. Consequently the effective NI around the flux path must remain constant; any increase in demagnetizing N_2I_2 must be balanced by an increase in N_1I_1, and if the ampere turns required for magnetization of the core are neglected,

$$N_1I_1 = N_2I_2 \qquad [94]$$

or

Magnetizing action = demagnetizing action

Then

$$\frac{N_1}{N_2} = \frac{I_2}{I_1} = a \qquad [95]$$

We shall see later that these ratios are not strictly true except for an ideal transformer, with no voltage drops, no core loss, and no magnetizing current.

115. Leakage Flux. When the primary winding is excited so that a flux is built up through the iron core, not all the flux goes around the magnetic core. That is, not all the flux cutting through the primary turns links with the secondary winding. The flux which links both primary and secondary is called the mutual flux, and its maximum value is abbreviated ϕ_m; the flux which cuts only the primary turns is called the primary leakage flux (ϕ_{l1}).

When the secondary winding is connected to a load so that a current of I_2 amperes flows, the ampere turns of the secondary oppose those of the primary. This results in the action previously described, but it also results in a leakage flux being built up around the secondary turns which does not link with the primary. An increased current increases the leakage flux in direct proportion.

To sum up: There are two mmf's in a transformer: that of the primary, N_1I_1; that of the secondary, N_2I_2. There are three fluxes: the mutual flux, linking both primary and secondary; the primary leakage flux, ϕ_{l1}, linking only the primary turns and varying with I_1; the secondary leakage flux, ϕ_{l2}, linking only the secondary turns and varying with I_2. Of course, not all turns are interlinked with the same leakage flux. Accordingly, the values ϕ_{l1} and ϕ_{l2} are equivalent values.

Each of these fluxes builds up a voltage according to the equation

$$E_{si} = 4.44 f \phi_{\max} N 10^{-8}$$

Just as the fluxes are considered as being made up of components, the voltages they induce are separated into their respective parts.

The voltages caused by leakage fluxes can be replaced by equivalent IX drops. This can be done because the leakage flux and consequently its induced voltage vary in proportion to I. Hence

$$4.44 f \phi_{\text{leakage}} N 10^{-8} = IX_l$$

where X_l is the leakage reactance of the winding. It is comparable with the leakage reactance of an alternator armature.

116. No-load Vector Diagram. If the secondary of a transformer is open the vector diagram is as shown in Fig. 86. In this diagram, E_2 is the voltage built up in the secondary winding. It is caused by the

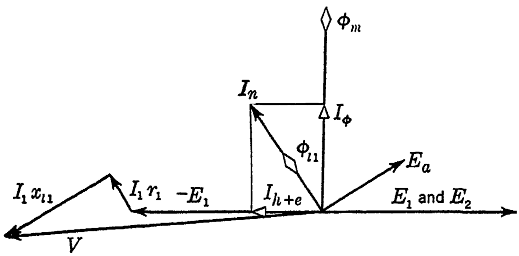


FIG. 86.

mutual flux ϕ_m which is the same flux that builds up the counter emf E_1 in the primary. A magnetizing current I_ϕ is necessary to produce ϕ_m. Core losses require a component of current in phase with the voltage, $-E_1$. This component, I_{h+e}, added vectorially to the magnetizing component I_ϕ, results in the no-load current I_n. The leakage flux reaches its maximum at the same time as I_n; consequently it is in time phase with I_n and is shown as ϕ_{l1}. The leakage flux builds up a voltage E_a, 90° behind it, in the same manner that E_1 or E_2 is built up 90° behind ϕ_m. E_a is replaced by its equivalent $I_1 x_l$; or strictly speaking, considering E_a as a counter voltage to be overcome, $I_1 x_{l1}$ is equal and opposite to E_a.

The resistance of the primary winding causes the voltage drop $I_1 r_1$ in phase with the primary current I_n. On account of the relationship between I_n, ϕ_{l1}, and E_a, the Ix_{l1} drop will always be shown as 90° ahead of $I_1 r_1$. The applied voltage V is used up in overcoming the leakage reactance and primary resistance drop and the induced voltage $-E_1$ in the primary winding.

117. The Vector Diagram of a Loaded Transformer. In this diagram (Fig. 87) the following symbols are used:

V = the voltage applied to the primary
ϕ_m = the mutual flux 90° from E_2 or E_1

I_n = the no-load current taken from the line by the primary winding
I_2 = the secondary, or load current
I_b = the current required in the primary to balance the secondary load current
I_1 = the total current taken by the primary
ϕ_{l1} = the primary leakage flux
ϕ_{l2} = the secondary leakage flux
E_a = the voltage generated by the primary leakage flux
E_b = the voltage generated by the secondary leakage flux
E_a is equivalent to $-I_1x_{l1}$
E_b is equivalent to $-I_2x_{l2}$
E_2 = the voltage generated in the secondary by the mutual flux
V_t = the terminal voltage applied to the load
θ_2 = the pf angle of the load

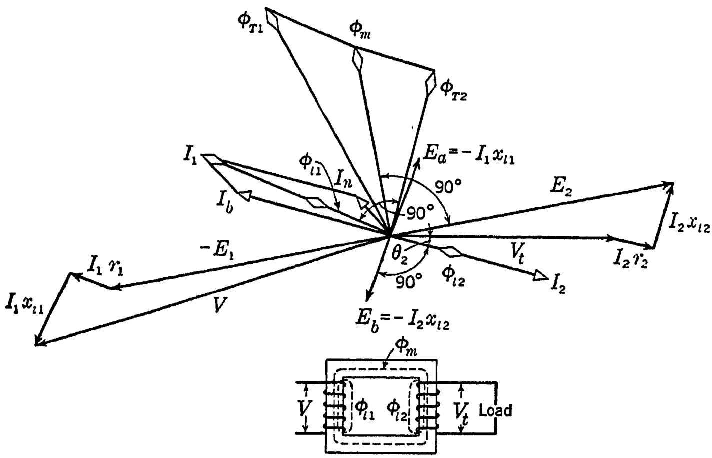


FIG. 87. Vector diagram of a transformer, showing the components of flux and the individual voltages which they induce.

This diagram can be simplified to that of Fig. 88 which shows the more conventional diagram with some of the components neglected. In this diagram the relationships existing in the loaded transformer can be built up as follows:

If the secondary current is I_2 and its terminal pf angle is θ_2 (with the rated terminal voltage V_t) then, adding the secondary resistance drop I_2r_2 and the secondary leakage drop I_2x_2, vectorially, gives the secondary induced emf E_2. This is induced by the mutual flux which also induces in the primary winding the voltage $-E_1$. Numerically $-E_1$ is

equal to aE_2. To set up the mutual flux ϕ_m requires the exciting current I_n which leads ϕ_m by the core-loss angle ψ.

The ampere turns set up in the secondary winding by I_2 must be balanced and opposed by the primary current I_b, equal to I_2 divided by a. The resultant primary current I_1 is then the vector sum of I_b and I_n. Adding the primary resistance drop I_1r_1 and primary leakage reactance drop I_1x_1 to the emf $(-E_1)$ then gives the necessary primary terminal voltage. Of course, in operation the primary terminal voltage is the

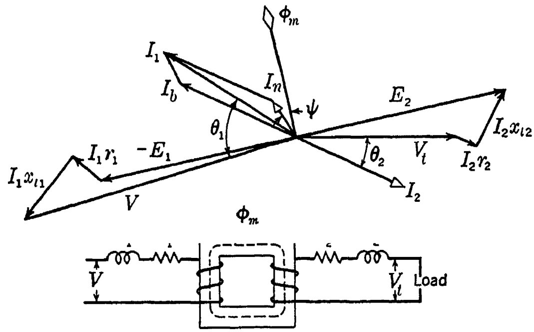


FIG. 88.

supply, and when the reactions or voltage drops occur, as explained above, the secondary terminal voltage is the dependent value.

The true ratio of transformation is defined by the Standardization Rules of the A.I.E.E. as the *turn ratio*. That is:

$$\frac{N_1}{N_2} = a$$

Consequently

$$a = \frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{I_2}{I_b}$$

Since the terminal voltage is nearly equal to E_2, and the applied voltage is nearly equal to $-E_1$, the ratio can be written as

$$a \approx \frac{V}{V_t} \text{ (approximately)} \qquad [96]$$

Similarly

$$a \approx \frac{I_2}{I_1} \text{ (approximately)} \qquad [97]$$

These ratios, already given in equations 92 and 95 though not strictly true, are useful and accurate enough for most purposes.

118. Equivalent Values. A given impedance will not produce the same effect when placed in the primary or the secondary circuit of a transformer, unless the ratio of transformation is unity.

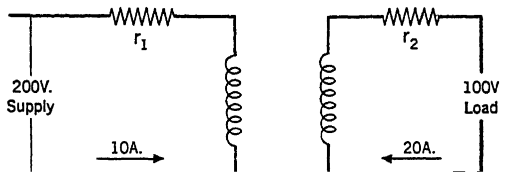


FIG. 89.

Refer to Fig. 89 for diagram and assumed values. Resistances r_1 and r_2 are inserted in the primary and secondary circuits, respectively. The per cent voltage drop due to r_1 is:

$$\frac{I_1 r_1}{V} = \frac{10 r_1}{200}$$

The percentage of voltage drop due to r_2 is:

$$\frac{I_2 r_2}{V_t} = \frac{20 r_2}{100}$$

For r_1 to have the same effect as r_2 it is necessary that

$$\frac{I_1 r_1}{V} = \frac{I_2 r_2}{V_t}$$

Transposing:

$$r_1 = \frac{I_2}{I_1} \times \frac{V}{V_t} \times r_2$$

Since

$$\frac{I_2}{I_1} = a$$

and

$$\frac{V}{V_t} = a$$

$$r_1 = a^2 r_2$$

In this example, since a is 2,

$$r_1 = 4r_2$$

This means that a resistance in the secondary would have to be $1/a^2$ times that needed in the primary to produce the same effect. Such an analysis holds also for reactances.

It is frequently necessary to obtain the values of the secondary voltage, current, resistance, or reactance in terms of the primary, or vice versa. This can be done by using the ratio of transformation in the following manner.

Step-up transformer:

Secondary in terms of the primary:
- Divide secondary emf's by a.
- Multiply secondary currents by a.
- Divide secondary resistance by a^2.
- Divide secondary reactances by a^2.

Primary in terms of the secondary:
- Multiply primary emf's by a.
- Divide primary currents by a.
- Multiply primary resistances by a^2.
- Multiply primary reactances by a^2.

For step-down transformers interchange the words "multiply" and "divide."

Example. A transformer with a turn ratio of 1 to 2 is rated at 220 to 440 volts, 25 kv-a, 60 cycles.

$$I_1 = \frac{25{,}000}{220}$$

$$113.6 \text{ amperes}$$

$$I_2 = \frac{25{,}000}{440}$$

$$= 56.8 \text{ amperes}$$

$$r_1 = 0.020 \text{ ohm}$$

$$r_2 = 0.076 \text{ ohm}$$

To convert these values to primary terms:

$$\frac{V_2}{a} = \frac{440}{2} = 220 \text{ volts}$$

$$I_2 a = 56.8 \times 2 = 113.6 \text{ amperes}$$

$$\frac{r_2}{a^2} = \frac{0.076}{4} = 0.019 \text{ ohm}$$

To convert the original values to secondary terms:

$$V_1 a = 220 \times 2 = 440 \text{ volts}$$

$$\frac{I_1}{a} = \frac{113.6}{2} = 56.8 \text{ amperes}$$

$$r_1 \times a^2 = 0.02 \times 4 = 0.08 \text{ ohm}$$

Since the secondary resistance is equivalent to a certain resistance in the primary, and the primary also has a resistance, the total resistance is the sum of r_1 and r_2 in primary terms. Hence in the above case:

$$r_1 = 0.020 \text{ ohm}$$

$$r_2 = 0.019 \text{ ohm in terms of the primary}$$

$$R_e = 0.020 + 0.019 \quad \text{or} \quad 0.039 \text{ ohm}$$

R_e is the equivalent resistance, or the total resistance in the circuit, referred to the primary. Equivalent resistances and reactances will be used in later analyses.

CHAPTER XIII

VOLTAGE REGULATION

119. Chapter Outline.

Solution of the Vector Diagram with the Various Constants of the Transformer Supplied.
- Voltage Regulation.
- Example.
  - Load of Unity Power Factor.
    - Lagging Power Factor.
    - Leading Power Factor.
- Percentage Reactance and Resistance.

120. Solution of the Vector Diagram. To draw the complete vector diagram of a transformer the following values must be known or suitable values assumed.

Rated kilovolt-amperes.
Ratio of turns.
Secondary voltage. (The A.I.E.E. Standards assume that the secondary voltage only is given.)
Secondary current and pf.
No-load current.
No-load pf. (The value of the core loss will do as well.)
Primary winding resistance and leakage reactance.
Secondary winding resistance and leakage reactance.

With these values known, use V_t as the reference vector and lay off I_2 at the angle corresponding to the load pf for which the diagram is to be drawn. Cos θ_2 then represents the pf of the load. Then proceed to lay off the remaining vectors in the order suggested below.

Draw:

I_2r_2	In phase with I_2
I_2x_2	Leading I_2 by 90°
E_2	By vector algebra,

$$E_2 = V_t + I_2(r_2 + jx_2)(\cos\theta_2 \pm j\sin\theta_2)$$

ϕ_m This is the mutual flux. If its value is not known it can be drawn as an indefinite line to help locate other vectors. It should lead E_2 by 90°. When E_2, N_2, and f are known, ϕ_m is found from the emf formula.

I_n This is the no-load current. It can be located by the no-load power-factor angle ϕ, or by the reading of core loss. The core loss, divided by E_1 (or approximately by V), gives a value of I_{h+e}. With I_n and I_{h+e} known, the direction of I_n is determined. The magnetizing current follows from this.

E_1 The induced primary voltage is in phase with E_2. It is shown in the diagrams as $-E_1$ (which is the component of

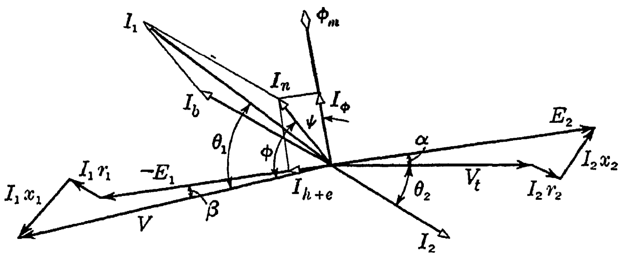


FIG. 90. Vector diagram of a transformer supplying a lagging current.

V required to overcome E_1); that is, $-E_1$ is 180° from E_2. Then $E_1 = (N_1/N_2)E_2$.

I_b The primary current required to balance the ampere-turns due to the load current; it should be drawn 180° from I_2. $I_b = (N_2/N_1)I_2$.

I_1 The total current of the primary is the vector sum of I_b and I_n.

I_1r_1 In phase with I_1.

I_1x_1 Leads I_1 by 90°.

V The applied voltage is the vector sum of $(-E_1)$, I_1r_1, and I_1x_1. If the ratio of transformation is calculated from N_1/N_2 the voltage determined for V need not, and usually will not, be the name-plate rating if rated secondary voltage is used as stipulated by the A.I.E.E. Standards. This difference is due to the voltage drops in the primary and secondary which are different for each value of current and which produce a different effect for each value of pf. $E_1/E_2 = N_1/N_2$; and obviously V/V_t cannot, in general, equal N_1/N_2, owing to the above-mentioned effects.

ψ The angle by which I_n leads ϕ_m.

Apart from those already mentioned, the principal uses of this diagram are the determination of regulation and short-circuit currents in power and lighting transformers, and of ratios and phase angles in instrument transformers.

The completed vector diagram is shown in Fig. 90. Generally it will be found in drawing such diagrams to scale, that a ratio of transformation greater than 2 exaggerates the values on one side of the diagram or else reduces the others. In starting such a diagram from the secondary side it is usually more convenient to convert all primary terms into equivalent secondary values or to use a different set of scales for drawing the primary and secondary parts of the diagram.

Expressed in vector algebra, the values on the diagram will be referred to V_t as reference vector.

$$E_2 = V_t + I_2(r_2 + jx_2)(\cos\theta_2 - j\sin\theta_2) \qquad [98]$$

The mutual flux ϕ_m will have the phase position represented by jE_2, and the exciting current I_n will have a phase position represented by $jE_2(\cos\psi + j\sin\psi)$.

The primary current I_b is required to balance the mmf of the secondary current I_2 in value and phase position.

$$I_b = \frac{-I_2}{a}(\cos\theta_2 - j\sin\theta_2) \qquad [99]$$

Adding I_b and I_n vectorially gives I_1 in value and phase.

The primary induced emf, $-E_1$, due to the mutual flux ϕ_m, is opposite to E_2 and is numerically equal to aE_2.

Then

$$V = -aE_2 + I_1(r_1 + jx_1) \qquad [100]$$

This is the applied voltage necessary to give a secondary terminal voltage of V_t, delivering a load current of I_2 amperes at a pf of $\cos\theta_2$.

121. Voltage Regulation. Assume that the name-plate rating of the secondary voltage is to be maintained as normal at full load. To do this the primary voltage will have to be increased over its rating (on all but leading power factor loads). If this primary voltage is maintained constant as the load is removed, the secondary terminal voltage will rise. This rise in voltage, divided by rated voltage of the secondary, expresses the voltage regulation. The equation becomes:

$$\text{Regulation (in percentage)} = \frac{V_t \text{ no load} - V_t \text{ full load}}{V_t \text{ full load}}\,100 \qquad [101]$$

At no load there are no I_2r_2 and I_2x_2 voltage drops, and there is present only the primary voltage drop caused by the small no-load current. Practically,

$$V_t = \frac{V}{a}$$

The regulation equation can also be expressed:

$$\frac{\frac{V}{a} - V_t \text{ full load}}{V_t \text{ full load}} 100 \qquad [102]$$

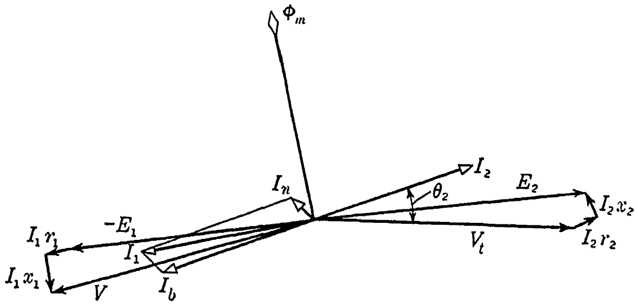


FIG. 91. Vector diagram of transformer supplying a leading current.

The conditions under which regulation is to be figured are specified as follows by the A.I.E.E.[1]

(a) Voltage, Current, and Frequency: Transformer regulation shall be determined for the rated voltage, current, and frequency.

(b) Load: When the regulation is stated without specific reference to the load conditions, rated load is to be understood.

(c) Wave Form: A sine wave of voltage shall be assumed in determining the regulation except where expressly specified otherwise.

(d) Power Factor of Load: The power factor of the load to which the regulation refers should be specified. If the power factor is not specified, 100 per cent power factor shall be assumed.

(e) Temperature of Reference: The regulation at all loads shall be corrected to a reference temperature of 75 degrees centigrade.

122. Example of Regulation. A transformer is rated at 2300/230 volts, 15 kv-a, and 60 cycles.

$N_1 = 1500$ turns	$N_2 = 150$ turns
$r_1 = 2.7$ ohms	$x_1 = 9.1$ ohms
$r_2 = 0.024$ ohm	$x_2 = 0.088$ ohm
$I_n = 0.15$ ampere	Core loss = 92 watts

[1] Standardization Rules of A.I.E.E., 13, pp. 13–350, May, 1930.

Assume that the transformer is operated at 80 per cent pf, lagging, and at rated output the secondary terminal voltage is 230.

$$I_2 = \frac{15{,}000}{230}(0.8 - j0.6) \quad \text{or} \quad 52.2 - j39.2$$

$$= 65.2 \text{ amperes}$$

$$E_2 = 230 + (0.024 + j0.088)(52.2 - j39.2)$$

$$= 234.7 + j3.78$$

$$= 234.7 \text{ volts}$$

$$-E_1 = 10 \times 234.7$$

$$= 2347 \text{ volts}$$

As the core loss is 92 watts,

$$I_{h+e} = \frac{W_c}{E_1}$$

$$= \frac{92}{2347} \quad \text{or} \quad 0.0392 \text{ ampere}$$

To determine ψ (refer to Fig. 90 for definition of angles):

$$\sin \psi = \frac{0.0392}{0.15} \quad \text{or} \quad 0.261$$

$$\psi = 15° \, 8'$$

Phase position of ϕ_m, from $jE_2 = j(234.7 + j3.78)$ must be such that ϕ_m is ahead of V_t by an angle whose tangent is 234.7 over -3.78. Since the tangent is -62.0, the angle is $90° \, 55'$.

Phase position of I_n is then $90° \, 55' + 15° \, 8'$ or $106° \, 3'$ ahead of V_t.

$$\text{Components of } I_n = 0.15(\cos 106° \, 3' + j \sin 106° \, 3')$$

$$= -0.04 + j0.14$$

Since $I_1 = I_b + I_n$ as vectors, and $I_b = (-I_2/a)$

$$I_b = -5.22 + j3.92$$

$$I_1 = -5.26 + j4.06 \quad \text{or} \quad 6.65 \text{ amperes}$$

Then as shown in equation 100:

$$V = -E_1 + I_1(r_1 + jx_1)$$

$$= -(2347 + j37.8) + (-5.26 + j4.06) \; 2.7 + j9.1)$$

$$= -2398.4 \text{ volts}$$

At no load, if the primary drops due to x_1 and r_1 are neglected, which is almost always permissible,

$$-E_1 = 2398.4$$

$$E_2 = \frac{2398.4}{10}$$

$$= 239.84 \text{ volts}$$

At no load,

$$V_t = E_2$$

$$V_t = 239.84 \text{ volts}$$

At full load and 80 per cent pf, lagging, V_t is 230 volts and the regulation becomes

$$\frac{\frac{V}{a} - V_t}{V_t} 100 = \frac{239.84 - 230}{230} 100$$

$$= 4.28 \text{ per cent}$$

It will be noted that although the transformer had a nominal rating of 2300/230 volts, so long as the turn ratio was 1500 to 150, the actual primary voltage must be higher than 2300 in order to give 230 volts across the secondary at full load on a lagging pf. At leading power factor the regulation frequently becomes negative, i.e., the secondary voltage increases with increase of load.

123. Regulation by the American Institute of Electrical Engineers Method. The Standards of the A.I.E.E. give a formula for regulation suitable for use in connection with test results, in terms of values so obtained. Reduced to machine constants, this formula is equivalent to

$$\text{Regulation} = \frac{IR_e}{V}\cos\theta + \frac{IX_e}{V}\sin\theta + \frac{1}{2}\left(\frac{IX_e}{V}\cos\theta - \frac{IR_e}{V}\sin\theta\right)^2 \quad [103]$$

R_e and X_e are equivalent values in the same terms in which I and V are expressed.

Example. For the transformer of the previous article, with constants in secondary terms:

$$R_e = \frac{2.7}{a^2} + 0.024 \quad \text{or} \quad 0.051 \text{ ohm}$$

$$X_e = \frac{9.1}{a^2} + 0.088 \quad \text{or} \quad 0.179 \text{ ohm}$$

$$\text{Regulation} = \frac{65.2 \times 0.051}{230} \times 0.8 + \frac{65.2 \times 0.179}{230} \times 0.6$$
$$+ \frac{1}{2}\left(\frac{65.2 \times 0.179}{230} \times 0.8 - \frac{65.2 \times 0.051}{230} \times 0.6\right)^2$$
$$= 0.0425 \quad \text{or} \quad 4.25\%$$

124. Percentage Reactance and Resistance. In dealing with power systems or distribution circuits, the terms *percentage reactance* and *percentage resistance* are frequently used. These have been discussed under Alternators. When used with transformers, they are defined similarly.

Percentage reactance is the reactance drop in volts at normal current and frequency expressed as a percentage of the rated voltage.

$$\% X = \frac{IX}{V} \times 100 \qquad [104]$$

For the example of Article 122:

$$X_e = 0.079 \text{ in secondary terms}$$

$$\% X = \frac{65.2 \times 0.179}{230} 100 \quad \text{or} \quad 5.09$$

In the same manner:

$$\% R = \frac{IR}{V} \times 100$$

$$= \frac{65.2 \times 0.051}{230} \times 100 \quad \text{or} \quad 1.45 \qquad [105]$$

Results are the same if primary terms are used throughout.

CHAPTER XIV

EQUIVALENT CIRCUITS

125. Chapter Outline.

Equivalent Circuits of the Transformer.
  Exact.
  Approximate.
  Regulation by Equivalent Circuits.
  Example.
Other Methods of Treatment.

126. Exact Equivalent Circuit.[1] It is of great assistance in the understanding of the physical nature of the phenomena that are being studied if the actual apparatus can be replaced, or represented, by an equivalent circuit which has the same properties. The transformer and its load are equivalent to the circuit shown in Fig. 92. This similarity was first

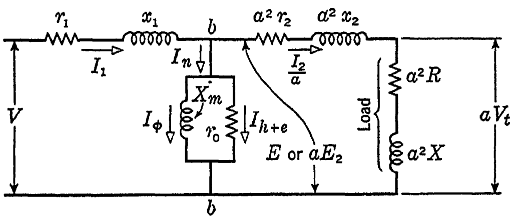


FIG. 92. Equivalent circuit of a transformer.

pointed out by Steinmetz. Faced with an actual machine or device for which an equivalent circuit is required, it is possible to build up step by step the correct equivalent circuit arrangement. This process will not be followed in detail here, in the belief that a brief study of the diagram will show how no-load and load changes on the transformer produce effects exactly similar to those brought about in the equivalent circuit.

In the circuit, R and X are selected as the load values necessary to cause rated secondary current (at the assumed pf) to flow, when voltage

[1] A. Boyajian, "A New Theory of Transformer and Auto-transformer Circuits." *Gen. Elec. Rev.*, February, 1929.

D. R. MacLeod, "New Equivalent Circuits for Auto-transformers and Transformers with Tapped Secondaries," *Gen. Elec. Rev.*, February, 1929.

V_t is impressed across them. This circuit is shown with all values referred to the primary. Hence

$$\frac{I_2}{a} = \frac{aV_t}{\sqrt{(a^2R)^2 + a^2X)^2}} \tag{106}$$

and

$$\cos \theta_2 = \frac{a^2R}{\sqrt{(a^2R)^2 + (a^2X)^2}} \tag{107}$$

where θ_2 is the load pf angle for which the action is to be investigated.

X_m is selected as the value necessary to cause the normal value of magnetizing current to flow through it when E_1 is applied across bb.

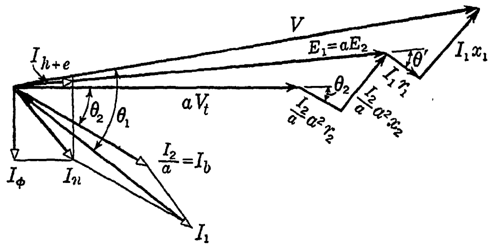


FIG. 93. Vector diagram of the equivalent circuit of Fig. 92 in primary terms.

The value of r_0 is so chosen as to make the power loss in it equal to the core loss of the transformer. That is,

$$X_m = \frac{E_1}{I_\phi} \tag{108}$$

$$r_0 = \frac{E_1}{I_{h+e}} \tag{109}$$

and, as vectors,

$$I_\phi + I_{h+e} = I_n \tag{110}$$

or

$$I_{h+e}^2 \times r_0 = \text{core loss} \tag{111}$$

The symbols r_1, x_1, etc., represent the same values used previously on the vector diagrams.

The true vector diagram of this circuit is shown in Fig. 92. If E_1 and the other primary voltage and current vectors are rotated counterclockwise through 180° this diagram will be the same as that of Fig. 88, representing a loaded transformer.

127. The Approximate Equivalent Circuit. Inasmuch as the no-load current of transformers is small, it causes very little voltage drop through r_1 and x_1. The exact equivalent circuit then can be simplified

by moving the no-load circuit to the left, so that it connects across V. This is shown in Fig. 94. This circuit assumes that the only voltage drops in the primary and secondary are those caused by the primary current required to balance the load current of the secondary. The

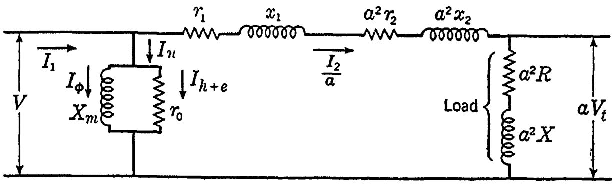


FIG. 94. Approximate equivalent circuit of a transformer in primary terms.

error this causes is ordinarily very small, and, for studying the effect of load on the voltage, the circuit is sometimes simplified to that of Fig. 95. Here the no-load circuit is dropped altogether and the following substitutions are made.

$$R_{\text{equivalent}} = r_1 + a^2 r_2$$

$$X_{\text{equivalent}} = x_1 + a^2 x_2$$

These are in terms of the primary.

If these circuits are solved from the primary with rated primary volts at V, the full-load voltage aV_t will be less than rated and the ratio will not express the true regulation. However, if aV_t is assumed as the normal value and the circuit is solved for V, then an approximately true value of regulation will be obtained. To do this it is often convenient to work from the secondary side. In secondary terms the values to be used are

$$V_t,\ R,\ X,\ I_2 \text{ and } \frac{V}{a}$$

Then

$$R_e = \frac{r_1}{a^2} + r_2$$

and

$$X_e = \frac{x_1}{a^2} + x_2$$

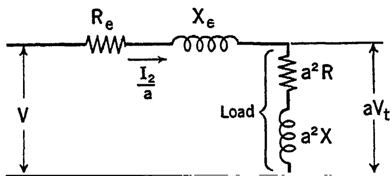


FIG. 95. A further approximation of the equivalent circuit, neglecting the no-load current.

These are in terms of the secondary. The regulation then becomes

$$\frac{\frac{V}{a} - V_t}{V_t} 100$$

The vector diagram for the circuit of Fig. 95 is shown in Fig. 96.

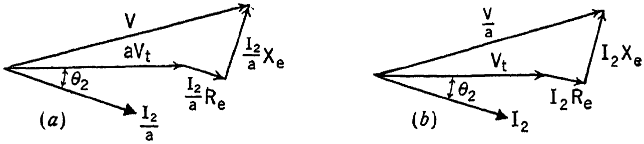


FIG. 96. Vector diagrams of the approximate equivalent circuit in (*a*) primary terms, (*b*) secondary terms.

128. Regulation by the Approximate Equivalent Circuit. The data of the transformer of Article 122 will be used on the approximate circuit and the regulation calculated.

$$V_t = 230 \text{ volts}$$

$$I_2 = 65.2 \text{ amperes}$$

$$R_e = \frac{2.7}{10^2} + 0.024$$

$$= 0.051 \text{ ohm}$$

$$X_e = \frac{9.1}{10^2} + 0.088$$

$$= 0.179 \text{ ohm}$$

In vector algebra:

$$\frac{V}{a} = V_t + I_2(R_e + jX_e)(\cos\theta - j\sin\theta)$$

$$= 230 + 65.2(0.051 + j0.179)(0.8 - j0.6)$$

$$= 240 \text{ volts}$$

(This and similar work can, of course, be done graphically.)

At no load, a voltage of 2400 on the primary results in 240 volts across the secondary. At full load, 0.80 pf, lagging, the secondary voltage is 230. The regulation is:

$$\frac{240 - 230}{230} \cdot 100 = 4.35 \text{ per cent}$$

This simple method gives results comparable with the more tedious solution of the exact circuit. It is much used for predicting the regulation from no-load tests.

129. Other Methods of Treatment. In addition to the methods given in previous chapters, two others are sometimes used for presenting the theory and calculation of transformer characteristics. The method given here uses the concepts of mutual reactance, primary leakage, and secondary leakage reactances, which were suggested by the late Professor Gisbert Kapp, of Birmingham University, and adopted by Steinmetz.

Professor André Blondel, of Paris, and Alexander Heyland, of Belgium,

used mutual, primary, and secondary leakage fluxes. The reasoning is: When the primary winding carries current, a flux ϕ_{I} is produced; of this, a part $\sigma\phi_{\mathrm{I}}$ is leakage and the remainder $\phi_{\mathrm{I}} = (1 - \sigma_1)\phi_{\mathrm{I}}$ reaches the secondary. If the secondary circuit is closed, the secondary current sets up a flux ϕ_{II}, of which $\sigma_2\phi_{\mathrm{II}}$ is leakage and $\phi_2 = (1 - \sigma_2)\phi_{\mathrm{II}}$ reaches the primary. The actual fluxes are then the vector resultants of the appropriate components. This method is much used outside of the United States.

The pioneer method, used by Fleming and by Bedell and Crehore, involves the setting up of the Kirchhoff law equations in either the vector or differential forms and the solving for the steady state. This method has been applied for many years in communication and related work. The last fifteen years have seen numerous applications to instrument transformers, multiwinding transformers, and induction motors.[2] All these methods are based on closely similar and related ideas. The use of vector algebra is an outgrowth of the solution by differential equations which was developed by Steinmetz in 1893. The methods used in this text are most closely related to the latter approach. For those who wish to read intelligently the work done in other parts of the world, or work for which no alternative analyses are available, some further explanations will be given. This material may be omitted where desired as only slight reference to the coupled circuit theory will be made elsewhere in this text.

130. Leakage Factors. Where the theory is built up on the basis of leakage fluxes instead of leakage reactances, the leakage reactances are replaced by so-called leakage factors, of which two variations are in use:

Heyland, about 1894, used these definitions:

Primary leakage factor:

$$\tau_1 = \frac{\text{primary leakage flux}}{\text{mutual flux}} = \frac{L_1 - M}{M} = \frac{x_1}{X_m} \qquad [112]$$

Secondary leakage factor:

$$\tau_2 = \frac{\text{secondary leakage flux}}{\text{mutual flux}} = \frac{L_2 - M}{M} = \frac{x_2}{X_m} \qquad [113]$$

Combined:

$$\tau = (1 + \tau_1)(1 + \tau_2) - 1 = \tau_1 + \tau_2 + \tau_1\tau_2 \qquad [114]$$

[2] A. T. Sinks, "Computation of the Accuracy of Current Transformers." *Trans. A.I.E.E.*, December, 1940.

L. Dreyfus, "The Pull-Out Torque of the Polyphase Induction Motor" (in German), *Archiv. Elektrotechnik*, Vol. 15, 1925.

Blondel about 1895 used the definition:

Total leakage factor:

$$\sigma = 1 - \frac{M^2}{L_1 L_2} = 1 - \frac{X_m{}^2}{X'_0 X''_0}$$

This follows from the theory of coupled circuits. The relation between the two is

$$\frac{-}{1 + \tau} \quad \text{or} \quad \tau = \frac{\sigma}{1 \qquad -} \tag{115}$$

As used above:

$$\begin{aligned} L_1 &= \text{total primary inductance in henries} \\ L_2 &= \text{total secondary inductance in henries} \\ M &= \text{mutual inductance in henries} \\ X'_0 &= \text{total primary reactance in ohms} \\ &= X_m + x_1 \\ X''_0 &= \text{total secondary reactance in ohms} \\ &= X_m + x_2 \end{aligned}$$

When used in connection with circle diagrams of current loci, σ is the ratio of the magnetizing current to the ideal short-circuit current that would exist if there were no losses. Also τ is the ratio of magnetizing current to the circle diameter. These relations apply to both transformers and induction motors.

131. Kirchhoff Law Equations.[3] For a pair of coupled circuits, the two simultaneous equations are

$$(L_1 D + r_1) i_1 + M D i_2 = V_1 \sin wt \tag{116}$$

$$M D i_1 + (L_2 D + r_2) i_2 = 0 \tag{117}$$

where $D = d/dt$, denoting the time derivative; the other quantities have been defined already. The steady state solution is obtained by replacing D by jw, dropping $\sin wt$, and solving simultaneously as algebraic equations. From this we obtain,

$$I_1 = \frac{V_1}{Z'_e + \dfrac{X_m{}^2}{Z''_e}} \tag{118}$$

[3] Frederick Bedell, "The Principles of the Transformer," Macmillan Co., 1896.

J. A. Fleming, "The Alternating Current Transformer," Electrician Printing and Publishing Co., London, 1895.

F. Bedell and A. C. Crehore, "Alternating Currents," Electrical World Publishers, 1892.

André Blondel, "Quelques propriétés générales des champs magnétiques tournants," *L'Éclairage Électrique*, August, 1895.

and

$$I_2 = \frac{-jX_m}{Z''_e} I_1 \tag{119}$$

where

$$Z'_e = r_1 + jX'_0 = r_1 + j(X_m + x_1) \tag{120}$$

$$Z''_e = r_2 + jX''_0 = r_2 + j(X_m + x_2) \tag{121}$$

The above equations for current denote the short-circuit values. When they are known, all transformer quantities (mutual flux, regulation, etc.) can be calculated. As these equations are of a fundamental nature, it is easy to extend their application to many classes of equipment operating on the transformer principle.

It is of interest to note that the equivalent impedance of the short-circuited transformer is given by the denominator of equation 118. From this, the equivalent resistance is

$$R_e = r_1 + \left(\frac{X_m}{Z''_e}\right)^2 r_2 \approx r_1 + r_2 \tag{122}$$

and the equivalent reactance is

$$X_e = X'_0 - \left(\frac{X_m}{Z''_e}\right)^2 X''_0 \approx x_1 + x_2 \tag{123}$$

These two equations are particularly convenient in dealing with those cases in which the magnetizing current is large, as in low-frequency or air-cored transformers and in induction motors. In such cases the approximations of equations 122 and 123 at the right are likely to be seriously in error. They assume, tacitly, the approximate equivalent circuit and the neglect of the magnetizing branch.

CHAPTER XV

CHARACTERISTICS FROM TEST DATA

132. Chapter Outline.

Characteristics from Test Data.
- Partial List of Tests Usually Made.
- Efficiency and Losses.
- Important Tests for Determination of Regulation and Efficiency.
  - Short-circuit Test.
  - Open-circuit Test.
- The Loading-back Method.

133. Determination of Characteristics. A preceding chapter, dealing with transformer regulation, showed how to use the values of resistances and reactances in calculation. In practice, these values must be determined in two ways:

(*a*) From test data taken, usually from no-load tests of the transformer. (Here the leakage reactances x_1 and x_2 cannot be individually determined. This difficulty is overcome sufficiently well in the manner shown later.)

(*b*) From calculations based on design data.

Of the outlined tests which follow, only two are required to predict the regulation and efficiency. Inasmuch as the test data can be used to obtain both of these important characteristics, a discussion of transformer losses and efficiency will be included here.

134. Transformer Efficiency. The efficiency of a transformer is the ratio of its output and input. Since output plus losses represents input, the efficiency equation becomes

$$\text{Efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} \qquad [124]$$

Efficiency from losses offers the most convenient and accurate method of determining what is known as the conventional efficiency.

Losses. It has previously been pointed out that the alternation of the core flux produces eddy-current and hysteresis losses in the iron. These

losses vary [1] with different exponents for flux density and frequency, and depend also upon the wave shape of the impressed emf. Inasmuch as the flux through the core of a constant-voltage transformer remains nearly constant, the core loss determined at no load can be assumed as a constant loss over the entire load range. With increased load the main flux actually decreases, but the leakage flux increases; the variation is very small and the effects more or less neutralize. The assumption of constancy is justified.

When the transformer supplies a load, the primary and secondary currents produce I^2r losses in their respective windings. These losses are obviously variable with the load. Usually it will be found that the copper losses in primary and secondary are approximately equal, as design based on this relation makes for the greatest economy of copper for a given total loss.

Equation 124 can be written:

$$\text{Efficiency } (\%) = \frac{V_tI_2 \cos \theta_2}{V_tI_2 \cos \theta_2 + I_1{}^2r_1 + I_2{}^2r_2 + \text{core loss}} 100 \qquad [125]$$

If $I_1{}^2r_1$ in equation 125 is replaced by $I_2{}^2(r_1/a^2)$ and the derivative of efficiency with respect to I_2 equated to zero, it shows that the efficiency reaches its peak at that value of current for which the variable losses equal the fixed losses. Approximately, this takes place at the current I_2, at which

$$I_2{}^2\left(r_2 + \frac{r_1}{a^2}\right) = \text{core loss} \qquad [126]$$

135. All-day Efficiency. The ratio of kilowatt-hours output to kilowatt-hours input over 24 hours is known as the all-day efficiency. It is an important figure in distribution transformers. Such transformers are connected permanently to the power lines and have a core loss regardless of their load. The all-day efficiency, then, is influenced by the division of the total losses between core and windings and also by the load factor of the transformer.

Numerically it can be expressed as follows:

Let P = the output for t hours, or $V_tI \cos \theta_2$
P' = the output for t' hours, or $V'_tI' \cos \theta'_2$
etc.

[1] See Articles 141 and 142. Also:

M. G. Lloyd, "Magnetic Hysteresis," *J. Franklin Institute,* July, 1910.

J. A. Ewing, "Magnetic Induction in Iron and Other Metals," Third Edition, London, 1900.

Let I^2R_e = the copper loss for t hours
I'_2R_e = the copper loss for t' hours
etc.
Let W_c = the core loss

Then the all-day efficiency is

$$\frac{tP + t'P' + t''P'' + \cdots}{tP+t'P'+t''P''+\cdots+tI^2R_e+t'I'^2R_e+t''I''^2R_e+\cdots+24W_c}100 \quad [127]$$

In the absence of more definite data, all-day efficiency is calculated for 4 hours of full-load and 20 hours of no-load, by N.E.M.A. Standards.

136. Transformer Tests.[2] A number of tests are usually made upon transformers to determine significant values. Among these are:

Ratio of Transformation. The true ratio is based on N_1/N_2. If the primary and secondary voltages are read at no load their ratio is very nearly equal to the true value. Measuring the ratio of I_2/I_1 on short circuit also gives fairly accurate results, especially if the transformer has little leakage flux and low core reluctance.

Resistance of the Windings. The recommended methods for measuring resistance of transformer windings are: drop of potential using a d-c ammeter and voltmeter; or Wheatstone bridge. Approximately rated value of current should be used in the former case and temperature correction should be made to 75 C.

Frequently it is sufficiently accurate to use the ohmic or conductor resistance in figuring losses. The leakage flux of a transformer has largely an air path, and this flux does not ordinarily produce large iron losses which are added to the conductor resistance as in the case of alternators. However, the eddy-current losses in the conductors plus these small iron losses of the leakage path are sometimes used in calculation for greater accuracy. Together with certain other small losses they are called the stray losses.

Reactance of the Winding. The reactance of primary or secondary windings cannot be measured directly. It is usually obtained by the short-circuit test, which gives what is called the equivalent reactance of both windings. This equivalent reactance is ordinarily used in connection with Fig. 96 for predicting the regulation from no-load tests and for predicting the short-circuit current.

$$I_{\text{short circuit}} = \frac{V}{Z \text{ equivalent in primary terms}} \quad [128]$$

[2] G. Camilli, "The Testing of Transformers," *Gen. Elec. Rev.*, Vol. 32, 1929; Vol. 33, 1930.

If it should be desirable to divide the total reactance into its proper proportion between the two windings, an approximation can be made on the assumption that x_1 and x_2 when referred to the same side are each equal to $\frac{1}{2}X_{\text{equiv}}$. The corresponding value for each winding is then found by multiplying or dividing by a^2.

No-load Current. The no-load current varies with the flux density and the permeability of the iron core, but for any given transformer on rated voltage and frequency it remains nearly constant. It can be measured directly.

137. Short-circuit Test. The short-circuit test, along with the open-circuit test, of which the description follows, form the two most important ones used to predict the behavior of a transformer when conditions are such that its rated load cannot be applied in the shop.

Connections are made as shown in Fig. 97*a*. The low-voltage side of the transformer is short-circuited through an ammeter. This meter may

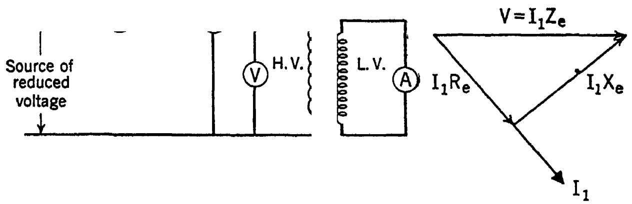


FIG. 97. (*a*) Connections for the short-circuit test. (*b*) Approximate vector diagram of the short-circuited transformer.

be omitted. The reduced voltage on the input is varied until normal current flows through either side of the winding. Under these conditions the equivalent impedance of the transformer is equal to the ratio of V_1/I_1. Inasmuch as any impedance in the secondary produces an effect on the magnitude of the short-circuit current, the impedance so determined is the equivalent value in terms of the input side. The resistance of the windings can be obtained roughly by direct measurement with direct current or from the readings of the wattmeter and ammeter. For the latter,

$$R_{\text{equivalent}} = \frac{\text{watts}}{I_1^2}$$

Such a determination includes the stray losses; these are usually negligible. The total input for this test represents the *load losses*.

With the equivalent impedance and reactance both determined in terms of the input, the equivalent reactance is

$$X_e = \sqrt{Z_e^2 - R_e^2} \qquad [129]$$

Or, from short-circuit data,

$$X_e = \sqrt{\left(\frac{V_1}{I_1}\right)^2 - \left(\frac{W}{I_1^2}\right)^2} \qquad [130]$$

Note that this process considers the transformer in terms of a simple, equivalent series circuit with no parallel magnetizing branch.

138. Open-circuit or Core-loss Test. If normal voltage and frequency are applied to one winding of a transformer and if the other winding or windings are "open-circuited," the watts input represent hysteresis and core eddy-current losses, I^2r loss in the winding to which the voltage is applied, and dielectric losses in the insulation. As the no-load current is relatively small, it is usually unnecessary to subtract the I^2r loss which it causes; consequently the no-load input can be taken as a measure of the core loss with fair accuracy. This loss remains constant at all loads. A diagram of connections is shown in Fig. 98. On this connection the wattmeter reads the loss in the voltage coil of its own meter and in the voltmeter. To eliminate this error, if the circuit is opened at X and the wattmeter read, this second reading may be subtracted from the previous reading to obtain the correct no-load loss. Such an error is appreciable in testing small transformers.

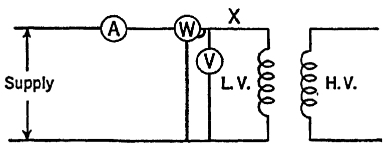


FIG. 98. Connections for the open-circuit test.

Another source of error in core-loss measurement lies in the wave shape of the applied voltage.[3] (See Article 144.) A sine wave is recommended, and for precision work any other wave shape should involve a correction to an equivalent sine basis. If the exact line voltage is not available for testing the transformer, an external resistance used to reduce the potential to name-plate value has the disadvantage of changing the emf wave shape from the sine. When possible, the use of such resistances should be avoided.

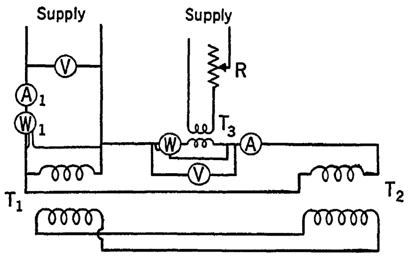
139. The Opposition or "Loading-back" Test. No extensive outline of test methods is given here, but a very common one for determining regulation, efficiency, and heating under actual load conditions will be

[3] An *iron-loss voltmeter* can be used to obtain the correct core loss regardless of the wave shape. When such a meter is connected properly in the test circuit and the voltage adjusted to the correct value as indicated by this meter, the wattmeter, measuring input in the usual manner, can be used to obtain the correct core loss. This loss corresponds to that which would have been indicated by the same effective value of voltage had the wave been sinusoidal.

See L. W. Chubb, "Method of Testing Transformer Core Losses, Giving Sine-wave Results on Commercial Circuits," *Trans. A.I.E.E.*, Vol. 28, Part 1, p. 417, 1909.

described. Where two similar transformers are available they can be tested by loading one on the other and connecting both to a source of supply. The only power required is that necessary for supplying the losses of both transformers and the small loss in the control circuit. This method is similar to that used on d-c motors and generators.

Connections are made as shown in Fig. 99. It is usually found to be more convenient to connect the low-voltage windings to the source of supply, and the high-voltage windings are then connected together so



T_1 and T_2 = the transformers under test
T_3 = the auxiliary transformer for varying the circulating current
R = resistance in primary of auxiliary transformer for voltage variation
W_1 reads no-load losses in both transformers
W reads total I^2R losses in both
A_1 reads magnetizing current of both
A reads circulating (load) current
V reads the equivalent IZ drop in both

FIG. 99. Connections for the loading-back test.

that their potentials are in opposition to each other. No current will flow through the secondaries, and the primaries will take a magnetizing current from the supply. Iron loss will be normal.

This balance can be upset by the introduction of a comparatively low-voltage a-c source in the primary circuit. Variation of this voltage will change the circulating current through the primaries and induce a proportionate secondary current. No instruments need be connected in the high-voltage circuit.

If the input is measured at no load and again when rated current is circulating through the windings, calculations can be made of no-load losses, copper losses, or total losses from which the efficiency can be obtained. Continued circulation of rated current enables data to be obtained for a heat run. The comparatively small amount of energy which must be utilized for tests of large transformers and the difficulty of obtaining a load large enough for direct loading make this method a very useful one.

CHAPTER XVI

DIVERS TOPICS ON TRANSFORMERS

140. Chapter Outline.

Divers Topics on Transformers.
- Eddy-current Losses.
- Hysteresis Loss.
- Shape of the No-load Current Wave.
- Shape of the Electromotive-force Wave and Its Influence.
- Initial Current Rush.

141. Eddy-current Losses. The losses in an iron core are usually obtained by measurement, using various means perfected for that purpose. The loss can be separated into eddy-current and hysteresis components by a simple experimental method. It is sometimes convenient, however, to calculate the eddy-current losses by a mathematical method which will be described here.

The power dissipated in an iron core by eddy currents is subject to the same laws which govern power losses in any circuit.

$$\text{Eddy-current loss in a section of lamination} = i^2r$$

where i = the assumed sine wave of current circulating through a section of lamination
r = the resistance of the iron path

The magnitude of the circulating eddy current is fixed by the voltage induced in the laminations and by their resistance. The voltage induced depends upon the rate of change of flux, and this in turn depends upon the maximum value of the flux wave and the frequency. Hence it follows that eddy-current loss varies as the square of the product of flux and the frequency, and by those constant factors which would influence the resistance, viz., conductivity of the iron and the dimensions of the laminations.

$$P_e = k_eB_m{}^2f^2 \text{ watts} \qquad [132]$$

In the analysis which follows we will derive the values "absorbed" in k_e.

Figure 100 shows a section of lamination with the flux entering perpendicularly to the surface of the section. The path of the eddy currents is shown as *abcg*, down the entire depth D of the lamination. If the maximum value of the total flux through the lamination is ϕ_m, the flux enclosed by the current around *abcg* at a distance x from the center is the fraction $(2x/d)\phi_m$. This neglects the ends, which procedure is usually permissible. Then the flux acting at any distance x is

$$\phi_{\text{enclosed}} = \phi_m \frac{2x}{d} \qquad [133]$$

Uniform flux density is assumed, but we shall see later the error of this assumption.

The emf induced follows the usual equation for sinusoidal flux variation.

$$E = \frac{2\pi}{\sqrt{2}} \times \phi_{\text{enclosed}} \times f \times N \times 10^{-8} \qquad [134]$$

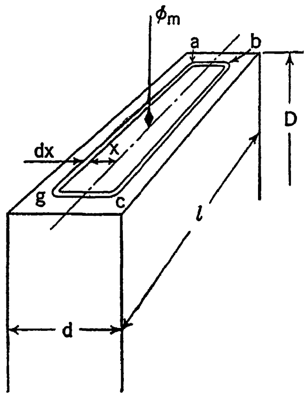


Fig. 100.

The path *abcg* represents one turn. Combining equations 133 and 134,

$$E = \frac{2\pi}{\sqrt{2}} \times \frac{2x}{d}\,\phi_m f\, 10^{-8} \qquad [135]$$

Neglecting again the path across the edges, we have the length of the current path as $2l$.

$$\text{The area of the path} = dx(\text{width}) \times D(\text{depth})$$

$$\text{The resistance of any conductor} = K\,\frac{\text{length}}{\text{area}}$$

$$\text{The resistance of this path} = K\,\frac{2l}{dxD} \qquad [136]$$

$$\text{Power} = i^2 r \quad \text{or} \quad \frac{E^2}{r}$$

From equations 135 and 136,

$$\frac{E^2}{r} = \frac{\dfrac{4\pi^2}{2} \times \dfrac{4x^2}{d^2}\,\phi_m{}^2 f^2\, 10^{-16}}{K\,\dfrac{2l}{dxD}}$$

$$= \frac{4\pi^2 \phi_m{}^2 f^2 D}{d^2 l K}\, x^2 dx\, 10^{-16} \qquad [137]$$

The total eddy-current loss in the lamination will be

$$\int_0^{d/2} \frac{4\pi^2\phi_m{}^2f^2D}{d^2lK} 10^{-16}x^2dx = \frac{\pi^2\phi_m{}^2f^2Dd \times 10^{-16}}{6lK} \text{ watts} \quad [138]$$

The volume of the lamination is lDd. We will use these dimensions in centimeters. Hence the loss per cubic centimeter

$$= \frac{\pi^2\phi_m{}^2f^2Dd \times 10^{-16}}{6lK} \cdot \frac{1}{lDd} \quad [139]$$

Let B = the flux density. Then

$$\phi_m = B_m ld$$

$$\phi_m{}^2 = B_m{}^2l^2d^2 \quad [140]$$

Watts loss per cubic centimeter from equations 139 and 140

$$= \frac{\pi^2f^2B_m{}^2l^2d^2\, 10^{-16}}{6l^2K}$$

$$\frac{\pi^2d^2}{6K} f^2B_m{}^2\, 10^{-16} \quad [141]$$

This is our original equation for eddy-current loss with the addition of the constants:

d = the thickness of the lamination, in centimeters

B_m = the maximum value of the sine wave of flux, in lines per square centimeter

K = the resistivity per centimeter cube. It is about 10^{-5} ohm for ordinary transformer iron. It may be 4 or 5 times as large for silicon steel.

A sinusoidal variation in flux was assumed in the foregoing analysis. The I^2r loss involves the effective value of the wave and not the maximum value. Consequently, for waves of various shapes with the same maximum values, the eddy-current losses will be proportional to the squares of their respective form factors.

Our assumption of uniform flux distribution through the cross-section of the lamination is incorrect. The eddy currents circulating around the center of the lamination tend to force the flux to the outer edges. This tendency varies directly with the frequency. Actual values in ordinary transformer sheet show a variation in the flux density between the inner and outer sections of 0.014-in. laminations of about 3 per cent for 60 cycles and less than 1 per cent for 25 cycles.

Calculated values of eddy-current loss have a tendency to be pessimistic.

142. Hysteresis Loss. If a completely demagnetized piece of iron is subjected to an mmf, the flux density increases according to the relationship shown by curve OM on Fig. 101a. As the mmf is reduced, the flux does not decrease as readily as it increased. At zero excitation a residual flux OD persists. To demagnetize the iron completely requires an mmf ON in the opposite direction. This is known as the coercive force.

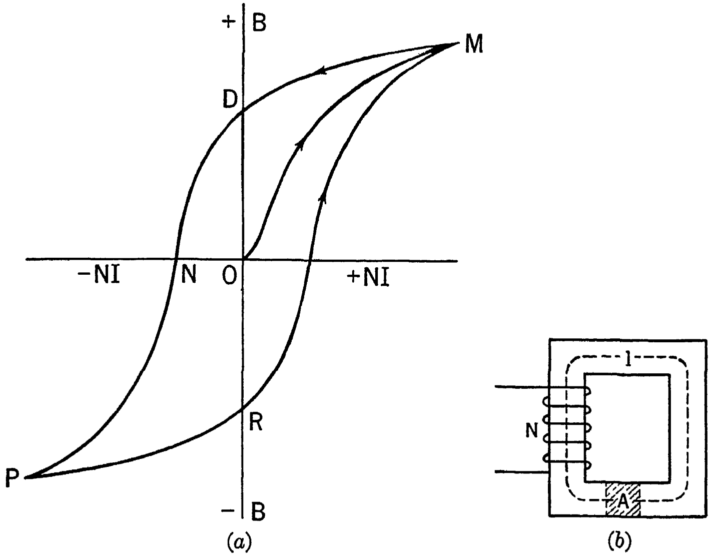


Fig. 101

Increasing the mmf in the opposite direction reverses the direction of flux which follows the relationship expressed by NP. On decreasing the mmf to zero and building it up in the positive sense, the flux follows curve PRM.

This is the familiar hysteresis loop [1] whose area is proportional to the work done in following through this cycle of magnetization and demagnetization.

One method of proof for the relationship between work done and area is based on the following analysis. Refer to Fig. 101b. The magnetizing coil N is placed on an iron core. The length of flux path is l centimeters. The cross-sectional area in square centimeters is A. A change

[1] Experimental methods of obtaining such curves can be found in V. Karapetoff and B. C. Dennison, "Experimental Electrical Engineering," Fourth Edition, John Wiley & Sons, Inc.

in current causes a change in ϕ and produces a counter emf in the coil. When the current rises from zero to its maximum value along the hysteresis loop, a counter emf is induced which opposes the flow of current. When the current falls to zero the flux induces an emf which tends to maintain the current. Hysteresis loss is the average *ei* going up minus the average *ei* going down.

Let w = the work done
di = the change in current in time dt

Work done during this change, in watt-seconds:

$$dw = eidt \tag{142}$$

But the self-induced voltage is

$$e = N\frac{d\phi}{dt}10^{-8} \tag{143}$$

The total work done in one cycle, requiring time T, is

$$W = \int_0^T N\frac{d\phi}{dt}idt \cdot 10^{-8} \tag{144}$$

Let B = the flux density in lines per square centimeter.
Then $\phi = AB$, and the mmf per unit length = H:

$$H = \frac{0.4\pi Ni}{l} \tag{145}$$

where i is in amperes.

Solving equation 145 for Ni,

$$Ni = \frac{Hl}{0.4\pi}$$

Substituting in equation 144,

$$W = \int_0^T \frac{Hl}{0.4\pi}A\frac{dB}{dt}dt\,10^{-8}$$

Since the volume of iron $V = Al$

$$W = \frac{V}{0.4\pi}10^{-8}\int_{-B_m}^{B_m} HdB \tag{146}$$

The $\int_{-B_m}^{B_m} HdB$ is the area of the hysteresis loop with a maximum flux density of B. Equation 146 gives the loss in joules per cycle. If the

actual loop is plotted from experimental data it can be evaluated as follows:

Obtain the area by a planimeter or otherwise.

Let unit length along the B axis represent b flux lines per square centimeter, and unit length along the mmf axis represent h gilberts per centimeter.

$$\text{Unit area} = \frac{bh}{4\pi}$$

$$\text{Hysteresis loss} = \text{units indicated on planimeter} \times \frac{bh}{4\pi}$$

in ergs per cubic centimeter

$$1 \text{ watt} = 10^7 \text{ ergs per second}$$

For many practical purposes the process just described is too inconvenient.[2] In such cases it is customary to use the following method.

The hysteresis loss in watts can be expressed approximately by Steinmetz's empirical equation:

$$P_{\text{hysteresis}} = \eta f V B^{1.6}\, 10^{-7} \text{ watt} \qquad [147]$$

where η = Steinmetz coefficient
f = frequency in cycles per second
V = volume or weight, depending upon the values used for η
B = the maximum flux density in lines per square centimeter

Values of η:

Annealed electrical steel: 0.001 to 0.004 erg per cubic centimeter per cycle

Annealed silicon steel: 0.0006 to 0.00095 erg per cubic centimeter per cycle

143. Shape of the No-load Current Wave. If a sine wave of voltage is applied to the primary of a transformer, the flux wave will vary as a sinusoidal function of time, but the no-load current wave will be distorted owing to the hysteresis loop. Figure 102*a* represents the hysteresis loop taken to the same maximum of flux density as used in the transformer of this analysis.

To produce a flux density of NP requires ON ampere turns per centimeter

$$N'P' = NP$$

[2] When this is the case, it is no longer customary to separate hysteresis and eddy losses, but to use the much more convenient *iron-loss curves*, in which watts loss per pound (or per cubic inch) are plotted against flux density.

The ampere turns per centimeter are plotted as $N'A$ in Fig. 102*b*. To produce a flux density MR requires OM ampere turns per centimeter.

$$OM = M'B$$

$$MR = M'R'$$

In brief, the various abscissas of *a* are plotted as ordinates to determine the shape of the current wave on *b*. This is continued until a sufficient

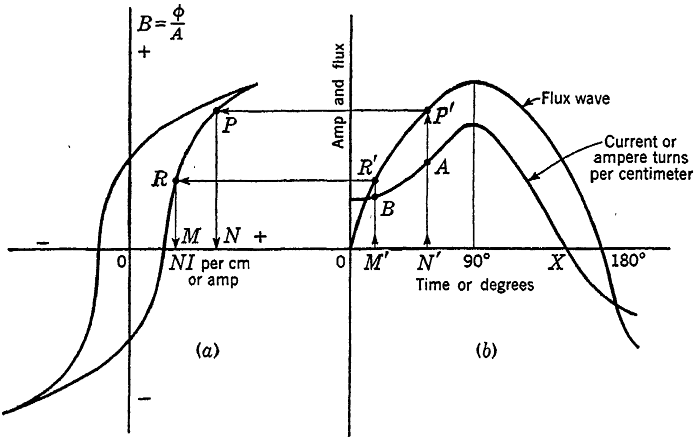


FIG. 102.

number of points is obtained. The use of a suitable constant changes the wave BAX from ampere turns per centimeter to amperes.

Such a wave represents the magnetizing component and the hysteresis component of the no-load current. It reaches its maximum at the same time as the flux wave, but the two waves do not go through zero simultaneously.

The components are shown in Fig. 103. The hysteresis current supplies the hysteresis loss and will be in phase with $-E_{si}$ which agrees with the position previously given it on the vector diagram. The vector diagram is not strictly correct, however, as vectors of constant length are accurate for sinusoidal quantities only.

Eddy-current loss represents real power, and its current wave should also be in phase with $-E_{si}$. If the eddy-current wave is added to the hysteresis and magnetizing current curve, the resultant will be the no-load current of the transformer. This is shown in Fig. 104.

This no-load current can be considered approximately as made up of two sine components I_e and I_h and the non-sinusoidal component I_ϕ. The sum of the sine and non-sine waves is a distorted wave as shown.

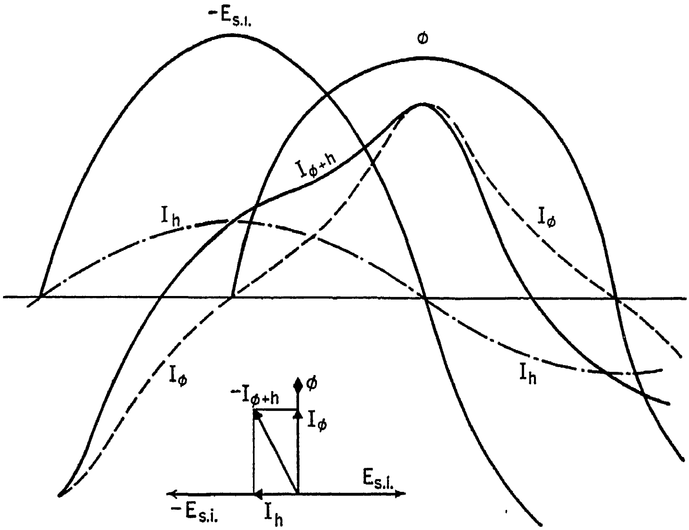


FIG. 103. The current curve derived from the hysteresis loop is made up of the magnetizing current and the hysteresis component.

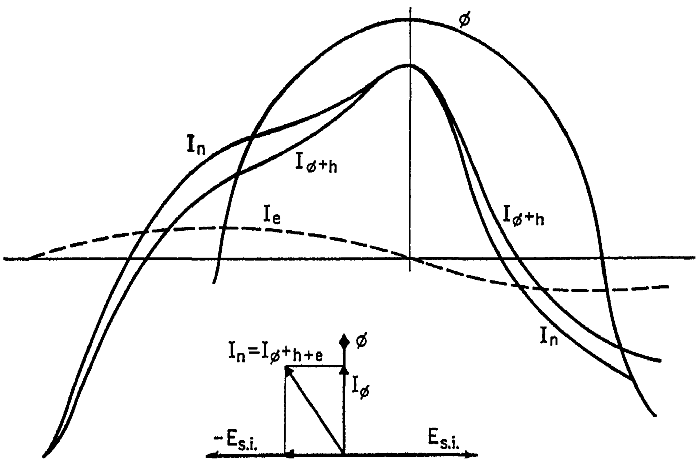


FIG. 104. The derived curve of Fig. 103 can be made to represent the true no-load current by the addition of the wave of eddy-current component.

For purposes of convenient analysis it is customary to replace the actual curve of exciting current by an equivalent sine wave which produces approximately the same effects.[3] An oscillograph record of transformer current is shown in Fig. 105.

It can be seen from this analysis that the component of no-load current commonly classified as that required to produce the flux in the transformer core could not possibly produce the required flux. During part

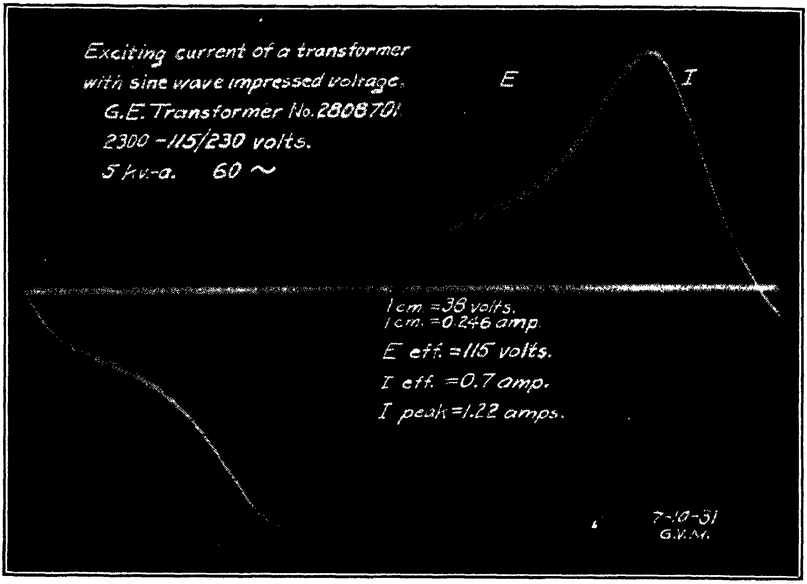


FIG. 105.

of the cycle it would be too small in magnitude because of the hysteresis loop. The true components to produce the flux are both the hysteresis component and the so-called magnetizing component.

144. Influence of the Shape of the Electromotive Force Wave. It was assumed in the previous article that the emf wave followed a sine function and resulted in a sinusoidal variation of the flux and a distorted wave of current. In case the emf wave is distorted, the resultant flux wave shape can readily be determined, and, with the hysteresis loop, the resultant current wave shape.

If the resistance and local reactance drops in the primary circuit of a transformer are neglected, the induced voltage is equal and opposite

[3] That is, by a sine wave which has the same effective value as the distorted wave which it replaces.

to the applied voltage. Hence a non-sinusoidal wave of applied voltage results in a non-sinusoidal induced emf. Since the induced emf depends upon $d\phi/dt$, the slope of the flux wave must accommodate itself to the value necessary to make the applied and induced voltages equal and opposite at all times.

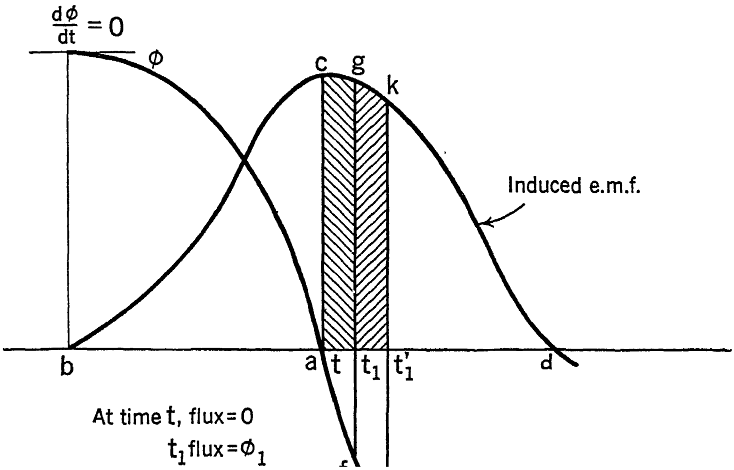


FIG. 106. Curves showing the relationships between flux and emf. To obtain an absolute value of flux read $acgt_1$ in square inches; multiply by $10^8/N_1$ times volts per inch times seconds per inch. The result is the instantaneous value of flux.

The induced voltage:

$$e = -N_1 \frac{d\phi}{dt} 10^{-8} \tag{148}$$

$$d\phi = -\frac{10^8}{N_1} edt \tag{149}$$

$$\phi = -\frac{10^8}{N_1} \int_t^{t_1} edt \tag{150}$$

where t represents the instant at which the flux is zero, and t_1 represents the instant the flux is at some value ϕ_1. Now the above integral represents the area under that part of the emf wave from the time of $\phi = 0$ to $\phi = \phi_1$. This is shown graphically in Fig. 106.

The induced emf lags the flux. The applied voltage is 180° from the induced, or equal and opposite to it. This agrees with previous vector diagrams.

It is obvious that when $d\phi/dt = 0$, the emf must be zero; also an equal quantity of flux must be added and taken away from the core during a half cycle. Hence,

$$\text{Area } abc = \text{area } acd$$

and a represents the zero point on the flux curve.

From equation 150 it can be seen that the cross-hatched area $acgt_1$ times $10^8/N_1$ represents the flux ordinate t_1f. Similarly the area $ackt'_1$ times $10^8/N_1$ represents the flux ordinate t'_1h. The factor $10^8/N_1$ can be neglected and the points on the flux wave plotted by proportion.

If a distorted emf wave with positive and negative loops of the same shape is applied to a transformer or reactance coil, is the flux wave equally distorted? In other words, what are the relative magnitudes of the harmonics in each? To answer these questions it is most convenient to follow the method of Fourier.

Let the equation of flux be:

$$\phi = \phi_1 \sin \omega t + \phi_3 \sin (3\omega t + \alpha) + \phi_5 \sin (5\omega t + \beta) + \cdots$$

Since

$$e = -N \frac{d\phi}{dt} 10^{-8}$$

Then

$$e = -[\phi_1 \cos \omega t + 3\phi_3 \cos (3\omega t + \alpha) + \cdots]\omega N\, 10^{-8} \qquad [151]$$

Assuming that equation 151 was the expression for the applied emf it will be seen that the flux and emf waves contain the same harmonics in the same phase relationships but the third harmonic of the flux is one-third as large as that of the voltage, the fifth harmonic is one-fifth as great, etc. Consequently, a distortion in emf does not produce equally bad effects on the flux.

145. Influence of the Shape of the Electromotive Force Wave on Iron Losses. The total iron losses can be expressed as:[4]

$$W = k_h f B_m^{1.6} + k_e f^2 k^2 B_m^2 \qquad [152]$$

where k_h = the hysteresis constant
k_e = the eddy current constant
k = the form factor

[4] This discussion is based on the analysis made by E. Arnold.

The effective value of the voltage induced in the transformer is:

$$E = 4kfN\phi_m\, 10^{-8}$$

Since

$$\phi_m = B_m A$$

$$E = 4kfNB_m A\; 10^{-8} \qquad [153]$$

where A is the cross-section area of the core.

Using the constant C to absorb some of the factors in equation 153 it can be written:

$$E = CkfB_m$$

or

$$B_m = \frac{E}{Ckf} \qquad [154]$$

Substituting equation 152, the iron losses are

$$W = \frac{k_h f}{(Cf)^{1.6}} \frac{E^{1.6}}{k^{1.6}} + \frac{k_e}{C^2} E^2 \qquad [155]$$

At constant frequency and constant effective voltage the eddy-current losses are independent of the wave form of the voltage curve, but the hysteresis losses become smaller with increasing form factor k.

Peaked voltage waves thus lead to smaller, and flat-topped waves to greater, iron loss than sinusoidal voltage waves. For peaked voltage waves the flux wave is flat topped with a lower maximum; conversely for flat-topped voltage waves the flux curve is peaked.

Relative values for hysteresis losses for various form factors are shown in Table IX. The hysteresis loss for a sine wave is taken as 100 per cent.

TABLE IX

HYSTERESIS LOSSES

Form factor	1.00	1.05	1.11	1.15	1.20	1.25	1.30	1.35	1.40
Hysteresis loss in percentage	118	109	100	94.5	88.5	82.2	77.6	73.3	69.3

Peaked wave forms were commonly used in the early days of electrical engineering, since they resulted in reduced iron losses. Such peaked waves, however, have component higher harmonics which are more likely to result in resonant phenomena, voltage surges, and inductive interference with communication circuits. Higher harmonics have

bad effects on motor action. As the crest value exceeds the effective value more for peaked than for sine waves, use of the former would require greater care in insulation. But since the advantage of the transformed current for long lines resides in the high voltage, and since the strength of insulating materials and air against brush discharges is limited, the peaked wave form has been abandoned. The sine curve is thus the most favorable wave form for alternating current.

146. Initial Rush of Current. When the primary circuit of a transformer is suddenly connected to a supply, a transient rush of current may take place. This rush depends upon the point in the cycle at which the switch is closed, the value and direction of the residual core flux, the shape of the saturation curve, and the normal flux density used. As silicon steel, which has roughly the same permeability as good non-alloy steel (but lower loss), permits higher magnetic densities, transformers with cores of this material are more subject to excessive rushes. Under some conditions the starting current may reach dangerously high values. The extent of the initial current rush will now be discussed.

Approximate Method.[5] The approximate value of the magnetizing-current rush can be determined as follows (this neglects the limiting effects of primary impedance drop and the effects of saturation):

Obtain the saturation curve for the transformer and also the number of turns on the primary winding.

To solve for $\phi_{\max}$ make use of the fundamental emf equation, thus:

$$E = \frac{2\pi}{\sqrt{2}} f N \phi_m \, 10^{-8} \qquad [156]$$

then

$$\phi_m = \frac{\sqrt{2} E \, 10^8}{2\pi f N} \qquad [157]$$

In instantaneous values this relationship can be expressed:

$$e = -N \frac{d\phi}{dt} 10^{-8}$$

also

$$e = E_m \sin 2\pi f t$$

[5] For a method using Froelich's equation for the magnetization curve, and an analysis based on "finite differences," see:

Woodruff's "Principles of Electric Power Transmission and Distribution," First Edition, Chapter XV, John Wiley & Sons, Inc.

T. D. Yensen, "Starting Currents of Transformers," Bulletin 55, University of Illinois.

Hence

$$E_m \sin 2\pi ft = N \frac{d\phi}{dt} 10^{-8}$$

Solving for $d\phi/dt$:

$$\frac{d\phi}{dt} = \frac{10^8 E_m \sin 2\pi ft}{N}$$

$$\phi = \frac{E_m 10^8}{N} \int_0^t \sin 2\pi ftdt$$

$$\phi = \frac{E_m 10^8}{N} \cdot \frac{1}{2\pi f} (1 - \cos 2\pi ft) \qquad [158]$$

The maximum value of flux will occur at the instant in which $\cos 2\pi ft = -1$. The voltage wave is then going through zero. Equation 157

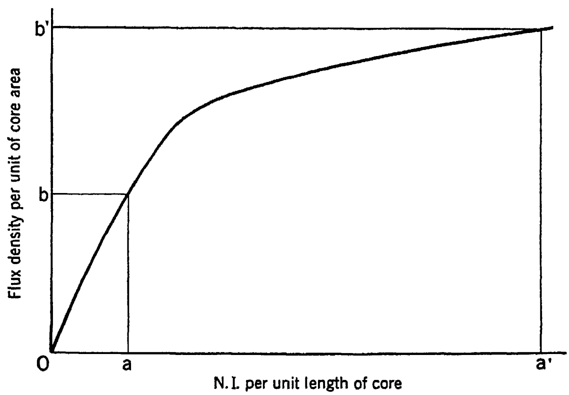


Fig. 107.

expresses normal flux; comparison shows that equation 158 is twice normal. Consequently the peak of the flux wave can be twice normal if the transformer is thrown on the line when the voltage wave is going through its zero point.

The saturation curve is shown in Fig. 107. Under a steady state the flux density will alternate between $\pm ob$, if b is the assumed point at which the iron is being operated. The magnetizing current will alternate between $\pm oa$. This is true if the primary is closed when the impressed emf is at its maximum in either direction. But if the primary circuit is closed when e is going through its zero point (see Fig. 108), the flux must rise through a range of 2ϕ. To double ϕ requires the greatly increased mmf oa'. Furthermore, if residual flux is present in a positive

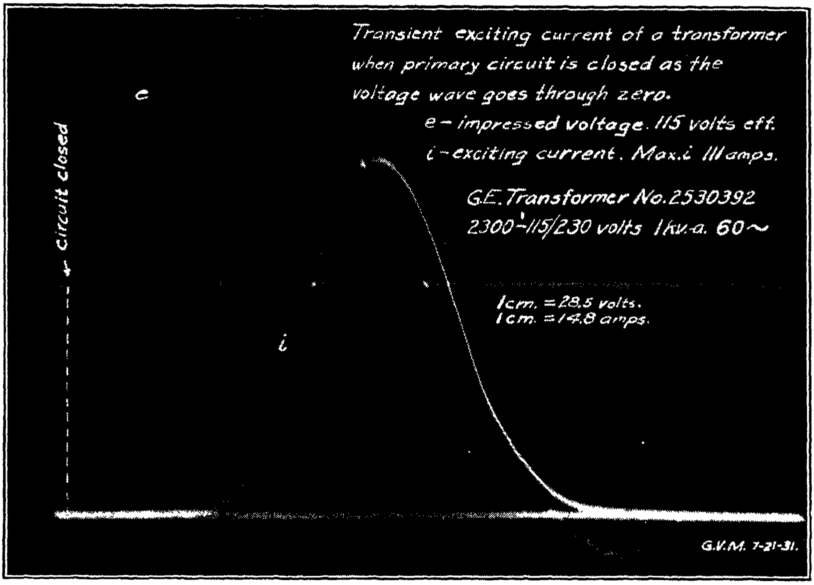


Fig. 108.

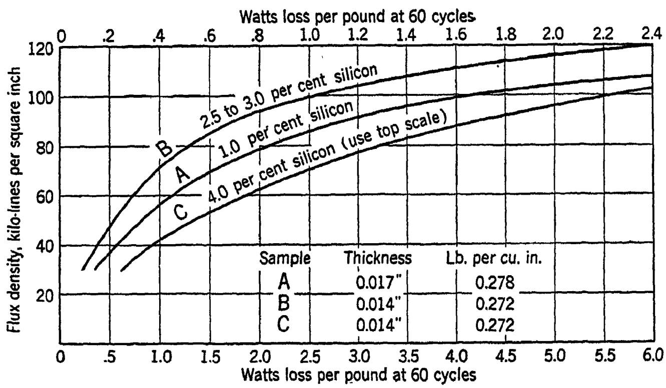


Fig. 109. Core-loss curves for sheet steel with various amounts of silicon.

sense, compared to the above diagram, the range of flux variation is $2\phi + \phi_r$ and the maximum possible peak of current is still greater than oa'. Residual flux in the opposite sense $(-\phi_r)$ reduces the current rush.

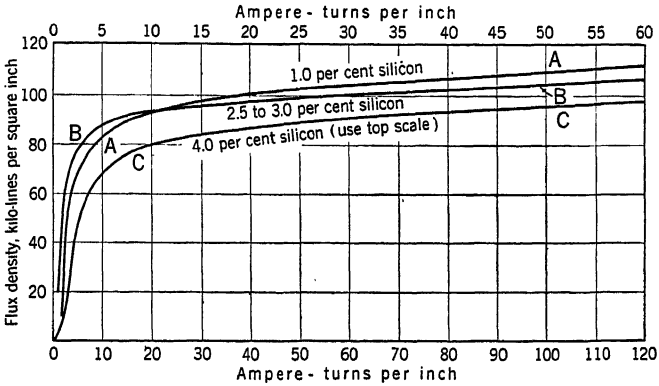


FIG. 110. Magnetization curves for sheet steel.

Closing the circuit at points other than zero on the emf wave gives intermediate values of current. The primary resistance and leakage reactance exert a considerable limiting effect on the value of the maximum peak of current.

CHAPTER XVII

PARALLEL OPERATION OF TRANSFORMERS

147. Chapter Outline.

Parallel Operation of Transformers.
- At No Load.
- Under Load.
  - Conditions to be Fulfilled for Successful Operation.
  - Example.

148. Parallel Operation at No Load. When transformer primaries are connected in parallel across the lines of a distributing system and have independently loaded secondaries, the operation of each is not affected by that of the other so long as the primary voltage is maintained and

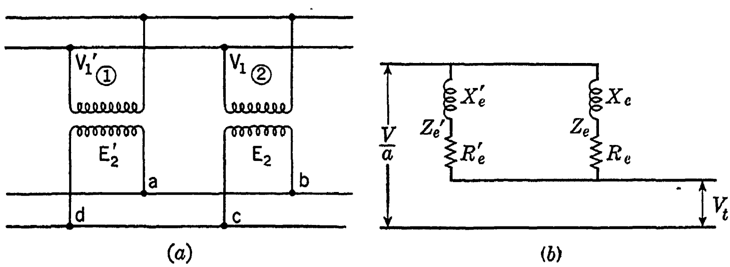


FIG. 111.

so long as the line-voltage wave form is not affected by either transformer. When the secondaries are also paralleled, however, the transformer characteristics must be properly coordinated if operation is to be successful. In the following analysis we will see what conditions must be fulfilled.

In Fig. 111*a* the primary applied voltages V'_1 and V_1 are obviously equal since both come from the same source. E'_2 and E_2 are the induced secondary voltages of transformers 1 and 2, respectively, at no load. Assume that the turn ratio of the two are not exactly equal. Then $E'_2 \neq E_2$, and a slight difference in voltage exists around the circuit *abcda*. A circulating current flows through this short-circuiting path,

limited only by the impedances of the circuit. The impedances of the circuit are, respectively,

$$Z'_2 = R'_2 + jX'_2$$

$$Z_2 = R_2 + jX_2$$

These are equivalent values, including the primary quantities in secondary terms.

Since E'_2 and E_2 depend upon both the turn ratios and the primary resistances and reactances, the unbalance of E'_2 and E_2 may also be exaggerated by unequal primary and secondary impedances. The secondary circulating current becomes

$$I_c = \frac{E_2 - E'_2}{Z_2 + Z'_2} \qquad [159]$$

Since the impedances are usually small, a slight unbalance may cause large circulating current and may result in overheating.

Figure 111*b* shows an equivalent network for a pair of transformers in parallel.

149. Parallel Operation under Load. Ratios Equal. If the paralleled transformers are loaded, the relation between secondary voltage and

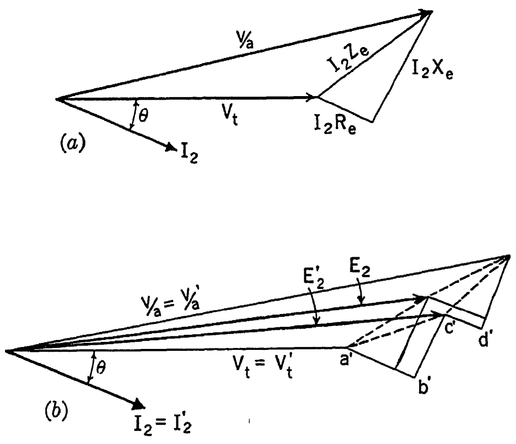


FIG. 112.

primary voltage referred to the secondary is given approximately by the vector diagram of Fig. 112*a*. This is based on the approximate equivalent circuit with the no-load current neglected.

R_e, X_e, and Z_e are equivalent values in terms of the secondary. Since both primaries and secondaries are paralleled, for equal ratios of turns,

$$\frac{V}{a} = \frac{V'}{a}$$

$$V_t = V'_t$$

As vectors,

$$\frac{V}{a} = V_t + I_2 Z_e \qquad [160]$$

and

$$\frac{V'}{a} = V'_t + I'_2 Z'_e$$

Consequently, as vectors,

$$I_2 Z_e = I'_2 Z'_e$$

and

$$\frac{I_2}{I'_2} = \frac{Z'_e}{Z_e}$$

This signifies that the load currents vary in the inverse ratio of the equivalent impedances. If two transformers of the same rating and ratio of transformation are to divide the load equally, their impedances must be equal. If one has twice the capacity of the other, proper division of load requires that its impedance be one-half that of the other. We shall see that the way in which the equivalent impedance is divided between primary and secondary in each individual transformer does not influence the load division.

In Fig. 112*b* both transformers have the same equivalent impedance. These impedances are divided into their respective parts. Thus

$a'b' = I'_2 R'_2$ of the first transformer secondary
$c'd' = I'_1 R'_1$ of the first transformer primary converted to secondary terms
etc.

Even though the primary applied voltages and the secondary terminal voltages are, respectively, the same, the voltages induced in the secondaries E'_2 and E_2 are not equal. Under load, however, this difference in voltage does not cause a circulating current through the secondary circuit as it is balanced by the different impedance drops of the secondary. Hence I'_2 still equals I_2 as determined by the equal equivalent impedance.

It can be seen further that, for the induced voltages of one secondary

to equal the induced voltages of the other, and for the two to be in phase, the following conditions must be satisfied:

$$\frac{R_1}{R_2} = \frac{R'_1}{R'_2}$$

and

$$\frac{X_1}{X_2} = \frac{X'_1}{X'_2}$$

150. Effect of Ratio of Resistance and Reactance. If the two transformers to be operated in parallel have equal equivalent impedances, they both may divide the current equally but need not operate at the

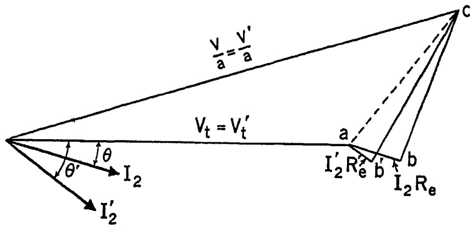


FIG. 113. Transformers operating in parallel with unequal ratios of R_e/X_e. The impedance drops are exaggerated.

same pf. This will occur when the ratio of R'_e/X'_e does not equal R_e/X_e. The effect of these ratios can be seen in Fig. 113.

It is assumed that

$$Z'_e = Z_e$$

(although $R'_e + jX'_e$ need not be identical with $R_e + jX_e$).

But the impedance drop triangles abc and $ab'c$ are obviously determined by the relation of R_e/X_e and R'_e/X'_e, respectively. If $I'_2R'_e$ and I_2R_e make different angles with the horizontal, this can be caused only by different pf angles for I'_2 and I_2. The transformers then operate at the pf's necessary to fulfill the conditions of the diagram. The total current divides so that, as vectors,

$$I'_2 + I_2 = I_{\text{load}}$$

For the transformers to be operating most effectively, i.e., with the minimum of secondary current for a given load, they should each operate at the same pf as that of the load. Under those conditions the arithmetical sum, as shown below, holds true:

$$I'_2 + I_2 = I_{\text{load}}$$

151. Summary.[1] To operate successfully in parallel with proper load division:

(*a*) Connections should be made with due regard to polarity.

(*b*) Voltage ratings should be identical.

(*c*) Transformation ratios should be equal.

(*d*) Equivalent impedances should divide in inverse proportion to the current ratings.

(*e*) Ratios of equivalent resistances and reactances should be equal.

Items *d* and *e* can be corrected by the addition of extra R or X to the circuit.

152. Load Distribution of Transformers of Unequal Rating in Parallel. Transformers connected for parallel operation can be analyzed as follows: The primes refer to the first transformer, and all quantities are to be treated as vectors. From the simple equivalent circuits it is obvious that

$$\frac{V'_1}{a'} = V'_t + I'_2 Z'_e \qquad [161]$$

$$\frac{V_1}{a} = V_t + I_2 Z_e \qquad [162]$$

$$I_T = I'_2 + I_2 \qquad [163]$$

Since both primary and secondaries, respectively, are paralleled,

$$V'_1 = V_1$$

$$V'_t = V_t$$

Z'_e and Z_e are the equivalent impedances of transformers 1 and 2, respectively, in secondary terms.

The unknowns are I'_2, I_2, V_t, or V_1. Select V_t as known.

Eliminate the primary voltages by equating equations 161 and 162.

$$a'(V'_t + I'_2 Z'_e) = a(V_t + I_2 Z_e) \qquad [164]$$

[1] The parallel operation of transformers for polyphase lines presents special problems which will not be covered here. A partial bibliography covering parallel operations includes:

W. M. McConahey, "Parallel Operation," *Elec. J.*, Vol. IX, p. 615, 1912.

W. V. Lyon, "Parallel Operation of Transformers," *Elec. World*, February, 1914.

Mabel Macferran, "Parallel Operation of Transformers Whose Ratios of Transformation Are Unequal" (Abstract), *J. Am. Inst. Elec. Engrs.*, Vol. 48, p. 809, 1929.

E. G. Reed, "Parallel Operation of Transformers," *Elec. J.*, Vol. XVI, p. 267, June, 1919.

N. Stahl, "Transformer Currents in Weakened Deltas," *Elec. World*, Vol. 55, p. 1382, 1910.

From equations 163 and 164,

$$a'I'_2Z'_e - aI_2Z_e = (a - a')V_t \tag{165}$$

$$I'_2 + I_2 = I_T \tag{166}$$

Solve equations 165 and 166 by determinants or by elimination:

$$I'_2 = \frac{\begin{vmatrix} (a-a')V_t & -aZ_e \\ I_T & +1 \end{vmatrix}}{\begin{vmatrix} a'Z'_e & -aZ_e \\ 1 & 1 \end{vmatrix}}$$

$$= \frac{(a-a')V_t + aZ_eI_T}{a'Z'_e + aZ_e} \tag{167}$$

$$I_2 = \frac{\begin{vmatrix} a'Z'_e & (a-a')V_t \\ 1 & I_T \end{vmatrix}}{\begin{vmatrix} a'Z'_e & -aZ_e \\ 1 & 1 \end{vmatrix}}$$

$$= \frac{a'Z'_eI_T - (a-a')V_t}{a'Z'_e + aZ_e} \tag{168}$$

At no load, $I_T = 0$, but owing to the unequal ratios the circulating current is

$$I'_2 = \frac{(a-a')V_t}{a'Z'_e + aZ_e} \tag{169}$$

$$I_2 = \frac{-(a-a')V_t}{a'Z'_e + aZ_e}$$

When the ratios of turns a' and a are equal:

$$I'_2 = \frac{Z_e}{Z'_e + Z_e} I_T \tag{170}$$

$$I_2 = \frac{Z'_e}{Z'_e + Z_e} I_T \tag{171}$$

$$\frac{I'_2}{I_2} = \frac{Z_e}{Z'_e} \tag{172}$$

At no load, $I_T = 0$, and $I'_2 = I_2 = 0$.

153. Example. Two transformers are to be operated in parallel to supply a load of 200 amperes, at 0.80 pf, lagging. Their ratings are as follows (note unequal ratios):

Transformer 1. 41,000 to 2400 volts 200 kv-a 60 cycles

$$R_e \text{ in secondary terms } = 0.40 \text{ ohm}$$

$$X'_e \text{ in secondary terms } = 0.30 \text{ ohm}$$

$$\text{Ratio of transformation } = 17.1$$

Transformer 2. 42,000 to 2400 volts 400 kv-a 60 cycles

$$R_e \text{ in secondary terms } = 0.20 \text{ ohm}$$

$$X_e \text{ in secondary terms } = 0.30 \text{ ohm}$$

$$\text{Ratio of transformation } = 17.5$$

Solution: Transformation ratios are not the same in each, nor is the ratio of R' to X' in one equal to the ratio of R to X in the other.

The total load current of 200 amperes, expressed as a vector, is

$$200(0.8 - j0.6) \quad \text{or} \quad 160 - j120$$

Then from equation 167:

$$I'_2 = \frac{(17.5 - 17.1)2400 + 17.5(0.20 + j0.30)(160 - j120)}{17.1(0.40 + j0.30) + 17.5(0.20 + j0.30)}$$

$$= \frac{1190 + j420 + 960}{10.34 + j10.38} = 124 - j84.0 \quad \text{or} \quad 149.4 \text{ amperes}$$

Power factor:

$$\cos\theta_2 = \frac{124}{149.4} \quad \text{or} \quad 0.83$$

From equation 168:

$$I_2 = \frac{17.1(0.40 + j0.30)(160 - j120) - (17.5 - 17.1)2400}{10.34 + j10.38}$$

$$= \frac{1665 - 960}{10.34 + j10.38} = 38.2 - j34.0 \quad \text{or} \quad 51 \text{ amperes}$$

Power factor:

$$\cos\theta_2 = \frac{38.2}{51} \quad \text{or} \quad 0.75$$

The circulating current at no load would be

$$\frac{(17.5 - 17.1)2400}{17.1(0.40 + j0.30) + 17.5(0.20 + j0.30)} = \frac{960}{10.34 + j10.38}$$

and hence the circulating current $= 46.2 - j46.4$ amperes

$$= 65.5 \text{ amperes}$$

Suppose that the ratios of the transformers had been the same: 42,000/2400. The results would have come as follows:

The circulating current at no load would be zero. From equation 170:

$$I'_2 = \frac{0.2 + j0.3}{(0.4 + j0.3) + (0.2 + j0.3)} 200(0.8 - j0.6)$$

$$= 76.5 - j36.6 \quad \text{or} \quad 84.8 \text{ amperes}$$

Power factor:

$$\cos \theta'_2 = \frac{76.5}{84.8} \quad \text{or} \quad 0.902$$

$$I_2 = \frac{0.4 + j0.3}{(0.4 + j0.3) + (0.2 + j0.3)} 200(0.8 - j0.6)$$

$$= 83.5 - j83.4 \quad \text{or} \quad 118.4 \text{ amperes}$$

Power factor:

$$\cos \theta_2 = \frac{83.5}{118.4} \quad \text{or} \quad 0.706$$

The total current is:

$$I'_2 + I_2 = (76.5 - j36.6) + (83.5 - j83.4)$$

$$= 160 - j120$$

This is the same as the load current of 200 amperes at 0.8 lagging pf. However, the arithmetical sum of the currents is

$$84.8 + 118.4 = 203.2 \text{ amperes}$$

This exceeds the load current because the pf of each of the two transformers is not the same as that of the load.

In exceptional cases where the exciting currents are very large, the more complicated equations for the more exact equivalent circuits may be set up and solved in terms of vector algebra. Such an analysis will be omitted here.

CHAPTER XVIII

SPECIAL TRANSFORMER TYPES

154. Chapter Outline.

Instrument Transformers.[1]
- Current Transformers.
- Potential Transformers.
  - Errors.
    - Ratio.
    - Phase Angle.

Autotransformers.
- Theory.
- Effective Ratios.
- Vector Diagrams.
- Comparison with Two-winding Type.
- Example of Characteristics.

Three-phase Transformers.

Constant-current Transformers.

Induction-voltage Regulators.

155. Current Transformers. Current transformers are used in a-c circuits to step down the current in accurate, known ratio for purposes of metering and for the operation of protective and regulating devices.

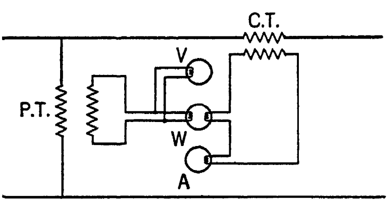


FIG. 114.

The primary winding of a current transformer is connected in series with the circuit, and the secondary is short-circuited through the measuring instruments. A typical diagram is shown in Fig. 114.

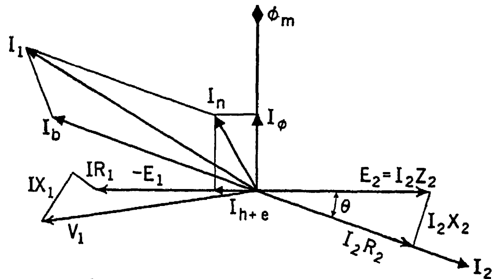
The secondary current of a transformer I_2 is directly opposite in phase from the primary current I_b required to balance the secondary mmf. But since the primary current includes another component, the no-load current, the secondary current

[1] L. T. Robinson, "Electrical Measurements on Circuits Requiring Current and Potential Transformers," *Trans. A.I.E.E.*, Vol. 28, Part 2, p. 1005.

will not be directly opposite in phase to the total primary current nor always bear a strictly constant ratio to it. The errors so introduced are called *phase angle* and *ratio errors*, respectively.

The vector diagram of Fig. 115 may be used to explain the reactions which occur in a current transformer with a secondary load.

The voltage built up in the secondary is used up in the IR and IX drops of the instruments and in the secondary winding of the transformer. To produce such a small voltage (E_2) requires comparatively



θ is determined by the constants of the meters connected in the secondary circuit

FIG. 115.

few flux lines. Consequently I_n will be very small and will add but little additional current to I_b.

It should be kept in mind that the voltage across the primary of a current transformer has nothing to do with the voltage rating of the circuit in which it is placed, but of course it must be well insulated with respect to the secondary and ground. The diagram shows that V_1 is made up of the impedance drops in the primary plus the impedance drops in the secondary circuit (referred to the primary). It is at most a few volts.

For a given I_1, when the impedance of the secondary load increases, it is necessary that E_2 increases also. To increase the induced voltage, the mutual flux must increase. If the impedance in the secondary is too high, the iron becomes almost saturated; then an excessive part of I_1 serves as exciting current. Under these conditions I_1 does not bear a constant ratio with respect to I_2 and shifts its phase position from that desired, viz., 180° from I_2.

It is possible to compensate for the ratio error by making the actual ratio of turns different from the nominal current ratio. Thus a transformer rated at 2500 volts, 100 to 5 amperes, might have an actual turn ratio of 4.98 to 100. The 2500 volts on the rating implies that the transformer is insulated to be safe on a line of such potential. Such

"compensation" overcomes ratio error at one value of load only. Phase-angle error cannot be satisfactorily compensated. Its error can be allowed for by calculation.

When the secondary circuit of a current transformer is opened and the primary winding carries current, there is no demagnetizing effect of the secondary ampere turns. The mutual flux increases because now all the current flowing through the primary winding is magnetizing.

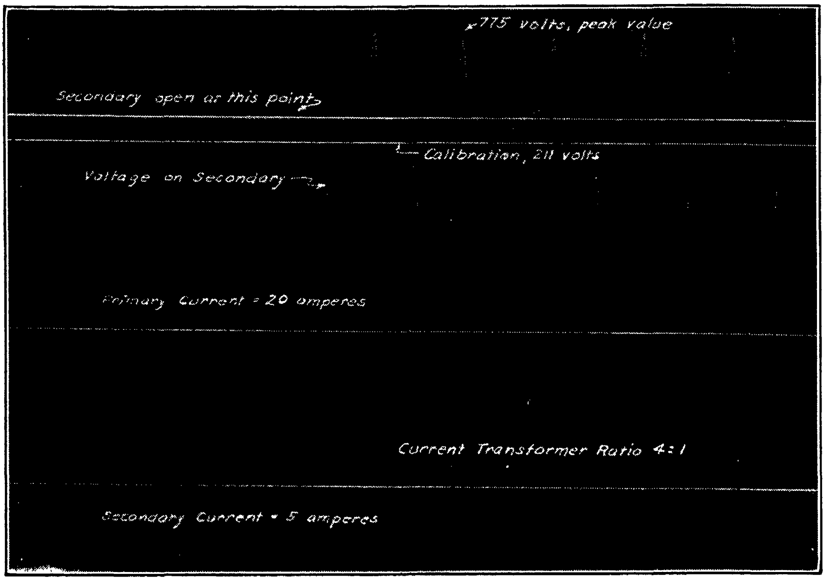


FIG. 116. Effect of opening the secondary of a current transformer carrying rated load. The secondary voltage above represents the voltage drop across a protective resistance of 4000 ohms

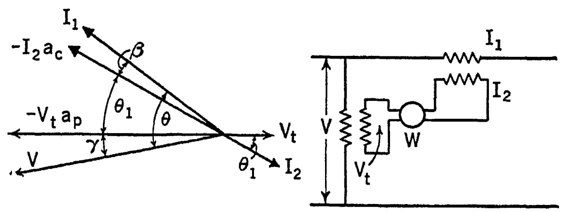
Owing to the large flux, which is limited only by saturation, the induced voltage of E_2 may become great enough to endanger life or puncture the insulation. For that reason, and because of the effect on the calibration of the transformer, the secondary winding should never be opened if the primary carries current. The increase in secondary voltage on open circuit is shown by the oscillogram of Fig. 116.

156. Potential Transformers. The potential transformer is connected across the lines of the circuit in which the measurements are to be made. The secondary circuit contains the voltmeter, voltage coil of a wattmeter, relay, or other device. Such a transformer differs very little from the ordinary transformer except in size. The ratio of voltages is not exactly equal to the turn ratio owing to the impedance drops of the winding. If these drops and the no-load current are small, only a slight

ratio error is introduced at loads within the rating of the transformer. A phase-angle error is introduced by reason of the fact that V_t is not directly opposite from V.

157. Errors.[2] In measuring current or voltage by means of instrument transformers only the ratio errors need be considered for accuracy. In metering power, not only the magnitude of the current and voltage but also the phase angle between them are important. Obviously, phase-angle errors of the instrument transformers affect the accuracy of power measurements, especially at low pf's.

Figure 117 shows the connection for a wattmeter and the instrument transformers. The wattmeter, reading from the secondary circuits,



True ratio of current transformers = a_c
True ratio of potential transformers = a_p

FIG. 117.

indicates $V_t I_2 \cos \theta_1$. The true power in the circuit is $VI_1 \cos \theta$. Owing to the phase-angle error, V_t when referred to the primary differs in position from V by the angle γ. Similarly, I_2 when referred to the primary is β degrees from I_1. Taking into account the ratios of both instrument transformers, the true power in the circuit is

$$VI_1 \cos \theta = a_p V_t a_c I_2 \cos \theta_1 \cdot \frac{\cos \theta}{\cos \theta_1} \qquad [173]$$

Obviously

$$\theta = \theta_1 \pm \gamma \pm \beta \qquad [174]$$

In most cases the phase angle for the potential transformer will be positive, especially if the secondary load is of high pf. At low-lagging pf, the constants of the transformer of normal design are such that the angle γ is negative. That is, $-V_t a_p$ leads V instead of lags (Fig. 117).

[2] E. B. Rosa and M. G. Lloyd, "The Determination of the Ratio of Transformation and of the Phase Relations in Transformers," *Bur. Standards Bulletin* 116.

J. B. Gibbs, "The Accuracy of Current Transformers," *Elec. J.*, Vol. 27, p. 204, April, 1930.

E. C. Wentz, "Ratio Error and Phase Angle in Current Transformers," *Elec. Eng.*, October, 1941.

The phase angle β of a current transformer varies from positive to negative, depending upon whether the secondary load of the transformer has a higher or lower pf than that of the no-load current.

158. The Autotransformer.[3] The autotransformer has a single continuous winding which is used for the input and output voltages; hence the supply and output voltages are not insulated from each other. The autotransformer is used for raising or lowering the voltage just as the ordinary transformer, but differs somewhat in its applications. Because of the danger to life, the autotransformer should not be used to transform from a dangerous voltage to a "safe" one.

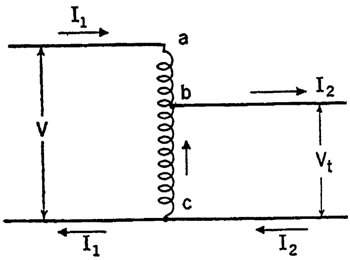


FIG. 118.

An analysis of the currents and voltages of the various parts of the winding follows (refer to Fig. 118).

V is the applied voltage, and the winding ac forms the primary. The portion of the winding shown as bc forms the secondary. Hence the winding bc is common to both primary and secondary, and results in economy of material.

Let the mutual flux linking all the turns be ϕ_m. The induced voltage for the entire winding is

$$E = 4.44 f N_{ac} \phi_{\max} 10^{-8} \text{ volt}$$

And since equal voltages will be induced in each turn, the ratio of transformation will be

$$a = \frac{N_{ac}}{N_{bc}} \qquad [175]$$

Hence the ratio of the induced voltages will be

$$\frac{E_{ac}}{E_{bc}} = \frac{N_{ac}}{N_{bc}} = a \qquad [176]$$

Neglecting the voltage drops,

$$\frac{V}{V_t} = a \text{ (approximately)} \qquad [177]$$

When the transformer is loaded, a secondary current I_2 flows in the direction assumed in Fig. 118. From Kirchhoff's law, as vectors,

$$I_2 = I_1 + I_{cb} \qquad [178]$$

[3] See also Articles 171, 173, and 174.

Obviously

$$I_1 = I_{ab}$$

$$\therefore I_2 = I_{ab} + I_{cb} \quad [179]$$

As in the ordinary transformer, the primary ampere turns must balance those of the secondary (except for the small magnetizing current). Hence

$$I_1 N_{ac} = I_2 N_{cb} \quad [180]$$

and

$$\frac{N_{ac}}{N_{cb}} = \frac{I_2}{I_1} = a \quad [181]$$

This relationship could also be suspected from the fact that IE output equals IE input (approximately), and hence the ratio of currents would have to be the same as the inverse ratio of the voltages.

These current equations have been developed on the external or line currents. Equations 178 and 181 show that currents flowing through the various parts of the winding will have other ratios than those displayed for the load values. Let us calculate the ratio of currents in the turns from c to b and from a to b. All currents are expressed as vectors. Later it will be shown that vectors are unnecessary.

By Kirchhoff's law,

$$I_2 = I_{ab} + I_{cb} \quad [182]$$

Transposing

$$I_{cb} = I_2 - I_{ab}$$

The ratio

$$\frac{I_{cb}}{I_{ab}} = \frac{I_2 - I_{ab}}{I_{ab}} \quad [183]$$

Since

$$\frac{I_2}{I_{ab}} = a$$

then

$$\frac{I_{cb}}{I_{ab}} = a - 1 \quad [184]$$

Hence the ratio of currents in the parts of the winding is $a - 1$.

What are the ratios of voltages in the parts of the winding ab and bc?

The voltages add so that, vectorially,

$$E_{ac} = E_{ab} + E_{bc} \quad [185]$$

Transposed:

$$E_{ab} = E_{ac} - E_{bc}$$

The ratio:

$$\frac{E_{ab}}{E_{bc}} = \frac{E_{ac} - E_{bc}}{E_{bc}} \qquad [186]$$

Since

$$\frac{E_{ac}}{E_{bc}} = a$$

$$\frac{E_{ab}}{E_{bc}} = (a - 1) \qquad [187]$$

The occurrence of this expression $(a - 1)$ in the voltage and current ratios signifies that the autotransformer with a turn ratio of a is equivalent in its transformation to the ordinary transformer with a ratio of $a - 1$. We shall see later how this affects the design economy.

159. Vector Diagrams of the Autotransformer. A voltage of V_{ac} (Fig. 119) is applied to the primary of a step-down autotransformer.

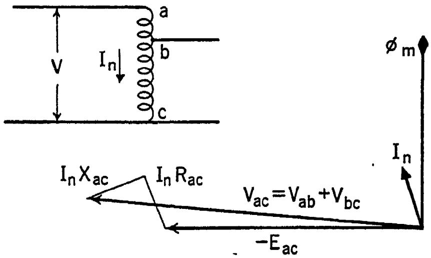


FIG. 119.

This voltage is divided into two parts, V_{ab} and V_{bc}. The self-induced voltage is E_{ac}, caused by the mutual flux ϕ_m. The setting up of this mutual flux requires the no-load current I_n. Resistance and reactance drops in the whole winding subtract from the applied voltage in the usual phase relationship. The balance of the applied voltage is used up in overcoming the self-induced voltage, E_{ac}.

When the transformer is loaded, the vector diagram becomes that of Fig. 120. The no-load current has been increased to an exaggerated value. Since the no-load current flows through the entire winding, it increases the current in ab and because of its relative direction with respect to I_{cb} it will reduce the current in winding cb.

$$I_2 = I_{ab} + I_{cb}$$

Since the no-load current adds and subtracts from the parts of the winding ab and bc, respectively, the vector I_2 is not changed in magnitude or position by its consideration. That is, had I_n been omitted entirely,

the vectors labeled "load component" would be used in place of I_{ba} and I_{cb}. The arithmetical addition of these vectors would have given I_2 also. This, coupled with the fact that the drops are relatively small in a well-designed autotransformer, helps explain why, on the previous equations, vector additions and subtractions are unnecessary. Practically, arithmetical additions of voltages and currents, respectively, give a true picture of the reactions. In small autotransformers for 60

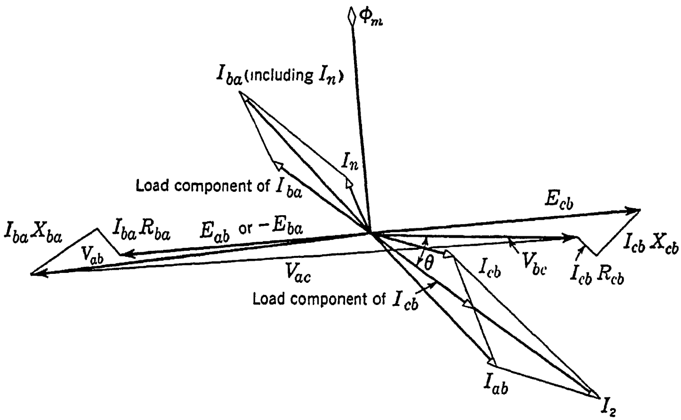


FIG. 120. Vector diagram of the autotransformer with the no-load current greatly exaggerated.

cycles the leakage reactance is usually negligible and the resistance alone then needs to be considered. This assumes that the part *ab* is wound concentrically with the part *bc* in cylindrical windings.

160. Comparison of Transformer and Autotransformer. In the ordinary transformer all the power is electromagnetically transferred from primary to secondary. Referring to Fig. 118, it will be seen that the voltage across *ab* is $V - V_t$. The current in *cb* is less than I_2, being $I_2 - I_{ab}$. Consequently the power transformed instead of being

$$VI_1 = V_tI_2$$

is only

$$(V - V_t)I_1 = V_t(I_2 - I_{ab}) \qquad [188]$$

From this it can be seen that the power transformed is

$$\frac{V - V_t}{V} \times \text{power output} \qquad [189]$$

or

$$\frac{a - 1}{a} \times \text{power output} \qquad [190]$$

If the ratio is changed so that b moves closer to c, $V - V_t$ becomes larger. The power transformed becomes greater and the autotransformer becomes relatively less economical in material. A curve expressing this relationship is given in Fig. 140.

If the ratio of transformation is a, the ratios applying to current, voltage, and turns, respectively, in the parts ab and bc of the windings are $a - 1$.

Thus to convert resistance of bc or secondary turns into equivalent primary resistances multiply by $(a - 1)^2$.

$$r_e \text{ in terms of primary} = r_{ab} + (a - 1)^2 r_{bc} \qquad [191]$$

Similarly

$$X_e \text{ in terms of primary} = X_{ab} + (a - 1)^2 X_{bc} \qquad [192]$$

The reactance is proportional to the turns squared.

Hence:

$$X_{ab} = kN_{ab}^2 \qquad [193]$$

$$X_{bc} = kN_{bc}^2 \qquad [194]$$

In primary terms,

$$X_e = kN_{ab}^2 + k(a - 1)^2 N_{bc}^2 \qquad [195]$$

For an ordinary transformer,

$$X_e = kN_{ac}^2 + ka^2 N_{bc}^2 \qquad [196]$$

The ratio of reactance of an autotransformer to that of an ordinary transformer becomes

$$\frac{X_e \text{ auto}}{X_e \text{ ordinary}} = \frac{kN_{ab}^2 + k(a - 1)^2 N_{bc}^2}{kN_{ac}^2 + ka^2 N_{bc}^2} \qquad [197]$$

Since:

$$\frac{N_{ac}}{N_{bc}} = a$$

$$N_{ac}^2 = a^2 N_{bc}^2$$

and

$$\frac{N_{ab}}{N_{bc}} = a - 1$$

or

$$N_{ab}^2 = (a - 1)^2 N_{bc}^2$$

Substituting in equation 197,

$$\text{Ratio of } X_e\text{'s} = \frac{kN_{bc}^2(a-1)^2 + kN_{bc}^2(a-1)^2}{ka^2N_{bc}^2 + ka^2N_{bc}^2} \qquad [198]$$

$$= \frac{2(a-1)^2}{2a^2}$$

$$= \frac{(a-1)^2}{a^2} \qquad [199]$$

Similarly it can be shown by a consideration of the relative outputs for the same heating, that the ratio of equivalent resistance of an auto-transformer to that of the ordinary two-winding type is

$$\frac{(a-1)^2}{a^2} \qquad [200]$$

On a percentage basis, however, the percentage of voltage drops, the percentage of regulation, and the percentage of losses are less for an auto transformer than for an ordinary transformer (with the same output) by the factor $(a-1)/a$.

161. Example. Two thousand volts are to be stepped down to 1200. The load is 100 kv-a at unity pf.

$$I_1 = \frac{100{,}000}{2000} = 50 \text{ amperes}$$

$$I_2 = \frac{100{,}000}{1200} = 83.3 \text{ amperes}$$

$$I_{bc} = I_2 - I_1 = 33.3 \text{ amperes}$$

The power transformed is:

$$\frac{a-1}{a} \times \text{Power output} = 0.4 \times 100 = 40 \text{ kv-a}$$

The voltage across ab is 800 and the current is 50 amperes. The voltage across bc is 1200 and the current is 33.5 amperes. Consequently the transformer is, in a sense, equivalent to one rated at 800/1200 volts, 40 kv-a.

162. Three-phase Transformers. Instead of using three single-phase transformers on three-phase circuits, a saving in space and cost may be effected by combining the magnetic circuits of the three into one. In general, two types of three-phase transformers are used: the core and

shell types, illustrated in Fig. 121. Other arrangements are shown in Fig. 122.

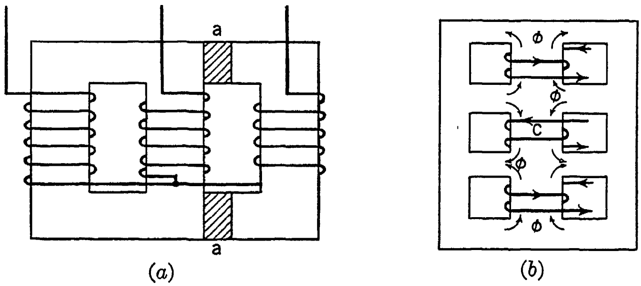


FIG. 121. Common type of three-phase transformers. (*a*) Core type, primary windings not shown. (*b*) Shell type; the three coils are not wound and connected in the same direction. If they were, the center section *C* would be acted upon by three mmf's 120 degrees out of phase and the resultant flux in *C* would be zero. The center coil is connected backward to bring about the flux directions indicated by the arrows.

The relative advantages of three-phase transformers can be outlined as follows:

(*a*) Smaller space, weight, and cost for a given output.

(*b*) A saving in high-tension bushings if only three leads are brought out from the case.

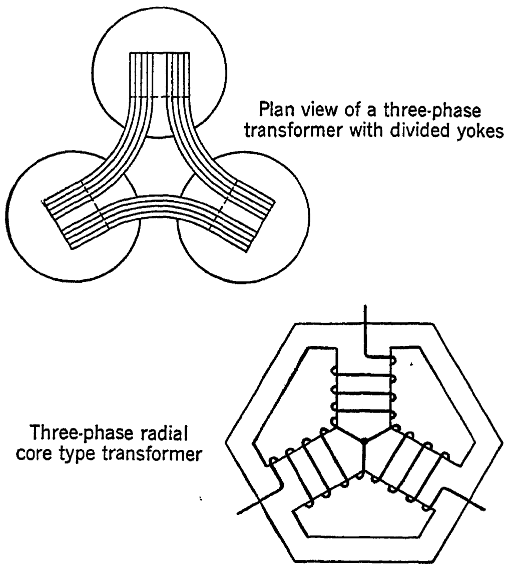


FIG. 122. Other types of three-phase transformer construction.

The disadvantages:

(a) More kilovolt-amperes are put out of service by an accident on one phase as the entire transformer must be taken off the line for repairs.

(b) Greater cost of spare units.

The ordinary single-phase transformer with windings on two legs of its core can be considered as two transformers with each leg transforming one-half of the kilovolt-amperes. If a third leg is added, we would then have a three-phase transformer with a rating of $\frac{3}{2}$ that of the original single-phase transformer. In addition the iron shown cross-hatched in Fig. 121 would need to be added to the core. Hence in designing a three-phase core-type transformer it is merely necessary to design a single-phase transformer of two-thirds the desired rating.

163. Constant-current Transformers. The usual electric circuit is of the constant-potential type in which the current varies as different loads are connected. The series street-lighting circuit represents the most common type of constant-current circuit in which the current is not a variable, but the voltage must vary directly as the number of lamps connected in series. Other methods have been developed for street lighting, but in general the constant-current series circuit presents the most frequent need for a special transformer to regulate for constant current.

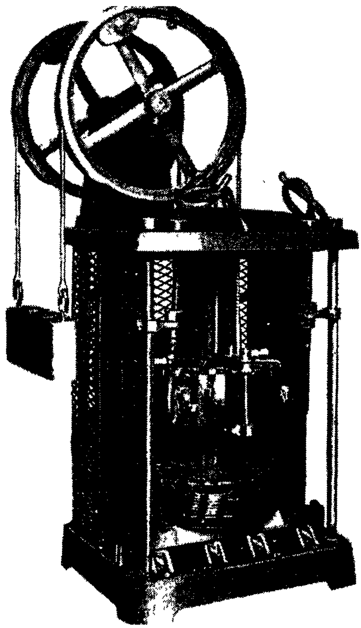


FIG. 123. Constant-current transformer. (*Westinghouse Elec. and Mfg. Co.*)

The constant-current transformer is designed so that the primary and secondary windings are free to move, relatively to each other.

Owing to the phase relationship existing between primary and secondary currents and leakage fluxes, the magnetic fields built around the coils are such as to force them apart when the secondary current increases. If they are free to move apart, the increased leakage flux of each results in large voltage drops in primary and secondary.

Figure 123 shows a common type of street-lighting transformer or "tub." The counterweight is such as to permit the secondary to rest on the primary winding at no load. This gives the maximum secondary

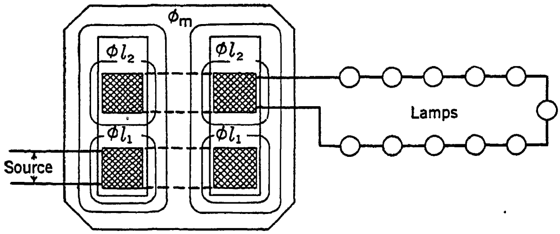


FIG. 124. Schematic diagram of the constant-current transformer and circuit.

voltage. Upon connecting the load, the repulsion between the two windings raises the secondary until its terminal voltage falls to the point necessary to force rated current through the load. The system is in equilibrium when this position is reached, when the current builds up the correct leakage flux to "float" the secondary coil in this position.

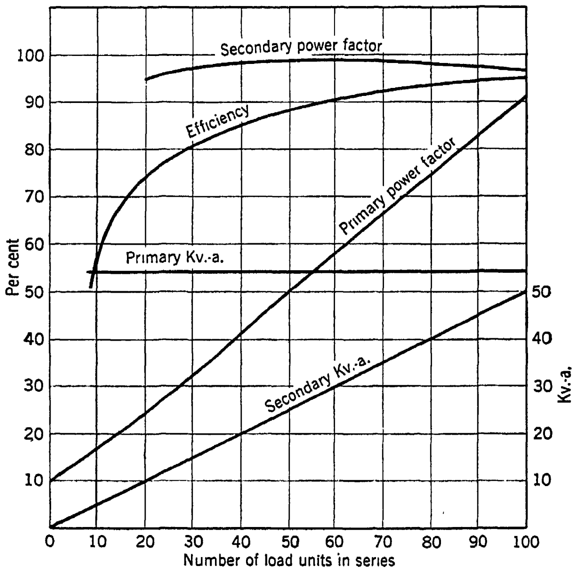


FIG. 125. Typical characteristics of a constant-current transformer.

If the units comprising the load are reduced in number, less voltage is required to force rated current through the circuit, or the same voltage would result in more current. Increased current increases the repulsion

force and moves the coils farther apart. That increases leakage reactance and reduces terminal voltage and current until an equilibrium is reached for this new load condition. Hence the transformer is self-regulating with practically constant output current over the whole working range. Adjustment of the counterweight permits change in current for which the transformer regulates.

164. Operating Characteristics. The vector diagrams of the constant-current transformer under two conditions of load are shown in Fig. 126.

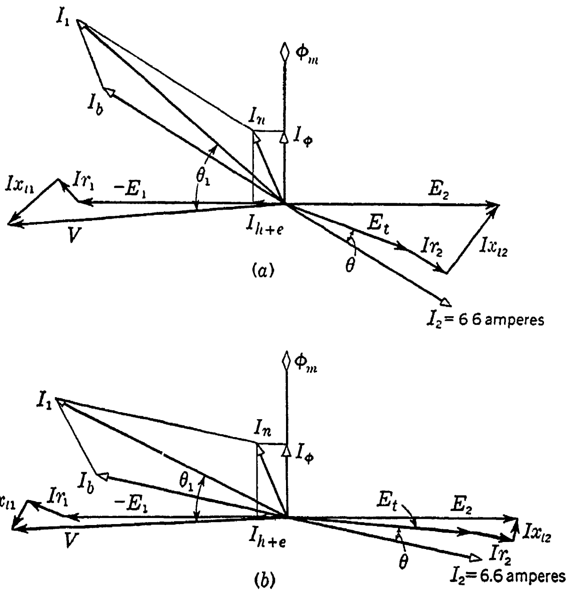


FIG. 126. Vector diagram of the constant-current transformer. (a) Half load. (b) Full load.

Inasmuch as I_2 is practically constant, the load component I_b of the primary current is constant. Change in the magnetizing current at various loads causes a slight variation in I_1. Except for this the constant-current transformer operates at constant primary current and voltage but with a variable primary pf. Even though the secondary load may be purely resistive, the primary pf of such a transformer is very low at light loads, owing to the leakage reactance. This feature is an objectionable one.

165. The Induction-voltage Regulator. This device is essentially a transformer with a variable ratio of transformation. It finds its greatest application on distribution circuits for the raising or lowering of line potential to maintain the terminal voltage at a constant value under all

load conditions. Voltage variation is obtained on the regulator by turning the primary inside the secondary core. This may be done by hand, but usually it is done automatically. A simplified diagram of its connection in a feeder line is shown in Fig. 127.

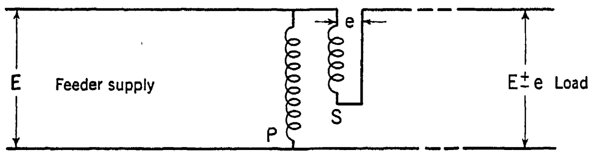
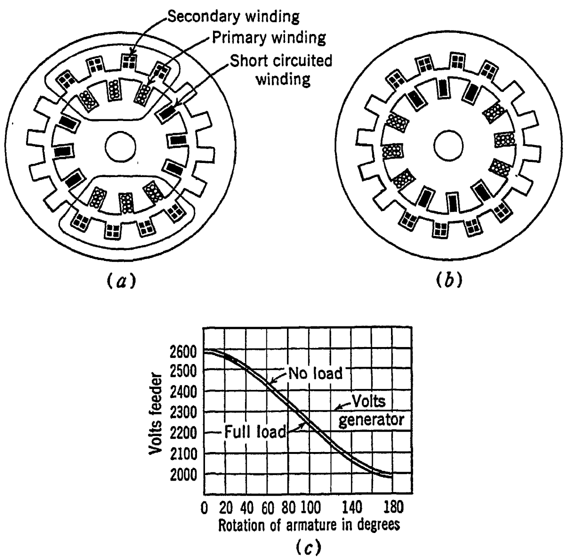


FIG. 127. Induction voltage regulator in a feeder line.

The single-phase construction is illustrated in Fig. 128. The rotor holds the primary winding and also a short-circuited winding. The center lines of these two windings are 90° apart. With the rotor in the position shown, the mutual induction between primary and secondary is a maximum, as is also the secondary voltage. If the ratio is 10 to 1,



(a) Maximum mutual linkages.

(b) Zero mutual linkages.

(c) Curves showing boosting and lowering of feeder voltage by regulator.

FIG. 128. Windings and voltage curve of the single-phase induction regulator.

the feeder of Fig. 127 would have its voltage raised 10 per cent by the regulator.

As the rotor is turned with respect to the stator, the number of flux lines linking the secondary reduces to a final value of zero at the position shown in Fig. 128b. Continued rotation increases the flux linkages and

the voltage until a maximum is again reached at 180° from the initial position. This voltage is reversed in phase with respect to the initial induced value. The result is that the line voltage would then be reduced, say, 10 per cent. We see then that such a regulator is capable of raising or lowering the voltage of a line by an ultimate value, depending upon the ratio for which the device is designed, with intermediate voltages achieved by turning the rotor to various positions.

Assume that the rotor is turned through, say, 45° from the position of maximum voltage. The mmf of half of the secondary coils will be

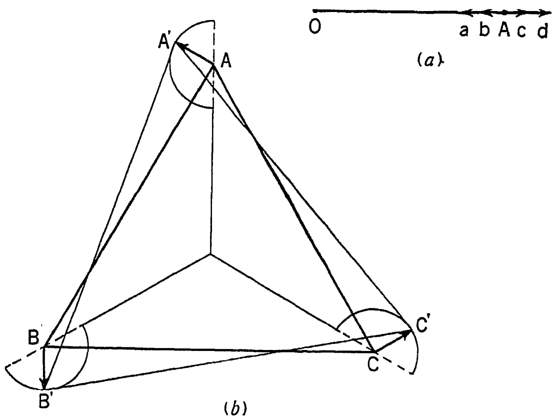


FIG. 129. Vector diagrams of the induction voltage regulator. (*a*) Vector diagram of the single-phase regulator. The original line voltage of *OA* is raised or lowered to *Oc, Od, Ob*, etc. (*b*) Three-phase Y-connected regulator. Original line voltages *A–B–C*. Boosted line voltages *A′–B′–C′*.

unopposed by an mmf of any of the primary coils, and the secondary leakage flux will be large. This raises the leakage reactance of the secondary and opposes the flow of the load current through its coils. If uncorrected, this would reduce the line voltage and pf. To correct this defect, a third or short-circuited winding—previously mentioned—is wound on the rotor in space quadrature to the primary. The current induced in it builds up an mmf which opposes the leakage flux and keeps down the leakage reactance. The short-circuit current varies with the angle between the primary and secondary. This short-circuited winding causes very little loss when sufficient copper is used.

The three-phase regulator is similar to that for single-phase lines, except that three windings each are necessary for the primary and secondary. These windings are spaced 120° apart around the rotor and stator, respectively. No short-circuited rotor winding is necessary.

The action of the flux system built up by the primary windings makes the three-phase regulator essentially an induction motor with the wound rotor blocked. A rotating or spinning flux results in the air gap, cutting the secondary windings. Consequently, for a fixed ratio of primary and secondary turns the induced voltage is constant regardless of the rotor position. However, varying this position does vary the vector relationship between the primary or line voltages and the induced voltages. The result can be seen in Fig. 129*b*. The semicircles are the loci of the secondary voltages which are added to the line. The three-phase regulator is unable to balance the voltages on unbalanced lines as it can supply an identical correction only, between any two lines. Two or three single-phase regulators are frequently used on three-phase lines as the voltage of any one regulator can be entirely independent of the others. Phase unbalance can be corrected also by phase balancers.

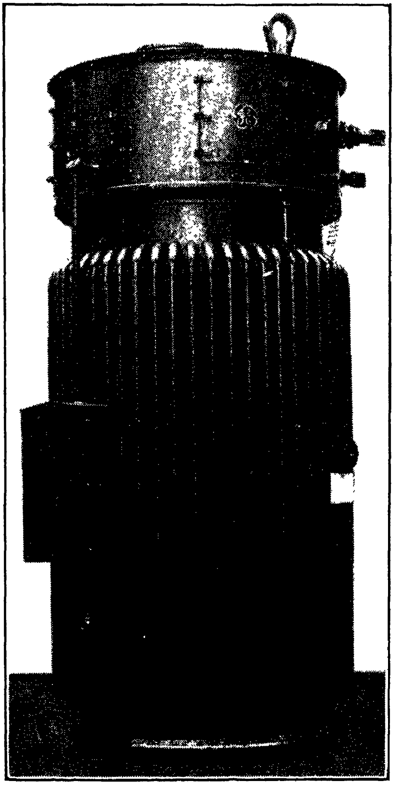


FIG. 130. Three-phase induction voltage regulator for outdoor service. (*General Electric Co.*)

Regulators are seldom manually operated; a small high-speed motor with a worm gear is generally used to turn the rotor to various positions. This motor is actuated from a contact-making voltmeter and from a line-drop compensator so that quick-acting, automatic voltage control can be maintained on a feeder line.

166. Operation of Line-drop Compensator. Let us assume that a feeder line connects a source of supply or transformer bank with a load center some distance away. As the load and pf vary, the voltage at the load will also vary, owing to line-drop variation. This condition is to be corrected by means of an induction-voltage regulator, equipped with line-drop compensation.

Examine the miniature circuit of Fig. 131, in which the current and voltage of *abc* are reduced in accurate ratio from the main feeder line. The percentage R and the percentage X of this miniature circuit are the

same as the respective percentage R and percentage X of the feeder. Then any change in load or pf, bringing about a change in terminal voltage V_t, will bring about a proportionate change in the miniature circuit. Voltmeter readings on this line-drop compensator mirror the voltage at the load end of the feeder line.

As a second step, let us assume that the usual indicating voltmeter is replaced by a *contact-making voltmeter*. For a crude example, imagine that an ordinary voltmeter is equipped with a set of contacts, one at, say, the 108-volt point and another at the 112-volt point. Now as the voltage at the end of the feeder line reduces, owing to load, the line-drop

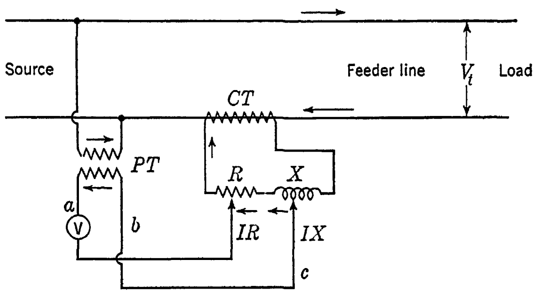


FIG. 131. Simplified connections of the line-drop compensator. The R and X in this miniature circuit have the same effect on the voltage read by this voltmeter as have the R and X of the feeder line on the terminal voltage V_t.

compensator shows a proportionately reduced voltage. The voltmeter swings to 108 volts, closing the contacts between the meter needle and the stationary 108-volt point. This in turn actuates a motor which turns the voltage regulator, thereby increasing the voltage supplied to the feeder line.

If the load reduces on the line, the rise in voltage, mirrored in the compensator circuit, causes the voltmeter needle to swing to the 112-volt point. Again the motor circuit is actuated, but this time so as to turn the motor in the opposite direction. The regulator rotor takes up a new position, supplying the line with reduced voltage. In this way the voltage at the end of the line is kept between predetermined limits.

The setup just described is a rough illustration to emphasize the principles involved. The actual contact-making voltmeter is a sturdy instrument bearing little superficial resemblance to an indicating meter and having adjustable contacts and provisions for reducing "overshooting" and "hunting." The small motor used through gears for driving the regulator is of special design for rapid reversal and acceleration.

CHAPTER XIX

TRANSFORMER CONNECTIONS. POLYPHASE SYSTEMS

167. Chapter Outline.

Transformer Connections.[1]
- Three-phase Systems.
  - Three Single-phase Transformers.
    - Delta-delta.
    - Delta-Y.
    - Y–Y.
    - Y-delta.
  - Two Single-phase Transformers.
    - Open-delta or V.
  - Transformations from Two to Three Phase.
    - T or Scott Connection.
      - Two-winding Transformers.
      - Autotransformers.
  - Autotransformer Connections.
    - Y.
    - Delta.
  - Six-phase Connections.

168. Delta and Y Connections.[2] Using three single-phase transformers, power may be transformed from one voltage to another by any of the following methods:

Primaries in Δ, secondaries in Δ.
Primaries in Δ, secondaries in Y.
Primaries in Y, secondaries in Y.
Primaries in Y, secondaries in Δ.
Mixed connections, such as the "extended delta," etc.

Delta-delta. In this case the ratio of line voltages, primary to secondary, is a, the ratio of transformation. The advantage of this connection

[1] See the series of articles by E. G. Reed in *Elec. J.*, Vol. 16, 1919, for further data on this subject.

[2] F. O. Blackwell, "Y and Delta Connections of Transformers," *Trans. A.I.E.E.*, Vol. 22, p. 385, 1903.

W. T. Taylor, "Transformer Practice," Second Edition, McGraw-Hill Book Co.

lies in its ability to operate at 58 per cent of normal rating in case of injury to one transformer. (See V Connection, Article 169.) Each transformer must be insulated for full-line voltage. Unbalance in load produces only a small unbalance in voltages.

Delta-Y. This connection is frequently used for stepping up voltage, as the secondary coils need not be insulated for the full voltage of the line. For each volt applied to the primary low-voltage winding, *a* volts will be

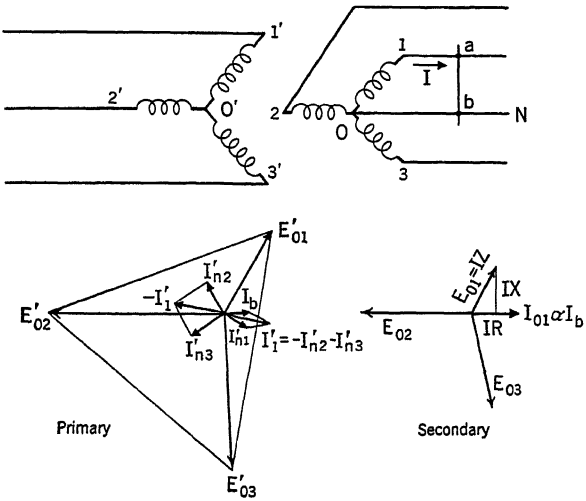


FIG. 132.

induced in the secondary between line and neutral, and 1.73*a* volts between secondary lines.

The grounded secondary neutral wire is commonly used with this connection, and on medium- or low-voltage installations single-phase loads are divided between neutral and outside lines.

Y–Y. This connection works satisfactorily on balanced loads and permits the neutral of each side to be grounded or carried through. If the primary neutral is not connected to the source, unbalanced load causes a roving neutral. That is, the voltage between line wires is fixed by the source, but the voltage between line and neutral point is not fixed on the transformers and can vary from E_{line} to zero with load unbalance.

In Y connections the vector sum of all the currents about the center point must be zero. If the neutral wire is connected from the source to the center of the transformer Y, each phase can work independently of the other; any unbalance in current is then carried by the neutral.

An extreme case is shown in Fig. 132. The transformers are connected Y–Y with the neutral carried on the secondary side only. A load, of such

low impedance as to be practically a short circuit, is connected on one of the secondary legs only. The voltage induced in this leg will be small, being equal to IZ, where Z is the external and internal impedance of the circuit Oab, and I is the current through this short circuit. The primary

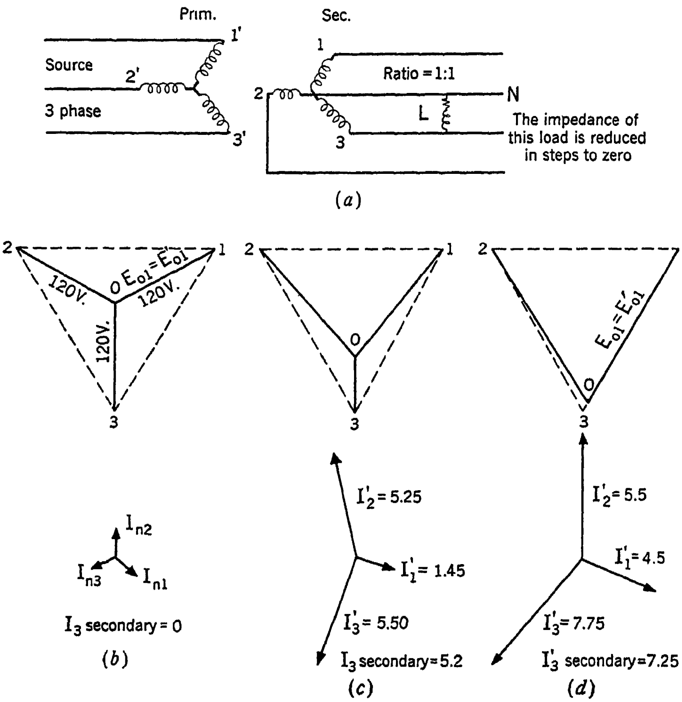


FIG. 133. Successive stages in the unbalancing of a Y–Y transformer bank. Transformers are rated at 1.5 kv-a, 115/115 volts. (*a*) Connections. (*b*) At no-load the primary and secondary voltages, respectively, are balanced. The "star" of primary currents is made up of normal no-load values. (*c*) The load is added at L, causing the neutral to rove. Load current is 5.2 amp and the primary currents are as shown. (*d*) Leg 03 is now short-circuited, causing a short-circuit current of 7.25 amp. I'_1 and I'_2 represent abnormal exciting currents as these transformers are now operating at over-voltage.

component of the short-circuit current is shown as I_b (for the sake of simplicity, the primary and secondary values are not shown opposite to each other on this diagram). The no-load current of transformer 1 is shown as I'_{n1}. Then the total current through the primary of the loaded or short-circuited transformer is I'_1. This is the vector sum of I_b and I'_{n1}. This current, flowing into the neutral point, must have a return circuit, and so divides between legs $02'$ and $03'$. But current cannot flow through

$0'2'$ and $0'3'$ if no load exists on these two transformers; all the current they require is the no-load value.

To understand how the circuit adjusts itself to satisfy these conditions we must see what happens to the voltages under such a load. E'_{12}, E'_{23}, and E'_{31} are all maintained by the source. Consequently the reduction in voltage of E'_{01} means an increase in E'_{02} and E'_{03}. This is shown in Fig. 133. As the voltage must increase on these two legs, the exciting current which they require will go to many times normal because of saturation. These exciting currents are I'_{n2} and I'_{n3}.

With no neutral line carried through on the primary side, the sum of the primary currents about the point 0 must equal zero.

That is, as vectors:

$$I'_1 + I'_{n2} + I'_{n3} = 0$$

or

$$-I'_1 = I'_{n2} + I'_{n3} \qquad [201]$$

Now I'_1 has been shown to be the current supplied to the short-circuited or heavily loaded transformer, and we see from equation 201 that it is limited to the vector sum of the no-load current of the other two transformers.

Figure 133 shows actual tests on three small transformers connected Y–Y. These results show such serious disadvantages as to prevent the use of Y–Y-connected transformers unless the primary neutral is used or absolutely balanced loads are assured on the secondary.

Y-delta. This connection is used very commonly for three-phase transformations, especially with the neutral carried through. When standard transformers are used on distribution networks the usual voltage across primary lines is 4000, as this gives 2300 volts to neutral and permits the use of such standard ratios as 2300/230 volts.

In selecting a scheme of connections for transformers on three-phase lines it is important to choose one that promises the least disturbance from harmonics of exciting current and harmonics of emf which are introduced if the former are not permitted to flow. This is discussed in Chapter XX.

169. Open-delta or V Connection.[3] A common method of transforming comparatively small amounts of three-phase power from one voltage to another uses the open-Δ or V connection, requiring two ordinary transformers. Not all the transformer capacity is effective in this connection, and hence the sum of the kilovolt-amperes of the transformers is greater than the kilovolt-amperes supplied. Analysis of the

[3] L. F. Curtis, "Regulation of V-connected Transformers," *Elec. World*, August, 1917.

diagram of Fig. 134 shows that 86.6 per cent of the transformer capacity is useful.

In the ordinary Δ connection of transformers, if the current in each secondary winding is I, the current in the lines is $\sqrt{3}I$. But on open Δ, line current and transformer current must be the same. This causes a phase difference of 30° between emf and current at unity pf in the individual transformers. Consequently the capacity is only $\frac{2}{3}$ cos 30°

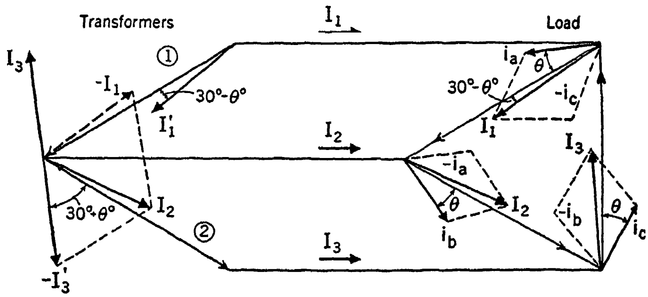


FIG. 134. Vector diagram of two transformers in open Δ, supplying a Δ-connected three-phase, reactive load. On the assumption that such a connection is capable of yielding a balanced supply to a load, such supply currents and voltages can be traced back into the open-Δ connection to determine the vector relationships which must exist therein.

(or $I/\sqrt{3}I$), which is 58 per cent of that available if the third transformer were connected.

If a balanced load is supplied by the V-connected transformers, the currents in the primary and secondary lines, respectively, will be approximately balanced also. It is only at unity pf of the connected load, however, that the two transformers operate at the same pf. As a result of the difference in pf's, the regulation of the two transformers will be different, and slight unbalancing in secondary voltages will result. These conditions may be conveniently analyzed as follows:

$$\dot{i}_a = \dot{i}_b = \dot{i}_c = \text{phase values of the Δ-connected load}$$

$$\cos\theta = \text{pf of load}$$

$$I_1 = I_2 = I_3 = \text{line currents}$$

As vectors:

$$\dot{I}_1 = \dot{i}_a - \dot{i}_c \qquad [202]$$

$$\dot{I}_2 = \dot{i}_b - \dot{i}_a \qquad [203]$$

$$\dot{I}_3 = \dot{i}_c - \dot{i}_b \qquad [204]$$

Line currents are identified at the load and then moved over to the supply transformers. Prime terms refer to secondary values of the V-connected transformers.

$$I_1 = I'_1$$

$$I_3 = I'_3$$

Since for balanced loads, vectorially:

$$I_1 + I_2 + I_3 = 0$$

Then

$$I_2 = -I'_1 - I'_3 \qquad [205]$$

Note that I_2, so derived, agrees with its position in the mesh of the load. The power equations can be written:

$$\text{Transformer 1 supplies } EI'_1 \cos (30° - \theta) \qquad [206]$$

$$\text{Transformer 2 supplies } EI'_3 \cos (30° + \theta) \qquad [207]$$

Obviously the load is equally divided only when $\theta = 0$. Furthermore, when θ is zero the available capacity of any transformer is

$$EI \cos \theta = 0.866EI \qquad [208]$$

Since the capacity is EI for ordinary connection, only 86.6 per cent of the capacity is made available by the open Δ.

170. T Connection with Two-winding Transformers.[4] Power may be transformed from two to three phase, or vice versa, by means of two two-winding transformers, T-connected. The diagram of connections and vector voltages is shown in Fig. 135. It will be seen that the windings of the transformer connected to the two-phase circuit require no special taps. The windings connected to the three-phase circuit require a 50 per cent tap in the "main" and an 86.6 per cent tap in the "teaser" transformer. Consider that the power is being transformed from two to three phase.

It is obvious that, to obtain balanced three-phase voltages, $E_1 = E_2 = E_3$, the voltage *ad*, which represents the altitude of the equilateral triangle of three-phase voltages, must be only 86.6 per cent of the base. The following equations hold for this connection:

$$E_{bd} = E_{dc} = 0.5E_{ca}$$

$$E_{ad} = 0.866E_{bc}$$

[4] For regulation diagrams and calculations see E. G. Reed, "Essentials of Transformer Practice," *Elec. J.*, Vol. 16, p. 28, 1919.

An historical sketch of the development of this T or Scott connection can be found in the article by Charles F. Scott, *Elec. J.*, Vol. 16, p. 96, 1919.

As vectors:

$$E_{ca} = E_{cd} + E_{da} \tag{209}$$

$$E_{ab} = E_{ad} + E_{db} \tag{210}$$

$$E_{bc} = E_{bd} + E_{dc} \tag{211}$$

Since

$$E_{ad} = 0.866E_{ab}$$

and

$$E_{cd} = 0.50E_{ab},$$

then

$$\alpha = 60°$$

Similarly,

$$\beta = 60°$$

The three line-voltage vectors for the three-phase circuit are 120° apart. In the secondary circuits at unity pf load the current in the teaser

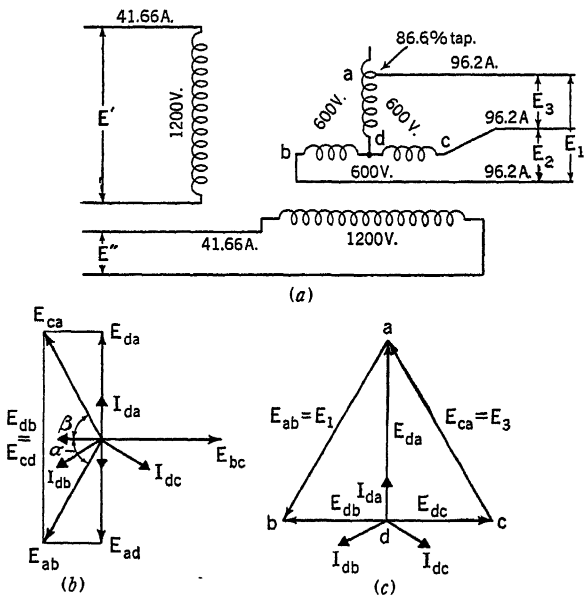


FIG. 135.

transformer is in phase with the teaser voltage; the currents in the halves of the main transformer are out of phase 30° from their respective voltages. This is because these currents are made up of two components: the main transformer load current and the superimposed teaser current.

171. T Connection with Autotransformers.[5] To transform power from two to three phase, or vice versa, T-connected autotransformers are commonly used where the two-phase and three-phase voltages are of roughly the same magnitude. A general case of conversion from three- to two-phase at a lower voltage is shown in Fig. 136. The three-phase supply is connected to two transformers of which 1 is the teaser and 2 is the main. It is assumed that the voltages are balanced and consequently the voltage across transformer 1 is only 86.6 per cent of that across 2. This is because

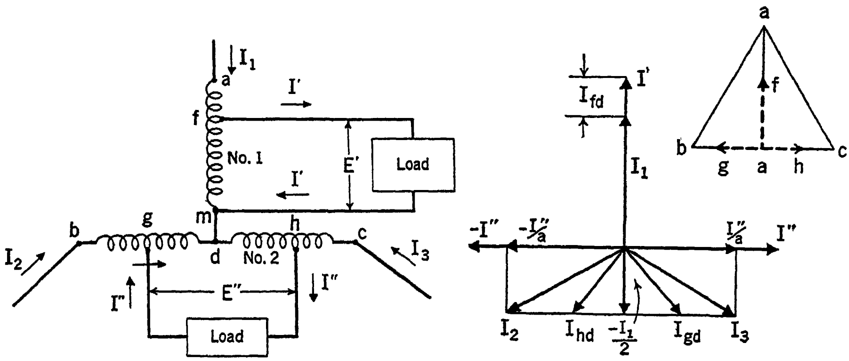


FIG. 136.

abc, considered as a vector diagram, forms an equilateral triangle with an altitude 86.6 per cent of the base. In practice both transformers are usually built identically so that extra winding would be above the point *a*. This is not shown here. Both transformers would be equipped with midtaps, as at *d*, and both with 86.6 per cent taps of which the point *a* represents one.

172. Relative Ratings. It has previously been pointed out under autotransformers, Article 160, that in using autotransformers it is not necessary that all the power be transformed. Instead, some of it flows through the circuit without transformer action. The ratio

$$\frac{\text{Kilovolt-amperes of autotransformer}}{\text{Kilovolt-ampere output}}$$

is a useful one in showing the economy and relative ratings permissible under the two different arrangements.

[5] E. P. Wimmer, "Phase Transformation with Auto-transformers," *Elec. J.*, Vol. 18, 1921.

E. G. Reed, "Phase Transformations with Auto-transformers," *Elec. J.*, Vol. 16, 1919.

Curves giving this ratio for three-phase to two-phase, four-wire transformation are shown in Fig. 137. They assume that teaser and main are not built with equal ratings.

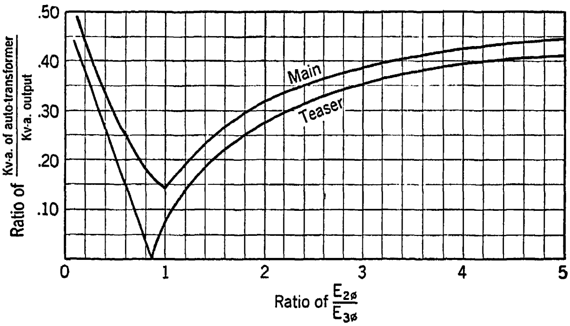


FIG. 137.

Example. Three hundred kilovolt-amperes are to be transformed from 2300 volts, three-phase, to 1200 volts, two-phase, by Scott-connected autotransformers.

$$\frac{E_{2\phi}}{E_{3\phi}} = \frac{1200}{2300}$$

$$= 0.521$$

From the curves:

$$\frac{\text{Kilovolt-amperes of autotransformer}}{\text{Kilovolt-ampere output}} = 0.19 \text{ for teaser}$$

$$\text{Ratio for main} = 0.28$$

$$\text{Output} = 300 \text{ kv-a}$$

$$300 \times 0.19 = 57 \text{ kv-a}$$

$$300 \times 0.28 = 84 \text{ kv-a}$$

The teaser, then, will be approximately the size of an ordinary transformer of 57 kv-a, and the main will be similar to an 84-kv-a transformer. Approximately 141 kv-a are all that are required to transform 300 kv-a. from 2300 volts, three-phase, to 1200 volts two-phase.

173. Autotransformers in Y connection. Autotransformers are frequently used to raise or lower the voltage of a three-phase system. One of the most frequent applications is that shown in Fig. 138*a* for the transforming from a standard voltage between lines to a standard voltage to neutral on a four-wire system.

A general case is shown in Fig. 138*b*. The ratio of transformation is

$$\frac{E_1}{E_2} = \frac{I_2}{I_1} = a$$

Since

$$E_1 = \sqrt{3}E_{01}$$

and

$$E_2 = \sqrt{3}E_{02}$$

the ratio for each leg of the autotransformer becomes

$$\frac{E_1}{E_2} = \frac{\sqrt{3}E_{01}}{\sqrt{3}E_{02}}$$

$$= \frac{E_{01}}{E_{02}} = a$$

The ratio

$$\frac{\text{Kilovolt-ampere autotransformer}}{\text{Kilovolt-ampere output}} = \frac{a-1}{a} \qquad [212]$$

It is obvious that the relationships already derived for single-phase autotransformers can readily be applied to this three-phase transformation.

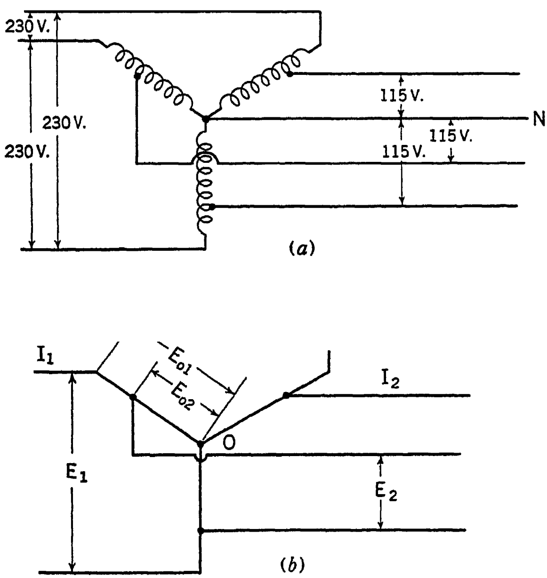


FIG. 138.

174. Autotransformers in Delta Connection. The current and voltage relationships in Δ-connected autotransformers are relatively more complicated. Figure 139 shows the connection diagram.

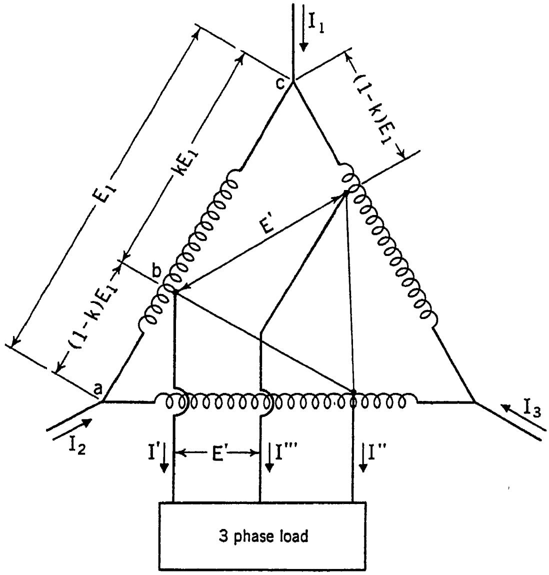


FIG. 139.

The three-phase voltages are to be reduced from E_1 to E' by means of the autotransformers. It is obvious that the ratio of transformation cannot be greater than 2 by this connection.

$$\frac{E_1}{E'} = \frac{I'}{I_1} = a$$

If the ratio of input to output voltages is a, then the turns cb and ca cannot be of the same ratio.

Let

$$\frac{cb}{ca} = k$$

Then k is the ratio of the individual autotransformers and must be determined to obtain the current and voltage relationships of the winding parts.

This can be determined as follows:

(*a*) By direct graphical measurement.

(*b*) By the following analysis, given here in brief form.

$$E' = \sqrt{[kE_1 - (1-k)E_1 \cos 60°]^2 + [(1-k)E_1 \sin 60°]^2} \qquad [213]$$

$$= \sqrt{(\tfrac{3}{2}k - \tfrac{1}{2})^2 E_1{}^2 + (1-k)^2 \tfrac{3}{4} E_1{}^2}$$

$$\frac{E'}{E_1} = \sqrt{(\tfrac{3}{2})^2 k^2 - \tfrac{3}{2}k + \tfrac{1}{4} + \tfrac{3}{4} - \tfrac{3}{2}k + \tfrac{3}{4}k^2}$$

$$= \sqrt{3k^2 - 3k + 1}$$

$$k - \frac{1}{2} = \pm \sqrt{\frac{1}{3}\left(\frac{E'}{E_1}\right)^2 - \frac{1}{12}} \qquad [214]$$

Since

$$\frac{E'}{E_1} = \frac{1}{a}$$

Equation 214 can be written

$$k = \tfrac{1}{2} \pm \sqrt{\tfrac{1}{3}(1/a^2) - \tfrac{1}{12}} \qquad [215]$$

For balanced loads the current and voltage relationships are:
In magnitude:

$$I_1 = I_2 = I_3$$

$$I' = I'' = I'''$$

$$E_1 = E_2 = E_3$$

$$E' = E'' = E'''$$

$$I_{ab} + I_{cb} = I' \qquad [216]$$

$$I' = aI_1 \qquad [217]$$

$$I_{bc} = (1-k)I' \qquad [218]$$

$$I_{ba} = I'k \qquad [219]$$

$$E_{cb} + E_{ba} = E_{ca} = E_1$$

$$E_{cb} = kE_1 \qquad [220]$$

$$E_{ba} = (1-k)E_1 \qquad [221]$$

The ratio of capacity to output in terms of a is

$$\frac{\text{Kilovolt-ampere autotransformer}}{\text{Kilovolt-ampere output}} = \frac{a^2 - 1}{\sqrt{3}a} \qquad [222]$$

175. Other Transformer Types and Connections. It is sometimes advantageous to supply simultaneously two three-phase loads, one at half voltage and the other at full voltage, from a transformer bank with Δ-connected secondaries having midtaps. A common problem then is that of determining the permissible loads that can be carried for different ratios and pf's of the half- and the full-voltage loads. The voltage regulation is also of interest, and it is generally found to be worse when the transformers are thus loaded than when loaded to capacity by the usual method.

This problem can be considered by a rigorous analysis of currents in the various parts of the windings for given loads. It will not be presented

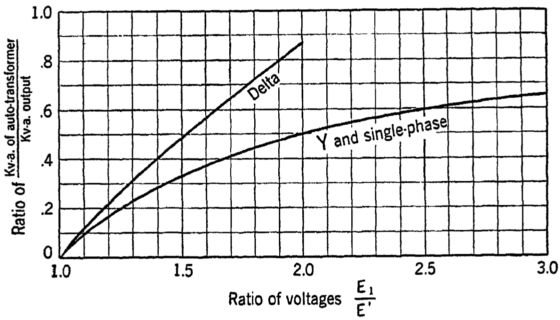


FIG. 140.

here in detail. The interested reader is referred to the analysis and to the useful curves prepared by Larson.[6]

Although no analysis will be given here concerning the three-winding transformer, its equations present an interesting problem. Such transformers, with two or even more "secondaries" are finding an increasing field of application, being useful for supplying various feeder circuits at different voltages from the same primary. Descriptions of applications and analytical methods can be found in the Bibliography.[7]

[6] Noble G. Larson, "Loads on Delta-connected Transformers with Mid-taps," *Elec. Eng.*, September, 1935.

[7] R. R. Lawrence, "Principles of Alternating Current Machinery," Second Edition, McGraw-Hill Book Co.

A. S. Langsdorf, "Theory of Alternating Current Machinery," McGraw-Hill Book Co.

H. P. St. Clair, "The Use of Multi-winding Transformers with Synchronous Condensers for System Voltage Regulation," *Elec. Eng.*, April, 1940.

A. Boyajian, "Theory of Three-circuit Transformers," *Trans. A.I.E.E.*, p. 508, 1924.

F. M. Starr, "An Equivalent Circuit for the Four Winding Transformer," *Gen. Elec. Rev.*, March, 1933.

176. Six-phase Connections. Consider a three-phase transformer, or three single-phase transformers, with split secondaries. For ordinary three-phase operation three secondaries may be paralleled as shown in

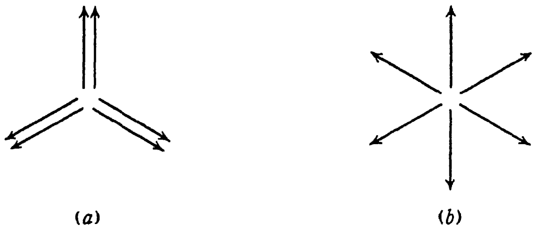


Fig. 141.

Fig. 141*a*. On the other hand, one-half of the secondary of each phase can be reversed, giving a six-phase relationship as shown in *b*. The complete transformer connections for this setup are shown in Fig. 144. The two neutrals can be interconnected. Figure 143 shows the double-Δ

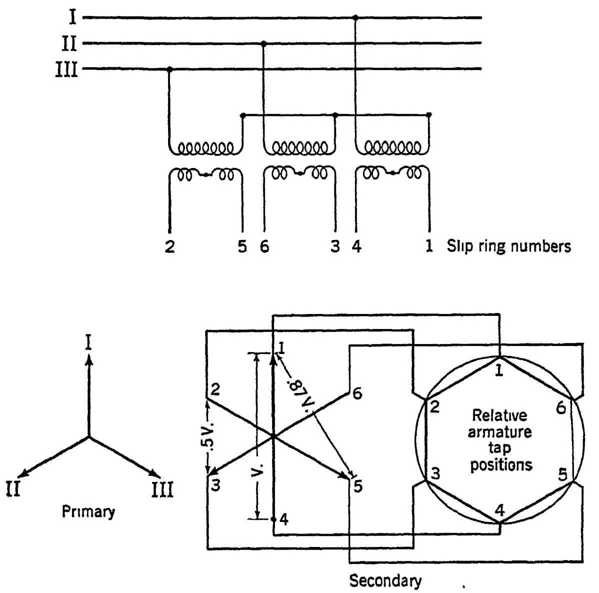


Fig. 142. Diametrical connection. Three-phase to six-phase.

connection for obtaining six-phase. Since one Δ is not tied in, with respect to the other, it is not strictly six-phase, but when connected to a six-phase load, the various voltages always maintain their correct relationships.

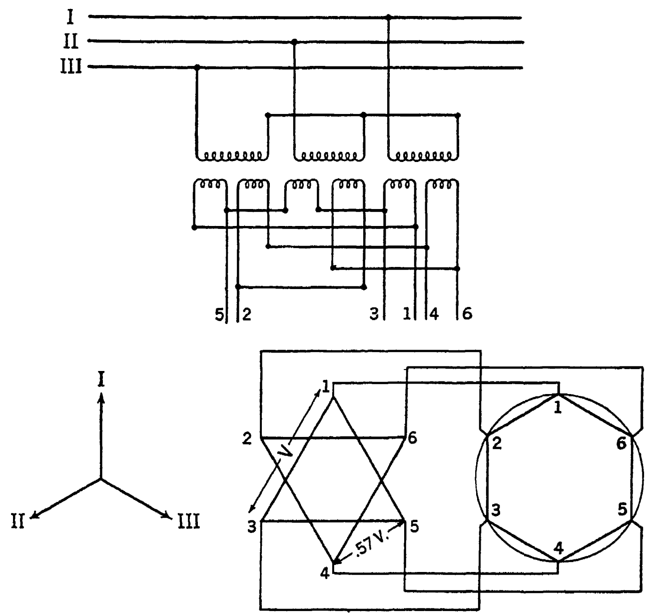


FIG. 143. Double Δ connection. Three-phase to six-phase.

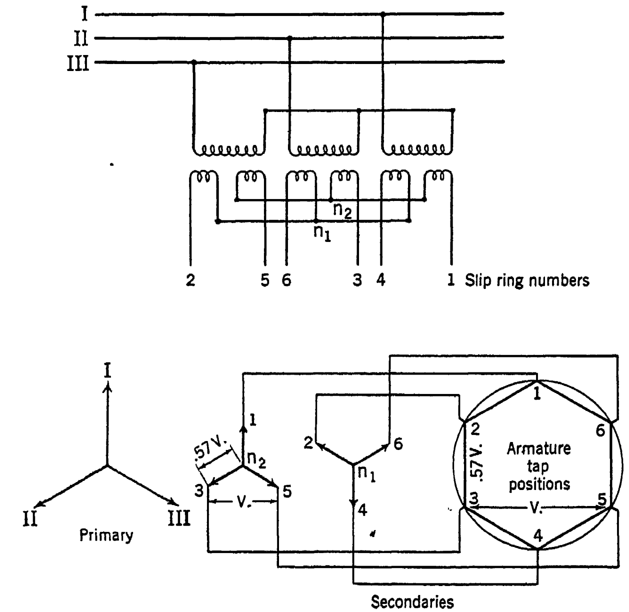


FIG. 144. Double Y connection. Three-phase to six-phase.

One of the most common methods of obtaining six-phase is by the diametrical connection shown in Fig. 142. Split secondary windings are not necessary, but no neutral point is provided on this connection.

Two transformers can be used to obtain six-phase if they are connected in the double-T hookup shown in Fig. 145. To obtain the neutral point,

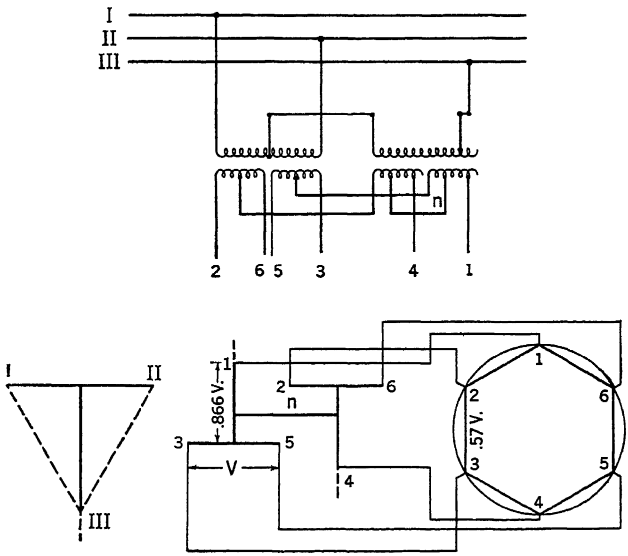


FIG. 145. Double T connection. Three-phase to six-phase.

a special secondary tap is needed. This tap may be eliminated if necessary, but the two T's are then not electrically connected, and true six-phase does not result. However, as with the double-Δ, when tied together on the same load, the correct phase relationships are maintained. This connection can also be used to transform from two- to six-phase.

All these primaries have been shown Y-connected. Delta connections can be used also, although under certain conditions they cause trouble from harmonics.

CHAPTER XX

HARMONICS IN POLYPHASE TRANSFORMATIONS

177. Chapter Outline.

Harmonics in Polyphase Transformations.[1]
- Y Connection with and without a Neutral.
  - Secondaries in Y or Delta.
- Delta connection.

178. Source of Harmonics. The influence of hysteresis loss and of saturation of the iron has been pointed out in Chapter XVI. Except for the resulting error in calculating or reading losses, the wave distortion caused by these phenomena is not very serious for single-phase transformers. In three-phase connections it is frequently an important factor in the choice of connections. To understand the effects of wave distortion on three-phase connections we will recapitulate the governing principles.

(*a*) The exciting current of a transformer is distorted to such a shape as to contain a pronounced third harmonic. (See Article 143.)

(*b*) If for any reason this third harmonic of current cannot exist, the core flux is not a sinusoidal function of time and the induced voltage wave is distorted. Usually the voltage wave is peaked and the flux wave made flat topped. The peaked voltage wave contains a pronounced third harmonic.

(*c*) In the discussion of Y-connected alternators it was pointed out that the displacement of 120° between fundamental emf's results in 3 times 120° between the third harmonics. The voltages between lines, being made up of the vector difference of the phase values, can contain no third harmonic. In dealing with Y-connected transformers, although the physical cause for the presence of the third harmonics is different, the same reasoning still holds. Consequently a third-harmonic emf does not appear at the line terminals.

[1] John J. Frank, "Observation of Harmonics in Current and in Voltage Wave Shapes of Transformers," *Trans. A.I.E.E.*, Vol. 29, 1910.

J. F. Peters, "Harmonics in Transformer Magnetizing Currents," *Trans. A.I.E.E.*, Vol. 34, Part II.

179. Harmonics in Y Connections. Assume three single-phase transformers are connected with their primaries in Y to a three-phase source of sinusoidal voltage.

Neutral. With the neutral carried through from the source to the neutral point of the primary connection, the third harmonic of the exciting current is carried independently through each phase and returns over

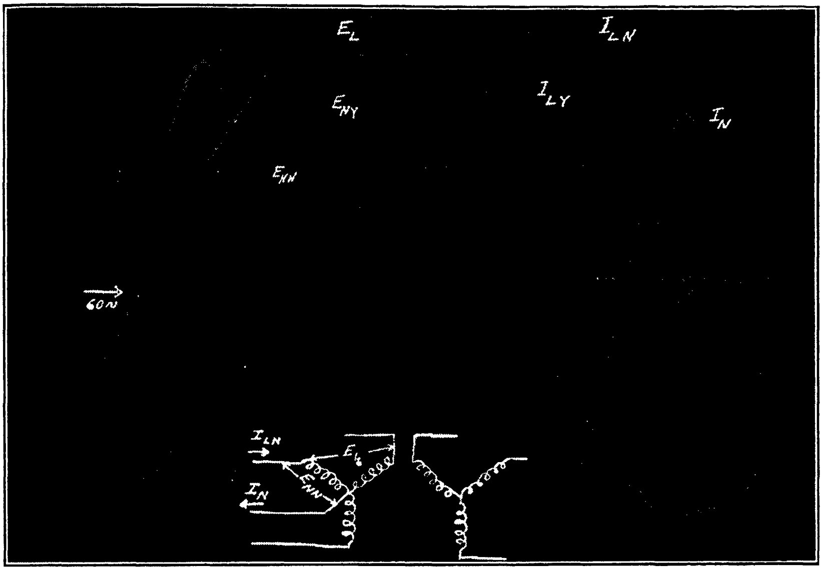


FIG. 146.

E_L = voltage between lines.
E_{NY} = voltage to neutral with neutral open.
E_{NN} = voltage to neutral with neutral closed.
I_{LY} = line current with neutral open.
I_{LN} = line current with neutral closed.
I_N = neutral current with neutral closed.

(*G. V. Mueller.*)

the neutral. These harmonics add directly on the neutral wire so that it carries three times the third harmonic current of one phase. If the neutral is grounded, inductive interference may occur with lines adjacent to the transformer feeders owing to the mutual magnetic flux interlinking both.

No Neutral. The third harmonic of the exciting current of any one phase is neutralized by that of any other. As a result, the flux and induced emf waves are distorted. The third harmonic of the emf's will show up between each outside line and the neutral point, but cannot

appear between lines. Third-harmonic voltages will be induced in the secondaries.

An illustration of these phenomena can be seen in the oscillogram of Fig. 146. The voltage between lines E_L is sinusoidal except for a slight tooth ripple from the generator.

With neutral closed from transformers to source, the voltage to neutral is E_{NN}. The relationship $E_L/\sqrt{3} = E_{NN}$ is obviously true. The

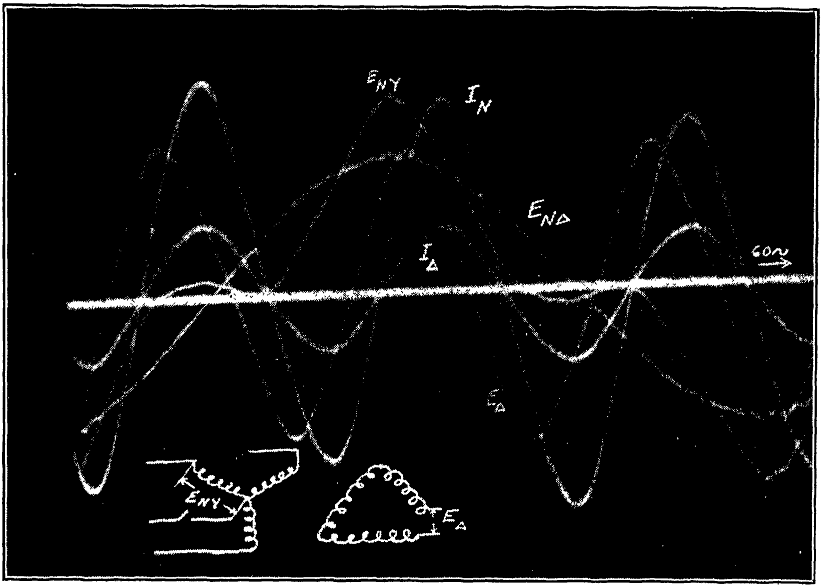


FIG. 147.

E_{NY} = voltage to neutral with neutral open and secondary delta open.
$E_{N\Delta}$ = voltage to neutral with neutral closed or delta closed.
E_Δ = voltage across corner of delta with neutral open.
I_N = current in neutral with neutral closed and delta open.
I_Δ = current in delta with neutral open and delta closed.

(G. V. Mueller.)

distortion caused by opening the neutral can be seen in the effect on the phase voltage E_{NY} which is considerably higher than $E_L/\sqrt{3}$. This changes the nominal ratio of the transformers and results in overstrain of the insulation. When selecting transformers whose windings are to be connected in Y without the neutral wire carried through, such information should be specified because more insulation is required.

The triple-harmonic current carried through the neutral wire can be seen as I_N in Fig. 146.

The Effect on the Secondaries. If the secondaries are Y-connected, no third harmonic will show up between lines. The reactions are similar to those stated for the primary.

When the secondaries are connected in Δ, a path is provided for the circulation of the third harmonics. This secondary third-harmonic current reacts on the primary magnetizing current and does away with the distortion of the primary emf's. The current of triple frequency, circulating in the secondaries, will be no larger than that needed in the primary exciting current. If the core is greatly saturated (which may occur when the load goes off at the receiving end of a transmission line) this component may be as much as 50 per cent of the normal exciting current.

Refer to Fig. 147. With the Δ open and the primary neutral open, the primary phase voltage is the distorted wave E_{NY}. Closing either the neutral line or the secondary Δ results in the emf wave to neutral $E_{N\Delta}$. The third-harmonic current, circulating around the Δ when closed, is shown as I_{Δ}. A voltmeter placed across the corners of the open Δ reads zero when the neutral is closed and E_{Δ} when the neutral is open.

Three-phase, Y-connected transformers are often provided with tertiary windings which are connected in Δ. They are used to permit the circulation of the triple-frequency current and supply the third-harmonic component to the magnetizing current. This does away with the objectionable rise in voltage from line to neutral.

180. Harmonics in Delta Connections. With primaries connected in Δ, each phase receives its voltage independently of the others. The third-harmonic component of the no-load current is free to flow through each winding. The supply lines will contain no triple-frequency current, owing to the combination in those lines of the harmonics of each phase. The third-harmonic current can be measured in the phases, however, by inserting an oscillograph element in the delta.

The delta connection permits sinusoidal emf and flux waves for each transformer on both primary and secondary sides.[2]

[2] For additional reading on the subject of harmonics in polyphase systems see:

Karapetoff and Dennison, "Experimental Electrical Engineering," Vol. II, Fourth Edition, Chapter XL, John Wiley & Sons, Inc. A bibliography is given with that chapter.

Berg and Upson, "Electrical Engineering, First Course," Chapter XXXII, McGraw-Hill Book Co., and the long list of papers on the subjects of harmonics in transformers and inductive interference.

POLYPHASE INDUCTION MOTORS

CHAPTER XXI

PRINCIPLES OF OPERATION

181. Chapter Outline.

The Induction Motor.
  Principles of Operation.
    The Rotating Flux.
    Slip.
    Torque.
    Starting.

182. Introduction. The induction motor was invented in 1888, by Nikola Tesla. It belongs to that general class of apparatus designated as *asynchronous;* i.e., it runs at other than the synchronous speed of its flux. Its good operating characteristics and its simple, rugged, and inexpensive construction have made it of great industrial importance.

The motor consists primarily of a frame or stator, holding the stationary winding to which the supply is connected, and a rotor about which is spread a second winding in which the current is induced. The stator winding is exactly like that used on alternators; the rotor winding usually (1) is of the squirrel-cage type or (2) consists of a winding, the ends of which are brought out to the external circuit by slip rings. A polyphase supply connected properly to the terminals of the stator winding results in a rotating flux in the air gap; this flux can be varied in speed only by changing the number of poles on the stator or by changing the frequency of the supply. Hence, without special means to bring about speed variation, the induction motor is practically a constant-speed device.

The rotating flux cuts past the bars or turns of the rotor and induces a voltage in them. As these turns form a closed circuit, a current flows through the rotor winding, building up a system of currents which follows after the rotating flux of the stator.

The torque results from the action of these currents on the air-gap flux; should the rotor turn at the same speed as the rotating flux, the relative speed between flux and conductors would be zero, no flux would be cut by the rotor turns, and the result would be zero torque. Hence an induction motor, even at no load, must lag a few rpm behind the rotating field (i.e., run at less than synchronous speed) to overcome the slight retarding torque of the ever-present losses. Occasionally, if the magnetic coercive force of the rotor iron is high, the rotor will reach exact synchronism at very light loads after the manner of a synchronous motor with weak excitation.

The addition of load to the motor causes the rotor to drop back still further behind synchronous speed, for by so doing the relative difference between flux and rotor speeds is increased, more emf is induced in the rotor bars, more current is built up in the rotor conductors, and increased torque to handle the increased load results. From the standpoint of the usual "force on a current-carrying conductor in a magnetic field" formula, the torque exerted by the rotor conductors is

$$Bli \cos \theta_2$$

If the air-gap flux density B, resulting from all phases, is practically constant, increased torque supplied for increased load can be explained as follows: Reduced rotor speed also causes increased rotor emf and current; increased i results in increased torque by the above formula. A further discussion of this idea is given in Article 206.

The reduction of speed with increased load is expressed as a "slip." This is the difference between synchronous and full-load speeds. It is used as a fraction or as a percentage, as follows:

$$\text{Full-load slip} = \frac{\text{synchronous speed} - \text{full-load speed}}{\text{synchronous speed}} \tag{223}$$

The slip depends upon the load and the motor design; it rarely exceeds 5 per cent at full load unless the motor is especially designed for speed reduction or high starting torque. High slip motors for hoist, punch press, and elevator service are commercially available, with slips of 10 to 15 per cent.

Inasmuch as the current is induced into the rotor by electromagnetic induction across the air gap from the stator winding, the induction motor is similar in its general theory to a transformer. The stator winding is frequently called the primary, and the rotor winding the secondary. It is possible, however, so to design the motor that the supply is connected to the rotating member; then the squirrel cage would be constructed on the stator. The action would be the same, but to avoid

confusion in the discussion which follows we will assume that the stator winding is connected to the supply, and hence is the primary.

183. The Rotating Flux. A simple illustration of the generation of a rotating flux by a polyphase source is given in Fig. 148. A two-phase

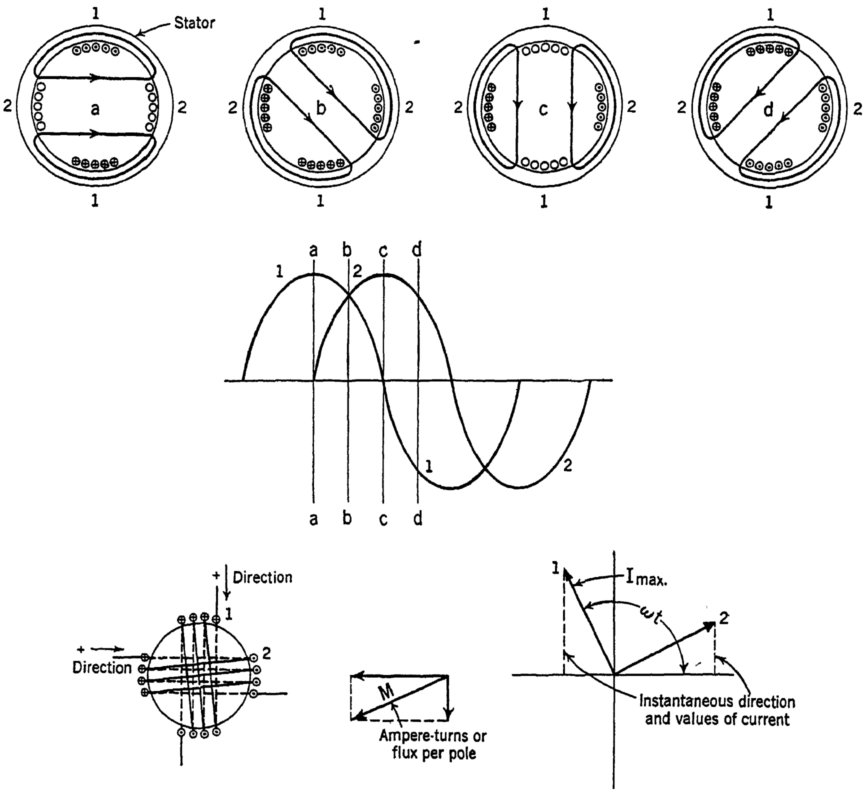


FIG. 148. Two-phase motor. (*a*) Two-pole, two-phase motor field. Supply currents are shown as 1 and 2, respectively. Various instants on the cycle are represented by the four diagrams of the stator winding. (*b*) The space and time relations, showing the action of the two phases considered as vectors.

supply is connected to the respective windings 1–1 and 2–2. The current through one phase winding builds up a pulsating flux which combines vectorially in space with that of the other winding. The resultant flux is shown at four successive intervals; its various positions show it to be a rotating flux, moving through the same number of electrical degrees in space as the current wave covers in time. To accomplish this, it is necessary that the phase windings of the two-phase motors be 90° apart; three-phase motors, 120° apart. The speed of the rotating flux is the syn-

chronous speed; i.e., it is governed by the same equation as previously used for alternator speed and frequency.

$$f = \frac{p}{2} \cdot \frac{\text{rpm}}{60}$$

or

$$\text{rpm} = \frac{f \times 120}{p} \qquad [224]$$

where p = the number of poles per phase.

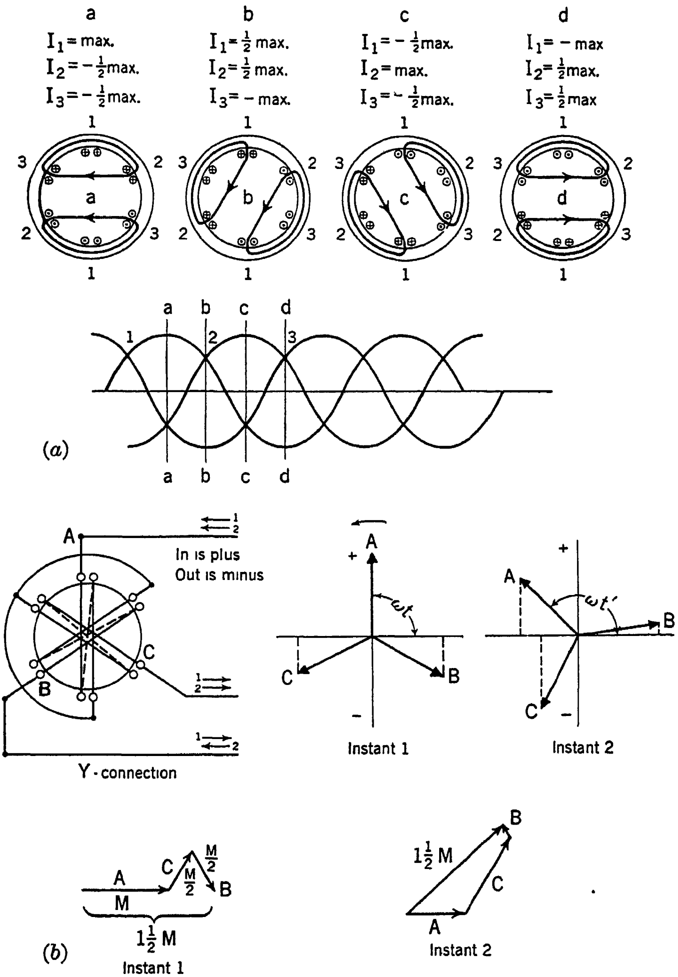


FIG. 149. Three-phase motor. (*a*) Two-pole, three-phase motor field. (*b*) The space and time relations.

Figure 149 shows a simplified winding layout of a three-phase motor stator. This is a 2-pole motor (2 poles per phase), and the instantaneous values of its supply current are shown by waves in *a* and by vectors in *b*.

The resultant flux of these windings at four different intervals agrees with the instands *a–b–c–d*, indicated on the waves.[1]

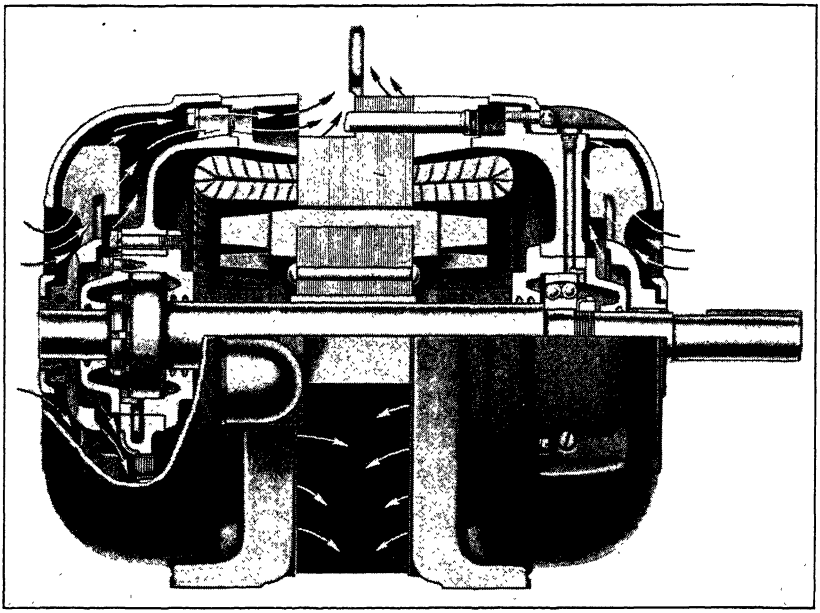


FIG. 150. Sectional view showing motor construction and ventilation. (*Fairbanks Morse & Co.*)

It will aid in the understanding of the induction-motor rotating field to point out that it is identical with the revolving mmf of armature reaction in a polyphase alternator. It has already been shown that, when load current flows through the alternator armature winding, armature reaction builds up a flux system, stationary in space *with respect to the field*, i.e., rotating at synchronous speed. In the motor, we introduce that current from the supply leads; what was before an armature reaction flux now becomes the rotating flux by which we get motor action. (See Chapter VII.)

[1] A simple method of constructing wire models by which the rotating fields can be illustrated is given in an article by George K. Parman, "Shadow Pictures Illustrating the Magnetic Fields in Induction and Synchronous Motors," *Gen. Elec. Rev.*, July, 1929.

184. The Rotor Winding. The squirrel-cage type of rotor is most generally used. The bars run through slots in the iron core and are connected at the ends to form a closed circuit. At standstill, the rotating flux sweeps past these conductors and induces in them emf's practically opposite in direction to the emf acting along the stator conductor directly outside. These emf's cause a flow of current through the bars and end rings, lagging behind, as determined by the resistance and leakage reactance of the rotor circuit. The current circulating through the rotor winding builds up poles of mmf around the periphery of the rotor. Examination of Fig. 151 will make clear the important conception that *as many poles are induced around the rotor as there are poles on the stator.* Hence, since the rotor winding is symmetrical, a squirrel cage for, say, a 4-pole induction motor would have built up, around its periphery, 2 or 6 poles, respectively, if placed in the stator of a 2- or a 6-pole motor. The mmf in ampere turns produced at any point by the rotor current is simply the sum of the instantaneous currents which flow around the point at that particular instant.

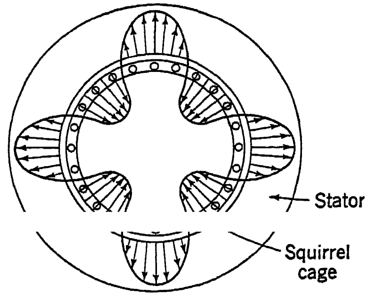
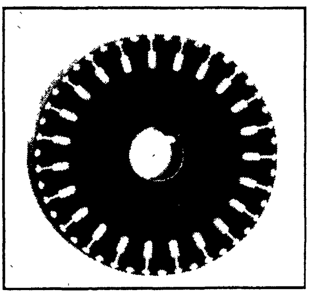
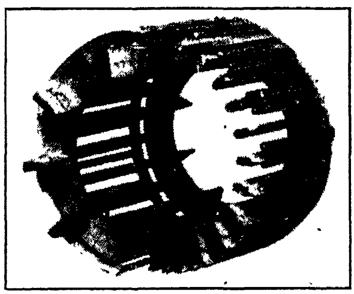


FIG. 151. Flux distribution in an 8-pole induction motor.



(*a*)



(*b*)

FIG. 152. Construction of cast-alloy, double-squirrel cage rotors. (*b*) Punchings removed by acid. (*General Electric Co.*)

The frequency of the rotor emf at standstill will be the same as that of the stator, for each have the same number of poles and the speed of flux change is the same through each. If the rotor should turn at synchronous speed and could still build up an emf it would be of zero frequency. A straight-line relationship exists between slip and rotor

frequency as shown in Fig. 153. At a slip of 5 per cent on a 60-cycle motor, the relative speed between rotating flux and rotor is such as to give a frequency to the rotor emf of 0.05 times 60, or 3 cycles per second.

A local leakage flux is built up around the rotor conductors, giving the rotor winding a leakage reactance. This reactance will vary with

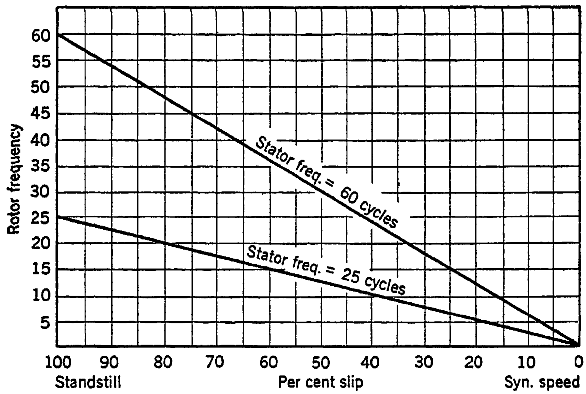


FIG. 153.

the frequency. Hence, on starting, the leakage reactance of the rotor is comparatively large and reduces as the motor gains in speed; it increases with the slip.

Combined with a constant value of rotor resistance, the increasing reactance will change the impedance and pf of the rotor circuit with the slip or load. High reactance of the rotor circuit at standstill means that

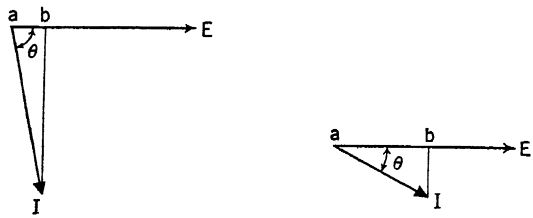


FIG. 154. If the starting torque is represented as proportional to the in-phase component and E and I, it can be shown to be increased by the addition of resistance in the rotor circuit.

the pf will be low when the motor is starting. The rotor current is then out of phase with the rotating flux, and under such conditions, to produce the required starting torque, the starting current may have to be very large. High starting current or low starting torque forms the serious objection to the use of the squirrel-cage induction motor in its simple form.

The most obvious method of correcting this condition is to increase the resistance of the rotor circuit. This will decrease the time phase angle between the flux and the rotor current and give a greater torque per ampere. In operation, high-resistance rotor windings cause increased I^2R losses, reduced efficiency, and increased slip with load. This is one of the problems of induction-motor design: to obtain high resistance in the rotor circuit for good starting torque, and low resistance under running conditions for efficient operation. It is met as follows:

(*a*) A compromise is made, designing the rotor to have fairly good starting torque with slightly decreased efficiency but large starting currents.

(*b*) A wound rotor is used. Instead of a squirrel cage with fixed electric constants, a conventional polyphase winding is placed in the rotor slots, the ends of which are open and brought out through slip rings to an external resistance. At starting, a comparatively large resistance can then be added to the rotor circuit to insure good starting torque at minimum current. After the motor has gotten up to speed, this resistance is cut out (unless it is to be used for speed reduction) and the slip-ring terminals of the winding proper are then short-circuited.

(*c*) Two squirrel cages are used on the same rotor core: one of high resistance and low reactance close to the rotor surface; the other of low resistance and high reactance. At starting, the rotor frequency is high and the high-resistance winding carries most of the current; the low-resistance winding is ineffective because of its high reactance. This gives good starting torque. When running, more of the current is carried by the low-resistance winding and the copper loss is not excessive.

(*d*) The rotor slots are narrow and deep and filled with a single rotor bar. The comparatively high frequency induced in the rotor at standstill causes unequal current distribution in these bars through local eddy currents. The current is crowded to the top of the bar. As the motor gains in speed, reduced rotor frequency permits the current to penetrate into the bar and the entire cross-section of the copper becomes effective. Hence full-load copper losses in the rotor circuit are not excessive.

(*e*) Resistances are used in the bars or end connections and mechanical devices controlled by hand or by centrifugal force are used to short-circuit them out after the motor gains speed. Such methods are no longer of commercial importance.

185. Methods of Analysis. The preceding discussion is an introduction to the more detailed induction-motor analysis which follows. Such analyses can be built up from various points of view. The motor can be considered a special transformer with a mutual air-gap flux inducing emf's in the primary and secondary windings. What might be called the

Kapp-Steinmetz basis assumes that both the primary and secondary windings have a local resistance and reactance and that the mutual air-gap flux requires a primary exciting current to set it up. The vector diagrams which follow will be built up from such a conception.

The Blondel basis starts with the idea that, of the primary flux (or ampere turns), only a fraction reaches the secondary, and the closed secondary winding does not affect this value. When the secondary carries current, it likewise sets up a flux, figured on the assumption of an open primary, of which only a part reaches the primary. The theory and diagrams are built around these premises. Both these bases have their advantages, and both lead to the same numerical results if applied in their most accurate form, but the former is more used in the United States. Other explanations are possible, but the use of the revolving field and the transformer principles is simplest.

CHAPTER XXII

CONSTRUCTION

186. Chapter Outline.

- The Stator.
  - Construction.
  - Effect of Pitch and Distribution.
- The Rotor.
  - Construction.
    - Squirrel-cage.[1]
    - Wound.
- Slots in the Rotor and Stator.
- Effect of the Air Gap.

187. The Stator. The stator of an induction motor is practically identical with the stator of a revolving-field alternator. The core is made up of laminations usually 0.014 to 0.025 in. thick. These may or may not be insulated by varnish. In small motors, or if low core loss is not so important, slightly thicker laminations are used.

Figure 155 shows a partially wound stator; the built-up laminations are held in the yoke by flanges. The ventilating ducts can be seen along the length of the core, spaced every 2 or 3 in. These are provided by the use of spacers placed between laminations.

188. The Rotor. Sheet steel laminations are used to build up the rotor core. In general the same material is used here as in the stator, but, owing to the lower frequencies of the rotor flux, thicker laminations could be used without excessive loss. In small motors the lamination is of one piece; in larger motors the laminations are segmented and dovetailed to a center spider. If the stator core contains ventilating ducts, an equal number is provided in the rotor core. Fan blades are ordinarily used on the ends of the rotor core to force circulating air through the machine.

189. Stator Windings. In general the same windings can be used on the stator of induction motors as were suitable for alternators.

[1] Frederick Miller, "Some Features of the Polyphase Squirrel-cage Induction Motor," *Gen. Elec. Rev.*, October, 1931.

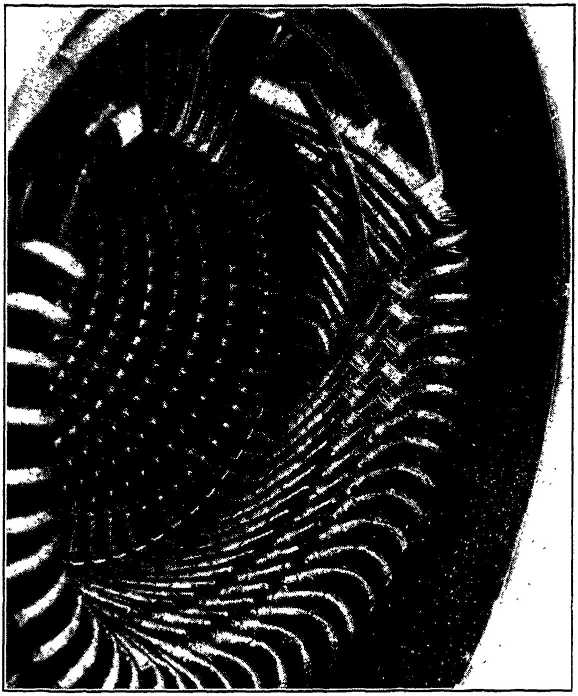


FIG. 155. Partially wound stator of an induction motor. (*General Electric Co.*)

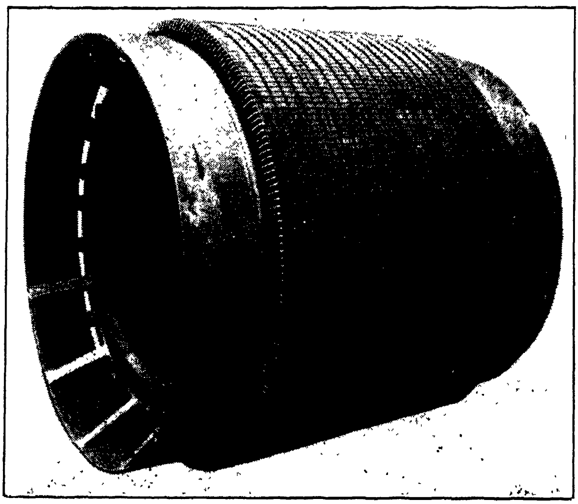


FIG. 156. Rotor ready for winding. (*General Electric Co.*)

The student is referred to Chapter III for a brief discussion of winding types. The double-layer winding is most frequently used on polyphase motors at present because of its greater ease of manufacture, assembly, and repair and because all coils are alike.

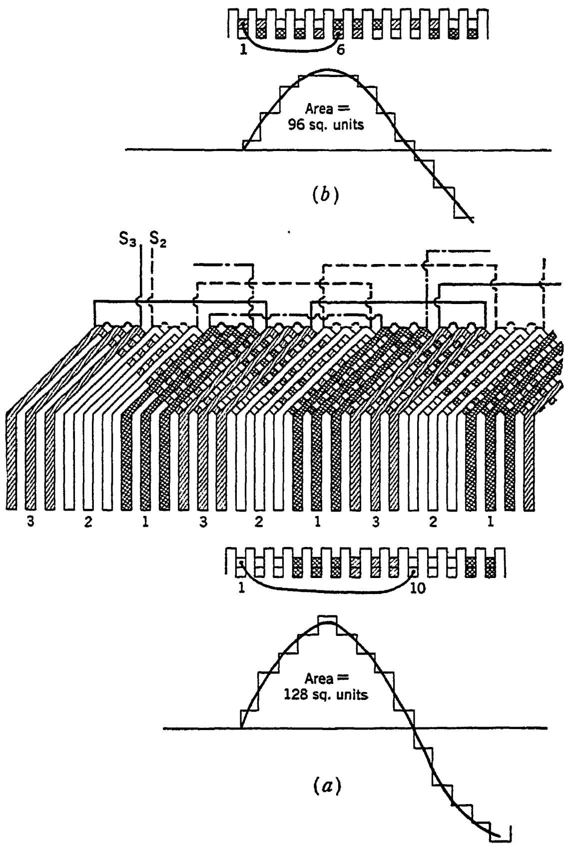


FIG. 157. (a) Section of a full-pitch winding and its mmf. (b) The pitch is shortened so that the coil throw is from slots 1 to 6.

One point which must be kept in mind in regard to induction-motor windings is the effect of short pitch or chording and the distribution of winding on the effective number of turns.

Windings are nearly always made short pitch because of reduced copper weight and winding resistance, as well as reduced leakage reactance and harmonic torque disturbances which result. Chording permits the same coil or coil form to be used with different numbers of poles. In the generator, a short-pitch coil brings about a reduction in the voltage

generated therein and in the mmf of armature reaction. On an induction motor a short-pitch coil gives less effective mmf per pole. The ratio of mmf for a short-pitch to mmf for a full-pitch coil is the pitch factor k_p, calculated exactly as the pitch factor for alternator windings.

Because all the windings per phase per pole are not concentrated in one slot, voltages built up in various coils of one phase winding in an alternator are so displaced as to give a reduced resultant. Similarly in the motor the displacements of the mmf's in space result in a reduced vector sum. The distribution factor k_d expresses the ratio of mmf built up by a distributed winding to mmf of a concentrated winding. It is calculated exactly as for the alternator. In either case, the winding behaves as if its effective number of turns were k_pk_pN.

The product k_pk_d is called the winding factor k_w. Thus if a motor winding consists of 200 turns, pitched and distributed so as to give a winding factor of 0.80, this would signify that the mmf of this winding is only 80 per cent of that which would be obtained if the coils were full pitch and if all windings per phase per pole were concentrated in one slot.

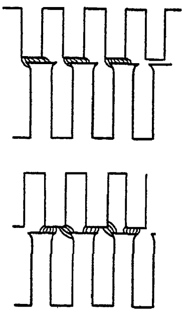


FIG. 158. As the rotor turns, the reluctance of the air gap varies.

The pitch and distribution are effective in limiting the magnitudes of the space harmonics which occur in the air-gap flux.

190. Effect of Slots. Common practice utilizes a number of stator slots equal to some multiple of the number of phases times the number of poles. Offhand it would seem necessary that this be true in order to use balanced voltages on each phase of the polyphase stator winding. To be able to utilize standard punchings in as wide a variety of machines as possible, it is often convenient to the manufacturer to use a layout in which the number of slots is not such a multiple.[2] These cases can be analyzed similarly to part slot windings on alternators, and can result in all phases as a whole being balanced.

The general tendency is to use a large number of slots, as this decreases the effect of variable air-gap reluctance. In Fig. 158 is shown a section of the stator and the rotor teeth in two different positions. The reluctance is obviously a variable at different points on the stator as the rotor turns. This pulsating reluctance results in pulsating flux, irregular torque, increased tooth losses, and noise. On the other hand, an increased number of slots, which will reduce the above effect, results in narrow teeth and increased complication and cost of manufacture.

[2] A more complete explanation of this and the possible combinations can be found in an article by E. M. Tingley, *Elec. Rev.*, p. 116, January, 1915, and in books on design (see note on p. 27).

The effect described above can be minimized by using partially closed slots (see Fig. 159). Such slots increase the working area of the stator or rotor surface without increasing the frame length (giving a reduced exciting current), but give higher leakage reactance to the windings. When used, they complicate the problem of winding, since the coils must

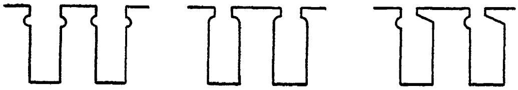


FIG. 159. Various forms of stator and rotor slots.

be fed through the narrow slot width, turn by turn, or if copper bars are used, pushed in through the ends. Both open and partly closed slots are much used for the stator. For the rotor, partly closed slots are used almost exclusively.

191. Rotor Windings. *The Squirrel Cage.* Copper, brass, or aluminum bars are used as the rotor conductors, and are short-circuited on the ends by end rings. The bars are welded, brazed, or bolted to the rings, although some manufacturers build up the rotor core and cast aluminum

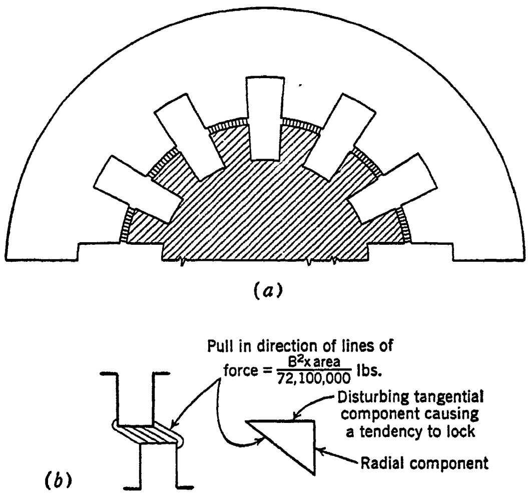


FIG. 160. Stator and rotor with an equal number of teeth. The rotor would lock in this position.

alloy bars into the slots with the end rings as integral parts. It is not necessary that these bars be insulated from the laminated rotor core.

When the number of slots on the stator has been fixed, it is then necessary to choose a suitable number of rotor slots, in order to avoid dead points or locking. Figure 160 illustrates a case with the same number of

stator and rotor slots; if one is any multiple of the other, equally bad results would follow.

The Wound Rotor. The same types of windings can be used for rotors as are used for stators. The flux resulting from the stator windings is a rotating flux with a definite number of poles; operation is only slightly affected by a change in the number of phases provided on the rotor winding so long as it is greater than one, but the number of poles must be the same on each. Three-phase rotors are standard for both three- and two-phase motors and are sometimes used in single-phase motors also.

Bar, strap, or wire is used for rotor windings, the last being used where many turns are desired. A large number of rotor turns increases the secondary voltage and decreases the current which must flow through the slip rings. The secondary voltage determines the insulation which must be provided; furthermore, the voltage and current influence the value of the resistance to be used across the slip rings for starting or speed control. The motor operation is not influenced by the number of turns, but the ratio of transformation is determined by consideration of secondary current, danger of high secondary emf at starting, and distance to secondary resistors.

192. Air Gap. The air gap is one of the two chief sources of the low pf at which induction motors operate. The air gap increases the magnetizing current necessary to set up the air-gap flux; this current lags with respect to the applied voltage. To improve this, it is necessary to use a small (but not too small) air-gap length; this presents mechanical difficulties. The shaft and frame must be rigid enough to prevent deflection and scraping of the stator and rotor teeth, and the bearings must be of a type which does not wear enough to permit such scraping. Stator and rotor must be circular and concentric. The windings, air gap, and slot details must be so selected that the exciting current and machine reactances conform to the performance desired. Reduced gaps may increase motor noise and tooth-face losses, and may prevent the machine from running up to speed.

193. Skew. The slots in the rotor are not always made parallel to the shaft, but are given a twist known as *skew*. The skew angle is illustrated in Fig. 189. Skew has the effect of eliminating noise and cogging; i.e., it results in a smoother torque curve for different positions of the rotor. Other results which may or may not be desirable are:

(*a*) Increase in the effective ratio of transformation between stator and rotor.

(*b*) Increased rotor resistance due to increased length of bars.

(*c*) Increased machine impedance at a given slip.

(*d*) Increased slip for a given torque.

CHAPTER XXIII

ANALYSIS OF OPERATION. THE EQUIVALENT CIRCUIT

194. Chapter Outline.

Analysis of Operation.
- Fundamental Electromotive Force Equation.
- Ratio of Transformation.
- Effect of Winding Resistance and Reactance.
- Analysis of the Vector Diagram.
- The Equivalent Circuit.
- Torque and Power Equations.
  - Exact.
  - Approximate.

195. Introduction. When the windings of a polyphase induction motor are connected to a suitable source of alternating current, the current drawn from the line must have a magnetizing component. This component is required to build up the flux necessary to satisfy the fundamental emf equation:

$$E_{si} = 4.44fN'\phi_{\max} 10^{-8} \qquad [225]$$

If E_{si} is the voltage to neutral, then N' will be the effective series stator turns per phase. So long as the applied voltage remains constant, the air-gap flux will be practically constant regardless of the load.

The applied voltage is "used up" in the following ways: (1) The counter emf built up in the winding, E_{si}, as given in equation 225, accounts for about 98 per cent in large, to roughly 70 or 75 per cent in small, motors. (2) The IR drop of the stator winding must be overcome. (3) The counter emf or self-induced voltage built up by the leakage fluxes cutting the stator and rotor conductors accounts for the remainder. This is calculated as an IX drop for convenience.

In the approximate formulas which follow in this chapter we will assume that the IR and IX drops of the stator winding are so small that E_{si} is equal and opposite to the applied voltage.

If the rotor is held stationary (i.e., blocked), the rotating flux of the air gap cuts the rotor windings and induces in them an emf governed by the equation (225). The rotor voltage depends upon the applied stator

voltage and the ratio of transformation between stator and rotor. So long as the rotor is blocked, the frequency of both is the same. The flux of the air gap, which we are considering here, is the mutual flux linking both rotor and stator windings. The ratio of transformation is different from that of the transformer because the number of physical turns in the stator and rotor is not a true measure of this ratio.

We have already seen from the discussion of coil pitch and distribution that an actual turn may have a reduced effect because of the way it is placed with reference to others. This was also brought out under the study of armature reaction in alternators. Now since it is not necessary that the stator and rotor be wound for the same number of phases, correction must be made for that fact as well. As in the transformer, the mmf of the secondary is balanced and opposed by a component of mmf from the primary. From the expression for the armature reaction of an alternator, we can write

$$\text{mmf} = m_2 N_2 k_{d2} k_{p2} I_2 = m_1 N_1 k_{d1} k_{p1} I_b$$

Substituting $aI_b = I_2$, it follows that

$$a = \frac{m_1 N_1 k_{p1} k_{d1}}{m_2 N_2 k_{p2} k_{d2}} \qquad [226]$$

where m_1 = the stator phases
m_2 = the rotor phases
N_1 = the actual turns per phase on the stator
N_2 = the actual turns per phase on the rotor
k_{p1} = the pitch factor of the stator winding
k_{p2} = the pitch factor of the rotor winding
k_{d1} = the distribution factor of the stator winding
k_{d2} = the distribution factor of the rotor winding

If the rotor is skewed, equation 226 must be divided by

$$\frac{\sin \frac{\alpha}{2}}{\frac{\alpha}{2}}$$

where

$$\alpha = \frac{\text{slots of skew}}{\text{slots per pole pair}} \times 2\pi \qquad [227]$$

Since the rotor circuit is closed, it acts (when the rotor is blocked) like the short-circuited secondary of a transformer, and the rotor current requires a balancing component in the primary winding. That is, the

secondary ampere turns must balance the primary ampere turns, except for that additional primary component needed for magnetizing. Hence:

$$I_b N'_1 = I_2 N'_2 \qquad [228]$$

or

$$\frac{I_b}{I_2} = \frac{N'_2}{N'_1} = \frac{1}{a} \qquad [229]$$

N'_1 and N'_2 are the *effective turns,* i.e., the actual turns corrected by the proper factors.

196. The Vector Diagram.[1] With the rotor blocked, the vector diagram per phase can be laid out as shown in Fig. 161.

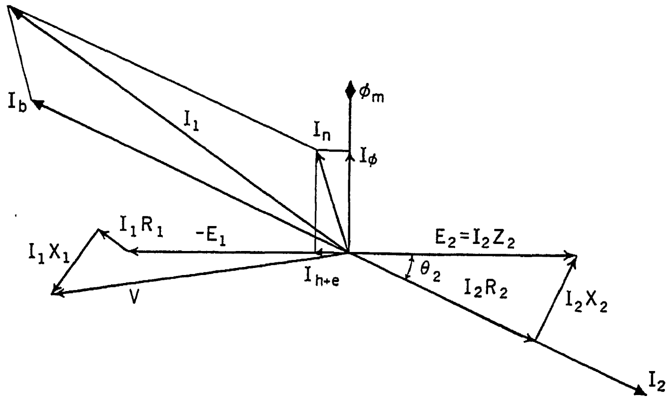


FIG. 161. Vector diagram of the induction motor with blocked rotor. θ_2 is determined by the resistance and leakage reactance of the rotor winding.

The maximum value of the air-gap flux per pole is shown as ϕ_m.

$-E_1$ is a component of the primary impressed voltage which is equal and opposite to the induced voltage of the stator, per phase, caused by the mutual air-gap flux, ϕ_m.

The no-load current is I_n, made up of two components: one quadrature component for magnetizing, I_ϕ; the flux causes eddy current and hysteresis losses in the iron which requires an in-phase component I_{h+e}.

The I_1X_1 drop represents the effect of the flux which links the stator turns only, giving rise to leakage reactance X_1.

The I_1R_1 drop is caused by the resistance of the stator winding.

As vectors:

$$V_{\text{applied}} = -E_1 + I_1R_1 + I_1X_1 \qquad [230]$$

[1] To calculate R_1, R_2, x_1, x_2, I_ϕ, I_n, etc., from drawings or machine dimensions, see works on electrical design; and the literature from about 1905 to 1925.

The voltage E_2, induced in the rotor winding per phase, is determined by the transformation ratio.

I_2 flows through the rotor turns, giving rise to the resistance and leakage reactance drops of the rotor circuit, I_2R_2 and I_2X_2, respectively. All the induced voltage is used up in overcoming these drops. That is,

$$E_2 = I_2Z_2$$

The rotor current I_2 requires a balancing component in the primary winding I_b, differing from it by the ratio of transformation.

The total current per phase on the primary is then the vector sum of the magnetizing component, the core-loss component, and the balancing component. That is, as vectors:

$$I_1 = I_b + I_\phi + I_{h+e}$$

197. Rotor Free. When the rotor is free to turn, the relative speed between the rotating flux and the rotor is reduced to the value:

$$\text{rpm (relative)} = \text{rpm (synchronous)} - \text{rpm (rotor)} \qquad [231]$$

The rotor frequency decreases with the slip as was shown in Fig. 153; or the frequency of the rotor currents can be expressed as

$$f_2 = f_1 s \qquad [232]$$

where f_1 = stator frequency
f_2 = rotor frequency
s = the slip as a fraction of synchronous speed

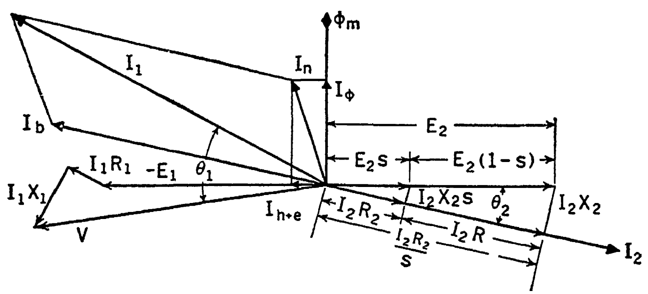


FIG. 162. Vector diagram of a loaded induction motor.

The low-frequency rotor currents build up an mmf which reacts upon that of the stator. This has previously been pointed out in connection with equation 228 and in explaining the stator current I_b. But if the rotor current is of a different frequency from that of the stator,

how can a simple explanation of their reactions hold? This can be explained as follows:

The mmf built up by the rotor is revolving at a speed, with respect to the rotor, of

$$\frac{120f_1s}{p}\text{ rpm} \tag{233}$$

The rotor speed with respect to the stator is

$$\frac{120f_1}{p}(1-s) \tag{234}$$

The speed of the rotor mmf with respect to the stator is the sum, or

$$\frac{120f_1s}{p} + \frac{120f_1}{p}(1-s) = \frac{120f_1}{p} \tag{235}$$

This sum is the same as the speed of the rotating flux or mmf of the stator in space. We can then say that any variable quantity of the secondary, at its reduced frequency, when referred to the stator, reacts at stator frequency. It is only through this fact that we can draw a load vector diagram containing both rotor and stator values.

The reduced frequency of the rotor when running with a slip s reduces the induced voltage of the rotor winding so that:

$$E_2 = 4.44f_1sN'\phi_m10^{-8}$$

Hence, if E_2 is the voltage induced in the rotor per phase when the rotor is blocked, at any slip the rotor voltage becomes E_2s.

198. Power Developed. When blocked, the rotor has a current:

$$I_2 = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \tag{236}$$

The pf of the rotor circuit is

$$\cos\theta_2 = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \tag{237}$$

With the rotor running, rotor current is

$$I_2 = \frac{E_2s}{\sqrt{R_2^2 + X_2^2s^2}} \tag{238}$$

If the reactance of the secondary per phase is measured at stator frequency and found to be X_2 ohms, then at a slip s it will be sX_2 ohms since reactance varies with the frequency.

The rotor current can also be expressed as

$$I_2 = \frac{E_2}{\sqrt{\dfrac{R_2^2}{s^2} + X_2^2}} \qquad [239]$$

This equation indicates that the moving rotor has an apparent resistance of R_2/s ohms. This idea will be developed more fully later, but at this point it is useful in indicating that the rotor input in watts per phase might be expressed as

$$\text{Rotor input} = I_2^2 \frac{R_2}{s}$$

or, using I_2 from equation 236,

$$\text{Rotor input} = \frac{E_2^2}{\left(\dfrac{R_2}{s}\right)^2 + X_2^2} \frac{R_2}{s} \qquad [240]$$

Since $R_2/s \div \sqrt{(R_2/s)^2 + X_2^2}$ equals the pf of the rotor ($\cos \theta_2$) when the motor is running, equation 240 can be written:

$$\text{Rotor input} = E_2 I_2 \cos \theta_2$$

The rotor copper loss in watts per phase is $I_2^2 R_2$, and therefore the power converted to mechanical form must be

$$\text{Rotor input} - \text{copper losses} = I_2^2 \left(\frac{R_2}{s} - R_2\right)$$

$$= I_2^2 R_2 \left(\frac{1 - s}{s}\right) \qquad [241]$$

$$= \frac{E_2^2(1 - s)}{\left(\dfrac{R_2}{s}\right)^2 + X_2^2} \left(\frac{R_2}{s}\right) \qquad [242]$$

Note from equation 241 that $R_2(1 - s/s)$ can be replaced by R since the value in paretheses is a numeric. This leads to the important conception that the rotor power converted to mechanical form can be represented as a resistance load in its rotor circuit.

Example. An induction motor carries a rotor current per phase of 10 amperes when loaded until the slip is 5 per cent. If the rotor effective resistance is 0.1 ohm per phase, determine the internal power developed per phase.

$$R = R_2\left(\frac{1-s}{s}\right)$$

$$= 0.1\left(\frac{1-0.05}{0.05}\right)$$

$$= 1.9 \text{ ohm}$$

$$P_r = 10^2 \times 1.9$$

$$= 190 \text{ watts, internal power developed per phase}$$

The true I^2R_2 loss in one phase of the rotor winding is

$$10^2 \times 0.1 = 10 \text{ watts}$$

This, added to the internal power developed, equals the total power transferred across the air gap per phase, or 200 watts, in this case.

The true $I_2{}^2R_2$ loss bears the same relation to the total power transferred across the air gap as the slip bears to the synchronous speed. That is,

$$\text{Percentage of slip} \times \text{power across air gap} = \text{rotor } I_2{}^2R_2 \text{ loss} \qquad [243]$$

This factor is used in determining the copper loss in squirrel-cage windings, the resistance of which cannot readily be measured. (When the squirrel-cage rotor quantities are referred to the primary, it is the same as replacing them by an equivalent polyphase winding.)

Equation 242 indicates the following:

(*a*) The internal power developed varies with the square of the induced voltage, for any given value of slip.

(*b*) In so far as E_2 is proportional to V, the internal power will vary with the square of the applied voltage. Error: The IR and IX drops of the stator winding prevent V from being equal to $-E_1 \cdot E_1$, of course differs from E_2 only by the ratio of transformation.

199. Vector Diagram. By making use of the relationships which have been developed in the above discussion, the vector diagram of Fig. 162, representing a loaded induction motor, can be drawn. Values are per phase and are referred to the stator.

The vectors representing stator quantities are similar to those of the diagram with the rotor blocked.

E_2 is the voltage induced per phase in the rotor if it were blocked.

E_2s is the voltage actually induced at a slip s.

I_2 is the rotor current per phase. This causes an I_2R_2 drop due to the winding resistance.

The I_2R drop is caused by the load, considering the mechanical load as a resistance.

The leakage reactances are larger in induction motors than in transformers on account of the air gap and the requirements of the mechanical construction.

In order to solve the induction-motor characteristics from such a vector diagram a number of the motor constants must be known. These are listed below. A more detailed analysis of the solution will be given in conjunction with the equivalent circuit.

$$R_1 \qquad R_2$$

$$X_1 \qquad X_2$$

$$I_{h+e} \qquad \text{slip}$$

The diagram can be drawn, assuming I_2 or E_2 and working through to the stator side. A value will then be found for V which will doubtless be different from the rated voltage per phase. It is then merely necessary to change the scale of all vectors on the diagram in the ratio

$$\frac{V_{\text{rated}}}{V_{\text{found}}}$$

The theoretically correct results can then be obtained.

200. Torque.

The general torque equation is

$$P_r = 2\pi \frac{\text{rpm}}{60} T \qquad [244]$$

where P_r = the internal power in watts developed by the rotor per phase. It is the power transferred electromagnetically across the air gap from which the rotor copper loss must be subtracted.

T = the torque in dyne-centimeters $\div 10^7$

rpm = the rotor speed

= synchronous speed $\times (1 - s)$

Since synchronous speed $= 120f/p$, the rotor speed can be expressed as

$$\text{rpm (rotor)} = \frac{120f}{p}(1 - s) \qquad [245]$$

Equation 244 becomes

$$P_r = 2\pi \frac{2f}{p} (1 - s)T \tag{246}$$

Solving for T,

$$T = \frac{p}{4\pi f} \frac{P_r}{(1 - s)} \tag{247}$$

From the previous analyses:

$$P_r = \frac{E_2^{\,2}(1 - s)sR_2}{R_2^{\,2} + X_2^{\,2}s^2} \tag{248}$$

$$T = \frac{pE_2^{\,2}}{4\pi f} \frac{sR_2}{R_2^{\,2} + X_2^{\,2}s^2} \tag{249}$$

Note that when practical units are used in the above equations, the value of torque is in units for which there is no name. In practical work the term *synchronous watts* is much used. This is how the term originated.

In the induction motor the revolving magnetic field turns at synchronous speed while the rotor turns at a lower speed equal to $1 - s$. This means that the power that would be developed if the rotor turned at synchronous speed could be represented by unity, while at slip s the rotor would then develop the power $1 - s$. The difference $1 - (1 - s) = s$ would represent the loss in the rotor copper, in a manner analogous to a slipping clutch. The torque can then be represented by the power that would be developed at synchronous speed, and called synchronous watts. Numerically this would be equal to the rotor input. The power converted to mechanical form is then equal to the product $1 - s$ times synchronous watts. From this fact, the developed torque in synchronous watts corresponding to a given power converted is

$$\text{Synchronous watts} = \frac{\text{watts converted to mechanical form}}{1 - s}$$

Synchronous watts can be reduced to pound-feet of torque by the relation:

$$\text{Pound-feet} = 7.04 \frac{\text{synchronous watts}}{\text{synchronous rpm}} \tag{250}$$

Or to ounce-feet:

$$\text{Ounce-feet} = 112.6 \frac{\text{synchronous watts}}{\text{synchronous rpm}} \tag{251}$$

It should be kept in mind that the power transferred across the air gap may be referred to as "electromagnetic" power, or torque; the

power remaining after the rotor copper losses are subtracted is the "developed" power at developed torque and rotor speed. After friction and windage losses are subtracted, the "output" power occurs at "pulley" torque and rotor speed.

To apply these principles:

Equation 242 indicates that the developed power is

$$\frac{E_2^2(1-s)}{\left(\frac{R_2}{s}\right)^2 + X_2^2}\,\frac{R_2}{s} \quad \text{or} \quad \frac{E_2^2 R_2 s(1-s)}{R_2^2 + s^2 X_2^2} \tag{252}$$

Dividing through by $1 - s$ gives developed torque:

$$\text{Synchronous watts} = \frac{sE_2^2 R_2}{R_2^2 + s^2 X_2^2} \text{ (per phase)} \tag{253}$$

Maximum torque can be determined by differentiating equation 249 or 253 and equating to zero; thus, for equation 249,

$$\frac{dT}{ds} = \frac{p}{4\pi f}\,\frac{(R_2^2 + X_2^2 s^2)(E_2^2 R_2) - E_2^2 s R_2 (2X_2^2 s)}{(R_2^2 + X_2^2 s^2)^2} = 0$$

$$R_2^2 + X_2^2 s^2 - 2X_2^2 s^2 = 0$$

$$R_2^2 - X_2^2 s^2 = 0$$

This signifies that the torque is a maximum when the slip is such as to make

$$sX_2 = R_2 \tag{254}$$

By substituting this in equation 249, the expression for maximum torque becomes

$$T_{\max} = \frac{pE_2^2}{8\pi f X_2} \text{ (dyne-centimeters} \div 10^7) \tag{255}$$

$$= \frac{E_2^2}{2X_2} \text{ (synchronous watts)} \tag{256}$$

Note that these equations use the internal voltage; when V is substituted for E they are only approximately true.

Equation 255 shows that the maximum internal torque is independent of the rotor resistance. But, if the rotor resistance is varied, the slip at which the maximum torque occurs will vary also since sX_2 must equal the new value of R_2. This is shown on the curves of Fig. 163, pertaining to a wound rotor induction motor in which different values of external

rotor resistance were used. Under the condition shown on curve 3, the starting torque (100 per cent slip) is a maximum but the torque is very low at small values of slip. This would provide good starting torque but poor speed regulation. Curve 4 represents excessive rotor resistance so that the maximum torque common to the other conditions is not reached unless the motor is driven against its normal rotation.

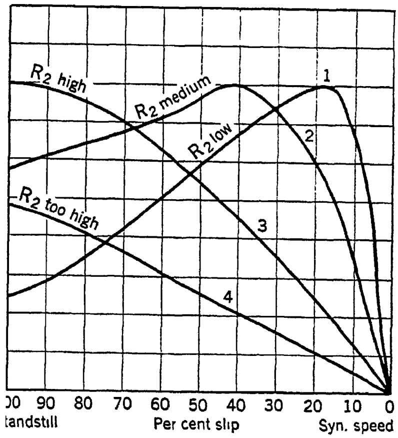


FIG. 163. Speed-torque curves of a wound-rotor induction motor with various values of rotor resistance.

201. Starting Torque. To start an induction motor, its terminals can be connected directly to the supply lines, provided excessive current is not required. If frequent starting is necessary, the troublesome voltage dips on the supply lines caused by excessive starting current may be a serious handicap. The starting current is limited only by the motor impedance. Since this is mostly reactance, the starting current is likely to be of low pf and so exaggerate the voltage regulation of the source. The magnitude of the starting currents is discussed later. The oscillographs of Fig. 164 illustrate the instantaneous rush of current.

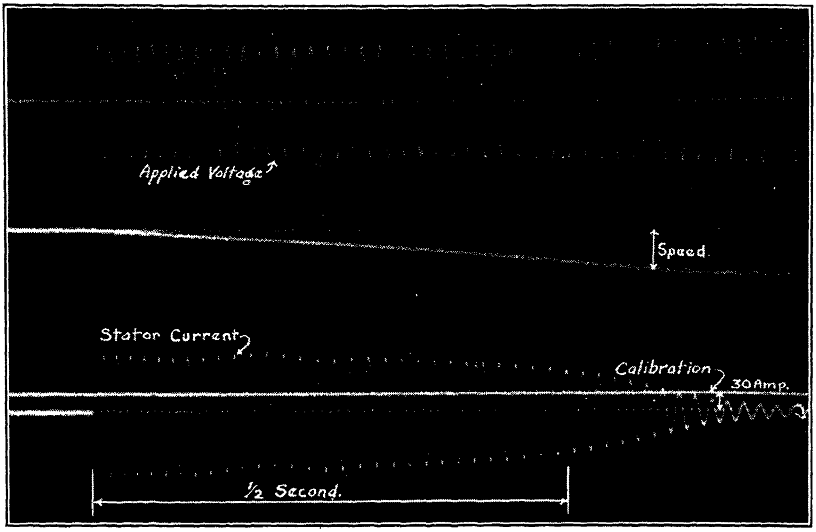
At standstill the rotor frequency and reactance are high. Since maximum torque occurs where

$$sX_2 = R_2$$

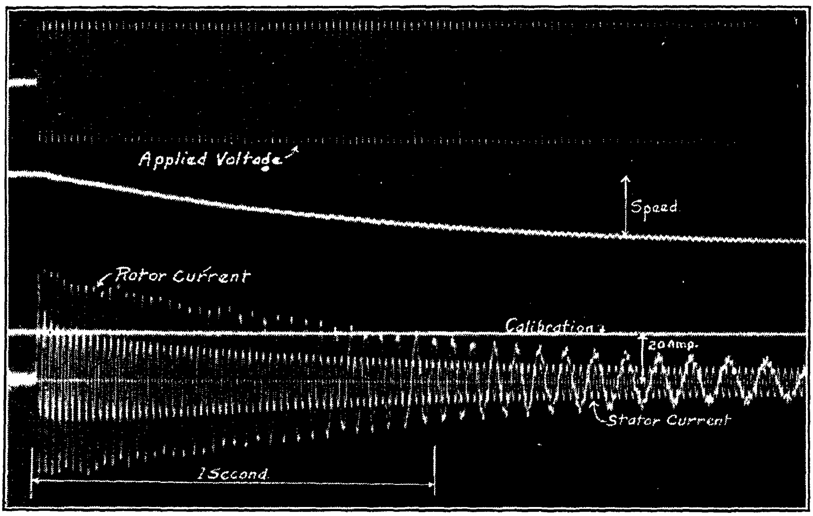
and the slip is 100 per cent at standstill, for maximum starting torque:

$$R_2 = X_2$$

If R_2 is made great enough to give this condition, the motor will have an excessive copper loss and slip under full-load operation. If the wound rotor induction motor is used, R_2 is added externally to give the greatest



(*a*) Starting current of a squirrel-cage induction motor. 5 hp. three-phase. 220 volt. 1800 synchronous rpm.



(*b*) Starting current of a wound-rotor induction motor. 5 hp. three-phase. 220 volt. 1800 synchronous rpm.

Fig. 164.

starting torque and then reduced under load. In the squirrel-cage motor a compromise must be made by the designer.

In general, in synchronous watts per phase:

$$\text{Starting torque} = \frac{E_2{}^2 R_2}{R_2{}^2 + X_2{}^2} \qquad [257]$$

202. Errors in the Torque Equations. The above analysis involving E_2 is in error because of the assumption that it is a constant value. The induced voltage of the rotor will depend upon the applied voltage and the ratio of transformation. If this ratio is unity, then E_2 equals E_1, but a constant applied voltage will not permit E_1 to remain constant at various loads, owing to the stator impedance drop. The above formulas are illustrative only; on motors with a large primary leakage reactance or with a large stator resistance, an analysis based on a constant value of E_2 may be in serious error.

The problem then becomes one of substituting for E_2 in the torque formulas an equivalent expression in terms of applied voltage and stator impedance drop. This will be done after the development of the equivalent circuit theory, and a series of useful equations of greater accuracy will be given.

203. The Equivalent Circuit. The polyphase induction motor can be represented by an equivalent circuit very similar to that used for a transformer. The circuit represents one phase only; usually in three-phase

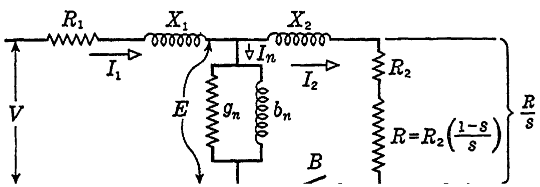


FIG. 165. The theoretically exact equivalent circuit, representing one phase of an induction motor. All secondary quantities are referred to the primary.

motors it is most convenient to have the circuit pertain to one leg of either a Y-connected or a Δ-connected stator. Use is made of the fact that a mechanical load on the motor is equivalent to a non-inductive load in the rotor circuit. Figure 165 shows such a circuit. The applied voltage (say, to neutral) is V, and the stator winding resistance and reactance are represented by R_1 and X_1, respectively. The rotor resistance and reactance are R_2 and X_2, respectively. R represents the load, so that

$$\frac{I_2{}^2 R}{746} = \text{horsepower developed} \qquad [258]$$

Opening the switch at B is equivalent to removing the load from the motor. Under such circumstances the motor would still take a no-load current to supply its losses. The constants of the circuit connected across E are chosen to represent the no-load condition. That is, g_n is such as to permit a current to flow through it equal to the eddy current and hysteresis loss current I_{h+e}; b_n is such that the current flowing through it equals the magnetizing current I_ϕ. Or:

$$\text{Core loss} = E^2 g_n \tag{259}$$

and

$$b_n E = I_\phi \tag{260}$$

Since E and V are nearly equal at no load, V can be substituted in the above equations with little error.

No provision is made in this circuit for the friction and windage losses of the motor. For strict accuracy, they should be subtracted from the power transmitted across the air gap, to give the net output.

To make use of this circuit it is necessary to know or estimate:

(*a*) The core and friction loss.

(*b*) The magnetizing current. This can be calculated from the no-load current and the no-load pf, or from the machine dimensions.

(*c*) The stator resistance and reactance.

(*d*) The rotor resistance and reactance.

(*e*) The ratio of transformation.

Obviously the circuit as shown in Fig. 165 has all its terms referred to the stator, and R_2 and X_2 must both be so referred by the use of the

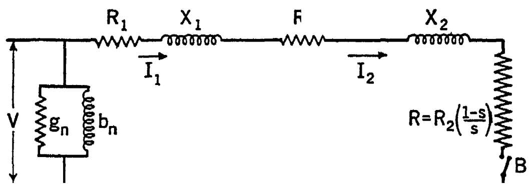


FIG. 166. The approximate equivalent circuit, representing one phase of an induction motor. Under certain conditions this may approximate the physical relationships more closely than the circuit of Fig. 165.

transformation ratio. The usual method of making use of such a circuit, with the above items known, is to assume a slip and solve for R. Then, by solving the circuit, I_2 and I_1 can be found for this condition. For that value of slip, the output (in watts) can be found as $I_2{}^2R$; the input is $VI_1 \cos \theta_1$. All the characteristic curves can be obtained such

as are shown in Fig. 175. Then another value of slip is assumed and other points obtained.

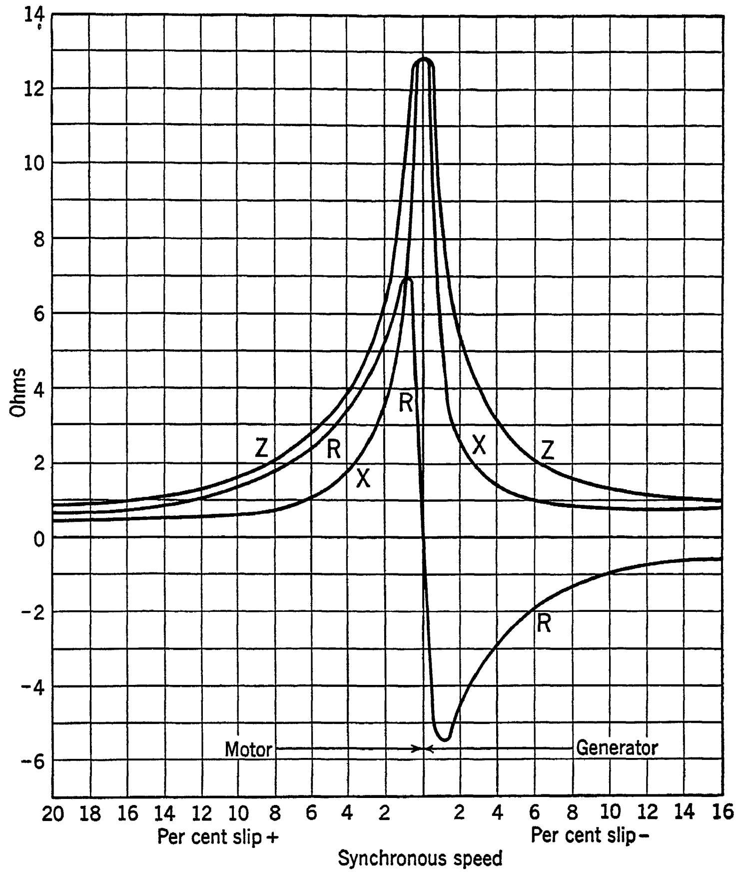


FIG. 167. If the induction motor is considered as a circuit, the circuit constants will vary with the slip as shown. Running above synchronous speed changes the motor to a generator and the resistance becomes negative

Unfortunately, by the above method it is only through trying different values of slip that the performance at rated output is obtained.[2] If a 25-hp motor were used, several trial values of s might have to be assumed

[2] Methods for determining performances with output as an independent variable exist. See:

Punga and Raydt, "Modern Induction Motors" (Hobart translation), p. 75, Pitman Publishing Corp., 1933.

L. Dreyfus, "The Pull-out Torque of the Polyphase Induction Motor" (in German), *Archiv. Elektrotech.*, Vol. 15, 1925.

until the solution showed such an output. This disadvantage can be eliminated by the use of an equation given by W. V. Lyon.

$$y = \frac{V^2}{P_0} - 2(R_1 + R_2) - \frac{Z_e^2}{\dfrac{V^2}{P_0} - 2(R_1 + R_2)} \qquad [261]$$

The slip at an output P_0 is,

$$s = \frac{R_2}{y + R_2} \qquad [262]$$

Z_e equals the equivalent impedance of the motor in terms of the stator as determined by the rotor blocked test or by calculation.

With all the values in equation 261 known, and P_0 as the assumed output in watts per phase for which the characteristics are to be determined, y and then the slip can be found for that output. Using s to determine R then enables the characteristics to be determined at a definite power output.

204. Other Torque Equations. Up to this point the torque equations have been in terms of counter emf E_2. More useful values are expressed in terms of applied volts per phase V, and we are now in position to consider these equations since the equivalent circuit can be used to show the relationship between V and E_2.

As vectors:

$$V = -E_1 + I_1(R_1 + jX_1) \qquad [263]$$

$$E_1 = aE_2$$

From the approximate equivalent circuit with the magnetizing branch moved to the machine terminals:

$$V = I_2\left[(R_1 + jX_1) + \left(\frac{R_2}{s} + jX_2\right)\right] \qquad [264]$$

Multiplying by s and eliminating the operator j:

$$Vs \approx I_2\sqrt{(R_1 s + R_2)^2 + s^2(X_1 + X_2)^2} \qquad [265]$$

Since

$$I_2 = \frac{E_2 s}{\sqrt{R_2^2 + X_2^2 s^2}}$$

Then

$$E_2^2 \approx V^2 \frac{R_2^2 + X_2^2 s^2}{(R_1 s + R_2)^2 + s^2(X_1 + X_2)^2} \qquad [266]$$

Equation 253 gave the developed torque as

$$T = \frac{E_2{}^2 s R_2}{R_2{}^2 + X_2{}^2 s^2} \text{ (synchronous watts)}$$

Substituting for E_2 from equation 266 gives

$$T \approx \frac{V^2 s R_2}{(R_1 s + R_2)^2 + s^2(X_1 + X_2)^2} \text{ (synchronous watts)} \quad [267]$$

This substitution gives torque in terms of the applied voltage, as derived from equation 264, and the approximate circuit. As such, it neglects the effect of the no-load current in giving a stator impedance drop. This error is likely to become large in slow-speed motors where the magnetizing current is usually relatively great.

If equation 267 is solved for maximum, it will be found that the maximum internal torque per phase will occur at a slip of

$$s_{mt} \approx \frac{R_2}{\sqrt{R_1{}^2 + (X_1 + X_2)^2}} \quad [268]$$

A more accurate expression, which still neglects only the effects of iron loss and saturation, is

$$s_{mt} = R_2 \sqrt{\frac{(X'_0)^2 + R_1{}^2}{(X'_0 X''_0 - X_m{}^2)^2 + (X''_0 R_1)^2}} \quad [269]$$

wherein:

$$X'_0 = X_1 + X_m \quad [270]$$

$$X''_0 = X_2 + X_m \quad [271]$$

The former value of slip, substituted in the torque equation, gives a more accurate expression for maximum torque per phase in synchronous watts:

$$T_{\max} = \frac{V^2}{2[R_1 + \sqrt{R_1{}^2 + (X_1 + X_2)^2}]} \quad [272]$$

Another formula for developed torque:

$$T = \frac{7.04}{\text{synchronous rpm}} \frac{mV^2 \dfrac{R_2}{s}}{\left(R_1 + \dfrac{R_2}{s}\right)^2 + (X_1 + X_2)^2} \text{ (pound-feet)} \quad [273]$$

The power developed internally is:

$$P = \frac{mR_2 s(1 - s)V^2}{(R_1 s + R_2)^2 + s^2(X_1 + X_2)^2} \text{ (watts)} \quad [274]$$

The power at the slip which gives the maximum torque:

$$P_{mt} = \frac{mV^2[\sqrt{R_1^2 + (X_1 + X_2)^2} - R_2]}{2\sqrt{R_1^2 + (X_1 + X_2)^2}\,[R_1 + \sqrt{R_1^2 + (X_1 + X_2)^2}]} \text{ (watts)} \quad [275]$$

This is not the maximum power; the maximum power occurs at a smaller slip, s_{mo}

$$s_{mo} \approx \frac{R_2}{R_2 + Z_e} \quad [276]$$

The maximum power at a slip s_{mo} is

$$P_m \approx \frac{mV^2}{2(R_e + Z_e)} \text{ (watts)} \quad [277]$$

The torque for the maximum power:

$$T_{mp} \approx \frac{mV^2(R_2 + Z_e)}{2Z_e(R_e + Z_e)} \text{ (synchronous watts)} \quad [278]$$

The resistance of the rotor circuit for maximum starting torque is

$$R_2 \approx \sqrt{R_1^2 + (X_1 + X_2)^2} \text{ (per phase)} \quad [279]$$

The starting torque in synchronous watts:

$$T_s = \frac{mR_2X^2}{Z_e^2} = m(I_{st})^2R_2 \quad [280]$$

When R_2 from equation 279 is substituted in equation 280, it gives the maximum starting torque. This is equal to the breakdown or pull-out torque, or the maximum torque shown by equation 272. The ratio of starting torque to pull-out torque is

$$\frac{T_s}{T_{\max}} = \frac{2R_2[R_1 + \sqrt{R_1^2 + (X_1 + X_2)^2}]}{Z_e^2} \quad [281]$$

The ratio of torque at maximum power to maximum torque is

$$\frac{T_{mo}}{T_{\max}} = \frac{R_2 + Z_e}{Z_e(R_e + Z_e)}[R_1 + \sqrt{R_1^2 + (X_1 + X_2)^2}] \quad [282]$$

The ratio of starting torque to torque at maximum power is

$$\frac{T_s}{T_{mo}} = \frac{2R_2Z_e(R_e + Z_e)}{Z_e^2(R_2 + Z_e)} \quad [283]$$

205. Examples. Some of the formulas just given will be illustrated by an example, using a 15-hp, 4-pole motor.

220 volts	three-phase	15 hp	1725 rpm
$R_1 = 0.15$	$R_2 = 0.18$	$X_1 = 0.31$	$X_2 = 0.31$
$X'_0 = 15.7$	Volts per phase $= 127$		
Friction and windage $= 240$ watts			

Slip at which maximum torque occurs (from equation 268):

$$S_{mt} = \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}} = \frac{0.18}{\sqrt{0.15^2 + 0.62^2}} \quad \text{or} \quad 0.282$$

Maximum torque in synchronous watts (from 272):

$$T_{\max} = \frac{mV^2}{2[R_1 + \sqrt{R_1^2 + (X_1 + X_2)^2}]} = \frac{3 \times 127^2}{2[0.15 + \sqrt{0.15^2 + 0.62^2}]}$$

$$= 102{,}500$$

$$T_{\max} = 7.04 \times \frac{102{,}500}{1800} \quad \text{or} \quad 120 \text{ lb-ft} \quad (118 \text{ by test})$$

Slip at which maximum power occurs. (From equation 276):

$$S_{mo} = \frac{R_2}{R_2 + Z_e} = \frac{0.18}{0.18 + 0.703} \quad \text{or} \quad 0.204$$

Maximum power in watts (from equation 277):

$$P_m = \frac{mV^2}{2(R_e + Z_e)} = \frac{3 \times 127^2}{2(0.18 + 0.703)} \quad \text{or} \quad 23{,}400$$

$$23{,}400 \text{ watts} = 31.4 \text{ hp}$$

Starting torque in synchronous watts (from equation 280):

$$\text{Starting torque} = \frac{mV^2R_2}{Z_e^2} = \frac{3 \times 127^2 \times 0.18}{0.703^2} \quad \text{or} \quad 17{,}580$$

$$\text{Starting torque} = 7.04 \times \frac{17{,}580}{1800} \quad \text{or} \quad 68.8 \text{ lb-ft } (96 \text{ by test}).$$

Starting current:

$$I = \frac{V}{Z_e} = \frac{127}{0.703} \quad \text{or} \quad 181 \text{ amperes } (182 \text{ by test}).$$

Check on starting torque:

$$\text{Starting torque} = mI^2R_2 = 3 \times 181^2 \times 0.18 \quad \text{or} \quad 17{,}580 \text{ synchronous watts}$$

The discrepancy between calculated and test values of starting torque requires some comment. Rapid heating of the rotor during starting-torque tests may account in part for this difference, as well as unpredictable skin effect in the rotor bars. Both would have a tendency to

raise the apparent resistance. The machine constants were calculated from design data and may be slightly different from actual values. These factors, along with possible space harmonics in the gap flux, may account for the differences. To continue:

Slip at full load (see Article 203):

$$\text{Watts developed} = 15 \times 746 + 240(F + W) \quad \text{or} \quad 11{,}440$$

$$P_0 = \text{watts per phase} = \frac{11{,}440}{3} \quad \text{or} \quad 3813$$

$$y = \frac{V^2}{P_0} - 2(R_1 + R_2) - \frac{Z_e^{\,2}}{\dfrac{V^2}{P_0} - 2(R_1 + R_2)}$$

$$= 3.401$$

$$\text{Slip} = \frac{R_2}{y + R_2} = \frac{0.18}{3.401 + 0.18} \quad \text{or} \quad 0.0502$$

$$\text{Full-load slip} = 0.0502 \times 1800 \quad \text{or} \quad 90.6 \text{ rpm}$$

$$\text{Full-load speed} = 1800 - 90.6 \quad \text{or} \quad 1709.4 \text{ rpm}$$

$$\text{Full-load torque} = \frac{15 \times 33{,}000}{2\pi \times 1709.4} \quad \text{or} \quad 46.0 \text{ lb-ft}$$

$$\frac{\text{Maximum torque}}{\text{Full-load torque}} = \frac{120}{46} \quad \text{or} \quad 2.60$$

$$\frac{\text{Starting torque}}{\text{Full-load torque}} = \frac{68.8}{46} \quad \text{or} \quad 1.49 \text{ (using calculated value)}$$

$$\text{Starting kilovolt-amperes} = \frac{3 \times 127 \times 181}{1000} \quad \text{or} \quad 68.9$$

$$\text{Starting kilovolt-amperes per horsepower} = \frac{68.9}{15} \quad \text{or} \quad 4.59$$

The above ratios are very significant, being used as a means of determining the suitability of motors for various applications and for contrasting designs. Standardizing bodies have placed various limitations on these ratios, as will be discussed in Chapter XXVIII.

206. Torque from Flux Density. The torque developed by a polyphase motor is the same for all positions of the rotor so long as there is no cogging or other defect. This can be seen from the simple physical conditions illustrated in Fig. 168 for a squirrel-cage rotor.

A "sheet of current" is built up in the rotor conductors, sinusoidally distributed as shown by the relative sizes of the circles, and following around after the pole flux. As both current and flux are invariable in time, the torque is constant even though the current shifts from conductor to conductor.

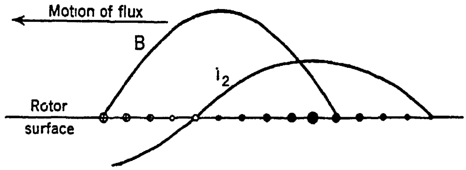


FIG. 168.

The formula for force produced on a conductor, carrying current in a magnetic field, can readily be applied to such a concept.

$$F = 0.1BLI$$

where F = force, in dynes
1 lb = 444,800 dynes
B = magnetic density, in lines per square centimeter, assumed uniformly distributed along the length of the conductor L
L = length of conductor in the magnetic field, in centimeters
I = current, in amperes
Z_r = total conductors on the rotor

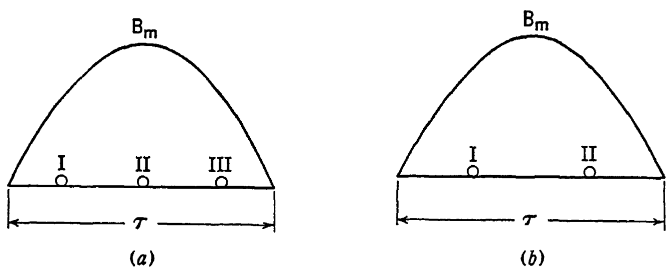


FIG. 169.

Assume a three-phase concentrated winding and unity pf in the rotor circuit. The tangential force on the rotor bars for the position shown in Fig. 169a is

$$F = 0.1\,\frac{B_m}{2} \cdot \frac{I_m}{2} \cdot L \cdot \frac{Z_r}{3} + 0.1B_mI_m \cdot L \cdot \frac{Z_r}{3} + 0.1\,\frac{B_m}{2}\frac{I_m}{2} \cdot L \cdot \frac{Z_r}{3}$$

$$= \tfrac{1}{20}B_mI_mLZ_r \text{ dynes}$$

The effect of distributing and chording the windings is to reduce the force by the factors k_p and k_d. Then,

$$F = \tfrac{1}{20} B_m I_m Z_r L k_p k_d \text{ dynes}$$

In effective values and kilograms:

$$F_{\text{kg}} = \frac{BLIZ_r k_p k_d}{9{,}810{,}000} \text{ kilograms}$$

In pounds (and inches):

$$F_{\text{lb}} = \frac{6.256}{10^8} \cdot B_m I L Z_r k_p k_d$$

To use flux per pole and rotor dimensions we have the following relationships:

$$\phi_m = \frac{2}{\pi} B_m L' \frac{\pi D}{P}$$

$$= \frac{2}{P} B_m L' D \text{ flux lines per pole}$$

where D = rotor diameter in inches
P = number of poles
L' = axial length of core in inches

Then the flux density in lines per square inch is

$$B_m = \frac{P\phi_m}{2L'D}$$

and the tangential effort in pounds is

$$F_{\text{lb}} = \frac{6.256}{10^8} \frac{P\phi_m}{2L'D} L' I Z_r k_p k_d$$

$$= \frac{3.128}{10^8} \cdot \frac{P\phi_m}{D} I Z_r k_p k_d$$

The torque in pound-inches is:

$$T_{\text{lb-in.}} = \frac{FD}{2}$$

$$= \frac{1.564}{10^8} (P\phi_m) Z_r I k_p k_d \qquad [284]$$

To consider the effect of the pf in the secondary circuit, multiply equation 284 by $\cos \theta_2$.

For a two-phase winding at the instant shown in Fig. 169*b*:

$$F = 2\left[0.1 \times \frac{B_m}{2} \times \frac{I_m}{2} L \frac{Z}{2} k_p k_d\right]$$

$$= \tfrac{1}{20} B_m I_m L Z k_p k_d \text{ dynes}$$

This is the same expression followed through for three-phase, and leads to the same torque formula. Of course, the torque for a given motor is the same when calculated on the basis of stator values or rotor values.[3]

[3] For a further example of the analysis of torque from a squirrel-cage winding, see: M. M. Liwschitz, "Starting Performance of Salient-pole Synchronous Motors," Supplement to *Elec. Eng. Trans.* Section, Vol. 59, pp. 913–919, and references listed, December, 1940.

CHAPTER XXIV

LOSSES AND EFFICIENCY. TEST DATA

207. Chapter Outline.

Losses and Efficiency.
- Discussion of Losses.
- Laboratory Tests for Their Determination.
  - Rotor Blocked.
  - Rotor Running with No Load.
- Test for Transformation Ratio. (Wound-rotor machine only.)
- Slip Measurement.
- Example of Characteristics from a Load Test.

208. Losses. The losses in an induction motor are:

(*a*) Core losses in the stator and rotor.
(*b*) Friction and windage losses.
(*c*) Stator copper loss.
(*d*) Rotor copper loss.

In most calculations the first two of these losses are assumed to be constant regardless of the load. Friction loss will actually vary slightly with the load, especially if the load is belted to the motor. This loss also varies with the speed, but, since the speed varies but slightly over the working range, an assumption of constancy involves little error.

The core loss is due to the main and leakage fluxes. The value and distribution of each of these vary with the load. The mutual flux decreases as the load increases, and the leakage fluxes increase with the currents. On account of saturation in parts of the iron path the leakage flux does not always vary in the same proportion over the entire current range. Some of the leakage fluxes in reality have no separate existences, but combine with each other and with the main flux to produce local distortions only. These distortions cause a change in the losses, most commonly an increase.

The loss due to the mutual flux is less, and that due to the leakage fluxes is greater, at heavier loads. Because of this partially neutralizing effect it is sufficiently accurate for many purposes to regard the core loss

as constant at all loads and speeds within the working range of the motor, under constant-voltage conditions. It can be assumed for relatively small changes that this loss varies directly as the applied voltage. The A.I.E.E. Standards suggest core-loss variation as the square of the voltage for low densities.

Except for the rotor tooth losses, practically all the core loss occurs in the stator, owing to the low frequency of the rotor flux.

Copper Losses. The stator-winding resistance can be calculated from the wire size and length, or measured directly by direct current. Eddy-current and hysteresis losses in the teeth, caused wholly by the stator currents, and eddy-current losses in the conductors themselves increase the effective resistance over the value obtained by d-c measurement. Skin effect is large enough so that it is not negligible, although it is not customary to make accurate calculations for it. The effect is noticeable in the secondary copper at high slips. In many cases, the effective a-c resistance may be taken as 1.15 to 1.35 times the d-c resistance.

The rotor copper loss in squirrel-cage designs cannot be measured directly. It can be separated from the total copper loss as determined by test through subtracting the calculated value of stator copper loss. A method of calculating the resistance of a squirrel-cage winding from design sheet data is given in Article 229.

The A.I.E.E. Test Code for Polyphase Induction Machines (No. 500, August, 1937) recommends:

The secondary I^2R loss should be determined from the slip, whenever the latter is accurately determinable, using either one of the following equations:

$$\text{Motor rotor } I^2R \text{ loss} = \text{secondary input} \times \text{slip}$$

$$= (\text{measured primary input—primary } I^2R \text{ loss}) \times \text{slip}$$

The slip is expressed as a decimal fraction.

209. The No-load Test. In its simplest form the no-load test can be made by connecting the induction motor to a supply of rated voltage and frequency and measuring voltage, current, and power input. A refinement on this method involves the varying of the voltage over a considerable range, giving curves such as are shown in Fig. 170.

The no-load power input represents core losses, friction, and windage, and a small stator copper loss. Because of the small slip, the rotor copper loss at no load can be neglected. Actually the rotor conductors carry some current even at exact synchronism because the flux wave is not exactly sinusoidal in time or in space; neither is the rotor perfectly concentric with the stator.

If it were possible to operate an induction motor at zero voltage, its

input would represent practically the windage and friction loss. For some purposes it is satisfactory to take friction and windage as equal to motor input at the lowest voltage at which it will run. By extending the curve of "voltage versus no-load loss" to the zero voltage axis, this condition can be predicted. Hence the intercept *OF*, shown in Fig. 170, represents friction and windage loss.

Usually the no-load current in the stator winding is large enough so that the I^2R loss it causes should be subtracted from the no-load

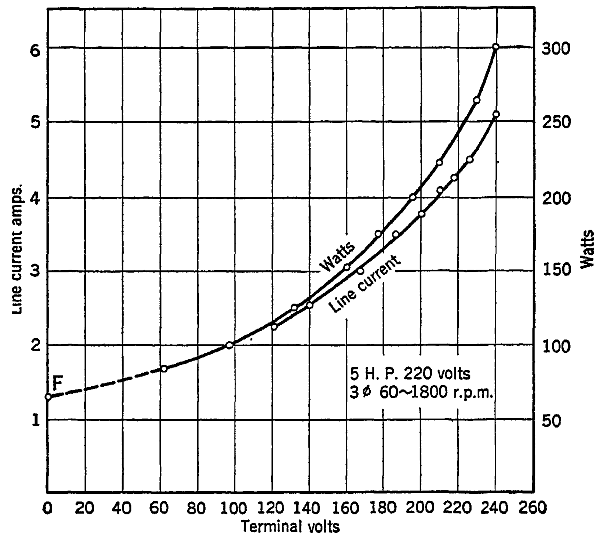


FIG. 170. The no-load run.

input to obtain these so-called constant losses: friction, windage, and core losses.

210. The Short-circuit or Rotor-blocked Test. The rotor is held firmly so that it cannot turn, and a reduced voltage is applied to the terminals of the stator winding. Voltage, current, and power are measured. The ampere reading is the short-circuit current, and, because full voltage on the stator terminals would cause excessive heating and mechanical stress, the applied voltage should be less than name-plate rating. The relationship between applied voltage and current under such conditions is approximately linear, and by taking a series of meter readings with varying voltage a curve can be plotted as shown in Fig. 171, and the short-circuit current predicted at rated voltage. For greater accuracy, the temperature of the stator winding should be measured during this test so that correction can be made to the recommended temperature of 75 C.

211. Example. No-load and short-circuit test data on a 5-hp, 220-volt, three-phase, 60-cycle, 1800-rpm induction motor are given in Figs. 170 and 171.

No load:

At 220 volts no load the total input is 244 watts.
The current is 4.35 amperes per line.
A resistance measurement gave $R_{75°} = 0.545$ ohm per leg.
$I_n^2R = 10.33$ watts per phase.
Friction, windage, and core loss $= 244 - 3(10.33)$
$= 213$ watts.
Friction and windage (OF in Fig. 170) $= 65$ watts (approximately).

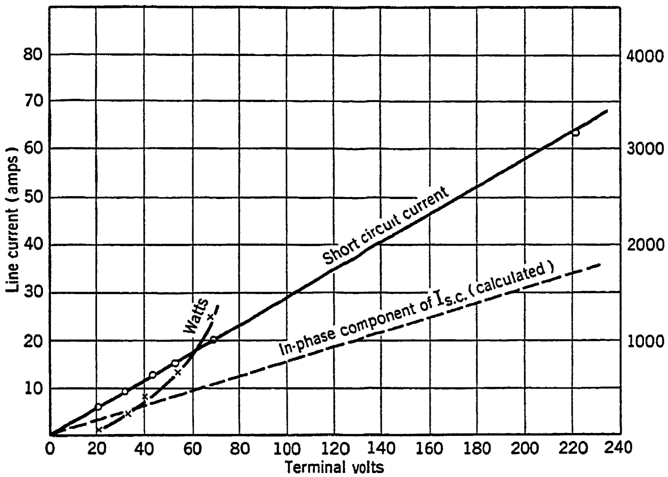


FIG. 171. The rotor-blocked test.

Short circuit:

By extending the curve of short-circuit current versus voltage, the probable current at 220 volts is read as 64.5 amperes.

On account of its shape the watts-versus-volts curve cannot be extended accurately to determine the power factor. The pf at short circuit can best be determined by calculating the in-phase component of the current. Since

$$P = \sqrt{3}VI\cos\theta$$

$$I\cos\theta = \text{in-phase component of current}$$

From the curves,

$$P = 1250 \text{ watts}, V = 68 \text{ volts}$$

$$I\cos\theta = \frac{1250}{\sqrt{3}\times 68}$$

$$= 10.6 \text{ amperes}$$

If several other points are calculated, a line can be drawn through them and the origin, giving the curve of in phase versus terminal volts. From this we find that,

with a short-circuit current of 64.5 amperes at 220 volts, the in-phase current is 34.5 amperes and the rotor-blocked pf is fixed.

These tests can be used to obtain further data on the machine constants.

$$X'_0 = X_m + X_1 \approx \frac{V}{I_{\text{no load}}} \tag{285}$$

$$= \frac{127}{4.35} \quad \text{or} \quad 29.2 \text{ ohms}$$

If we assume that the equivalent impedance of the motor is measured by applied volts and short-circuit current, the tacit assumption is made that the equivalent circuit is a simple series impedance. As a first approximation:

$$Z_e = \frac{\text{volts per phase}}{\text{short-circuit current}} \tag{286}$$

$$= \frac{127}{64.5} \quad \text{or} \quad 1.97 \text{ ohms}$$

$$R_e = \frac{\text{in-phase current}}{\text{total current}} \cdot Z_e \tag{287}$$

$$= \frac{34.5}{64.5} \times 1.97 \quad \text{or} \quad 1.054 \text{ ohms}$$

Since R_1 equals 0.545 ohm per phase, the corresponding rotor resistance assigned to each phase is

$$R_2 = 1.054 - 0.545 \quad \text{or} \quad 0.509 \text{ ohm}$$

The leakage reactance:

$$X_e \approx \sqrt{Z_e^2 - R_e^2} \tag{288}$$

$$= \sqrt{1.97^2 - 0.054^2} \quad \text{or} \quad 1.665 \text{ ohms}$$

Note that the effect of the magnetizing branch (thinking in terms of an equivalent circuit) is completely neglected by this method of calculation. Such neglect is particularly serious in motors that have long, radial air gaps or many poles. Many corrective methods have been developed for obtaining more accurate constants from test results.[1]

$$\text{Let } b = \frac{X_0 - X_e}{X_0} = 0.942$$

$$\text{Let } a = \frac{R_e - R_1}{X_e} = 0.305$$

Then the corrected value of leakage reactance is

[1] Without derivation, several ratios will be used here to illustrate a closer approximation to test constants.

See A. F. Puchstein, "Obtaining Test Constants for Induction Motors," under Letters to the Editor, *Elec. Eng.*, March, 1940.

$$X = \frac{1 - (1 - b)(a^2 + 1)}{b} X_e \quad \text{or} \quad 1.655$$

Corrected value of b:

$$\frac{29.2 - 1.655}{29.2} \quad \text{or} \quad 0.943$$

Corrected value of R_2

$$R_2 \approx \frac{R_e - R_1}{b} \quad \text{or} \quad 0.540$$

212. Ratio of Transformation. If the induction motor under test is of the wound-rotor type, its ratio of transformation can be determined by connecting a voltmeter across the terminals of its open-circuited rotor winding and by reading applied voltage V and rotor-induced voltage E_2. The rotor should be turned to various positions with respect to the stator during this test, and average values used for calculation. This ratio is inaccurate, owing to the differential leakage flux between stator and rotor turns. Such an error can be eliminated by the following procedure:

When the stator windings were connected to their rated voltage, the rotor voltage E_2 was found. Disconnect the stator winding and impress a voltage E_2 on the rotor. Then read the applied voltage E'_2 and the stator induced voltage V'. The correct ratio of transformation is then

$$a = \frac{V}{E_2}\sqrt{\frac{VE'_2}{V'E_2}} \qquad [289]$$

213. Efficiency.[2] When the losses of a transformer or an alternator have been determined, any load at any pf can be assumed and (with the correct value of losses) the efficiency can be calculated. In an induction motor such a procedure is not possible because for any load the pf is a dependent variable. It is necessary then to determine the pf at each load before the losses can be used. This can be done by:

(*a*) The solution of the equivalent circuit.
(*b*) The use of graphical methods such as the circle diagram.
(*c*) Load tests.

[2] T. H. Morgan and Paul M. Narbutovskih, "Stray Load Loss Test on Induction Machines," *Trans. A.I.E.E.*, Vol. 53, pp. 286–290, 1934.

Theodore H. Morgan and Victor Siegfried, "Stray Load Loss Tests on Induction Machines," *Elec. Eng.*, Vol. 55, May, 1936.

C. J. Koch, "Measurement of Stray Load Loss in Polyphase Induction Motors," *Trans. A.I.E.E.*, Vol. 51, pp. 756–763, 1932.

C. C. Leader and F. D. Phillips, "Efficiency Tests of Induction Machines," *Trans. A.I.E.E.*, Vol. 53, pp. 1628–1632, 1934.

Paul M. Narbutovskih, "Power Losses in Induction Machines," *Trans. A.I.E.E.*, Vol. 53, pp. 1466–1471, 1934.

In the load test, various loads are applied to the motor, and its input (volts, amperes, and watts) is measured. Accurate methods are necessary for measuring the slip. From such value of input the calculated losses are subtracted to give the output. These losses are determined as follows:

(*a*) Friction, windage, and the core losses are determined from the no-load run.

(*b*) The stator copper loss is calculated from direct measurement of the stator winding resistance.

(*c*) The rotor copper loss is determined from the relationship that exists between it and the slip (Article 208).

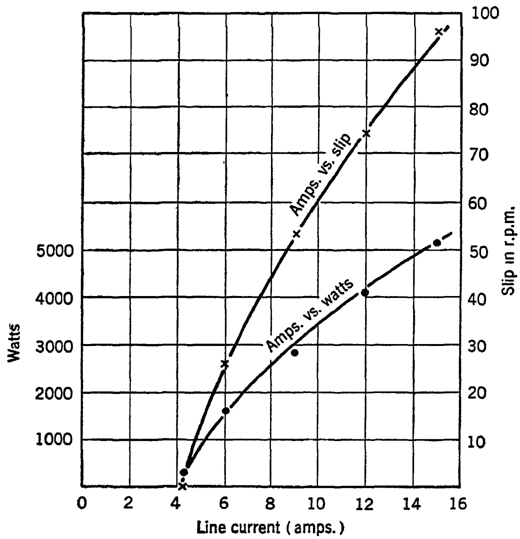


FIG. 172.

Such a method of determining the output from input minus losses is more accurate than direct measurement on account of the difficulty of making precise, mechanical power measurements.[3]

It has been shown that the copper losses of the rotor winding are proportional to the slip. Thus, at a slip of 5 per cent, the copper loss in the rotor is 5 per cent of the power transferred across the air gap. Hence, if there were no other losses, the efficiency of the motor would be $(1 - s)$ or 95 per cent. Since losses other than the rotor copper losses are always present, *the percentage of efficiency is always less than the speed in per cent of synchronism.*

[3] A more complete exposition of this method, and a description of slip meters, can be found in V. Karapetoff and B. Dennison's "Experimental Electrical Engineering," Fourth Edition, Chapter XXII, John Wiley and Sons.

214. Example of Efficiency from Losses. The same motor on which short-circuit and open-circuit characteristics were obtained was loaded and the input (watts, amperes, and volts) was read. The frequency and voltage were maintained at rated values. Values previously found as shown in Article 211:

$$R_1 \text{ per phase at } 75\text{ C} = 0.545 \text{ ohm}$$

$$I_n = 4.35 \text{ amperes per line}$$

$$P_n = 244 \text{ watts}$$

The slip was read on a slip meter. Results are given in Fig. 172. One point (at 14 amperes input) will be calculated to illustrate the method:

$$\text{Input per line} = 14 \text{ amperes}$$

$$I_1^2R_1 = 14^2 \times 0.545 \text{ (watts per phase)}$$

$$= 107 \text{ watts}$$

Total stator copper loss at 14 amperes per line = 321 watts
No-load input from curve or recorded data = 244 watts
No-load current = 4.35 amperes per line
$I_n^2R_1$ at no load = 31 watts
Friction, windage, and core loss = 244 − 31
= 213 watts
Input to stator (from curve or data) = 4800 watts (at 14 amperes)
Power transferred across air gap = 4800 − 213 − 321
= 4266 watts
(This assumes that constant losses occur in the stator.)
Slip at 14 amperes, from curve or data = 88 rpm
$= \frac{88}{1800} \cdot 100$ or 4.89 per cent
I^2R in rotor = 0.0489 × 4266
= 209 watts
Output = 4266 − 209
= 4057 watts

$$\text{Efficiency in per cent} = \frac{\text{output}}{\text{input}} \cdot 100 = \frac{4057}{4800} \cdot 100$$

$$= 84.7 \text{ per cent}$$

$$\text{Power factor} = \frac{4800}{\sqrt{3} \times 220 \times 14} \cdot 100$$

$$= 90.2 \text{ per cent}$$

Rotor speed = synchronous speed − slip speed
= 1800 − 88
= 1712 rpm

$$\text{Horsepower output} = \frac{4057}{746}$$

$$= 5.45 \text{ hp}$$

$$\frac{5.45}{1 - 0.0489} = 5.75 \text{ synchronous hp}$$

The above values for a slight overload condition are plotted on Fig. 202 along with the values determined from other assumed currents.

CHAPTER XXV

CIRCLE DIAGRAM

215. Chapter Outline.

Circle Diagram.
  Derivation.
    Construction.
      Obtaining the Motor Characteristics.
      Example.
Other Diagrams.

216. Introduction. A large number of graphical methods have been developed since 1894 for the analysis of induction-motor characteristics. They differ greatly in their accuracy and simplicity of application. The circle diagram most frequently found in American textbooks is that of Dr. A. S. McAllister. The derivation given in this chapter will make use of his method. It is based on the approximate equivalent network of Fig. 166. This network neglects the effect of the exciting current in causing a drop in the stator, and the effect of the stator resistance drop on the voltage. Even the more exact methods are based upon simplifying assumptions which are not completely justifiable and hence cannot represent the physical phenomena with absolute accuracy.

217. Derivation of the Circle Diagram. In the approximate equivalent circuit for the induction motor, the no-load admittance branch is shifted to connect across the terminals of the motor circuit. (See Fig. 166.) This means that the stator impedance drop caused by the no-load current is neglected. In most motors (excepting, say, fractional horsepower sizes or those with a large primary drop) the error of this assumption is small.

If a constant voltage V is applied to the circuit, the load current flowing through the stator and rotor windings will be

$$I_2 = \frac{V}{\sqrt{(R_1 + R_2 + R)^2 + (X_1 + X_2)^2}} \qquad [290]$$

This circuit requires that all rotor constants be converted to stator terms. Hence the rotor current I_2, when so converted, is the same as

the load component of the stator current I_b. The current flowing through this circuit is out of phase with the applied voltage by an angle whose sine is

$$\sin \theta = \frac{X_1 + X_2}{\sqrt{R_1 + R_2 + R)^2 + (X_1 + X_2)^2}} \qquad [291]$$

By combining equations 290 and 291 we get another expression for I_2:

$$I_2 = \frac{V}{X_1 + X_2} \sin \theta \qquad [292]$$

If the leakage reactances are assumed to remain constant regardless of the load, and the applied voltage is a constant, then equation 292

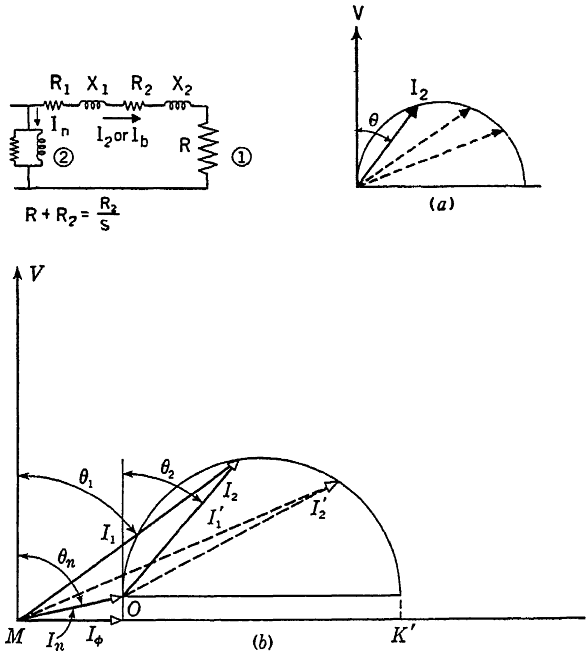


FIG. 173.

is the polar equation of a circle having a diameter of $V/(X_1 + X_2)$. By changing R (the load), the sine of θ will vary and cause I_2 to change in magnitude and direction with respect to the voltage V. Such a locus is shown in Fig. 173a. The entire current taken by such an equivalent circuit is not only I_2; the second leg requires the constant current I_n.

To agree with the physical facts in the motor, it is necessary that the change in load (or in R) must result in a variation in the current in I_2 with no change in I_n. The circuit fulfills this requirement, and by adding the current vector I_n, as shown in Fig. 173*b*, the circle diagram is also in agreement. The total motor current per phase is then the vector sum of I_2 and I_n, or I_1. The diagram shows three pf angles; the no-load pf angle is θ_n, the secondary pf angle is θ_2, and the motor as a whole has the pf angle of θ_1. The error of such an assumption is now obvious; the voltages V and E of Fig. 165 are assumed to be in phase and of equal magnitude.

218. Construction of the Diagram. Two tests are necessary (in addition to the resistance measurements of the stator windings) in order to predict the characteristics of an induction motor from laboratory data. These are the no-load and the rotor-blocked tests as previously described in Articles 209 and 210. The method of laying out a diagram is outlined below.

(*a*) The no-load current. Use the applied voltage per phase, V, as the reference vector. Determine the pf angle of the no-load current. Draw I_n behind V by θ_n degrees. The voltage and current scales can be chosen arbitrarily; the power and torque scales will be dependent upon the former two.

$$\text{Watts per inch} = V \times \text{amperes per inch}$$

$$\text{Synchronous watts per inch} = V \times \text{amperes per inch}$$

(*b*) Rotor blocked. Determine the rotor-blocked current and its in-phase component at rated voltage. To the current scale lay off I_{bl} so that *ab* on Fig. 174*b* represents the in-phase component.

(*c*) The points O and a must each lie on the locus circle. To draw the circle, determine its center by extending a horizontal line from O to K and erect a perpendicular bisector on the line connecting O and a. The intersection of this bisector with the horizontal extension from O is the center of the circle, shown as c.

(*d*) Using c as the center, draw the semicircle OPK.

(*e*) Draw the vertical line *ab* and divide it by the point d so that *ad* is to *dc* as the rotor copper loss is to the stator "added" copper loss.[1] Or, in terms of the resistances,

$$\frac{I_2{}^2R_2}{(I_{\text{bl}}{}^2 - I_n{}^2)R_1} = \frac{ad}{dc} = \frac{\text{rotor effective resistance per phase in stator terms}}{\text{stator effective resistance per phase}}$$

[1] Note that the original circular locus of Fig. 173*a* involved the rotor current. Because the stator losses have been added in this analysis, the term "added" losses is used. This will be clarified by contrast with some of the diagrams shown later.

If no resistance measurement can be obtained for the rotor, the point d can be determined as follows:

Any vertical line drawn from the base to the circle represents an in-phase component of current and hence is proportional to power.

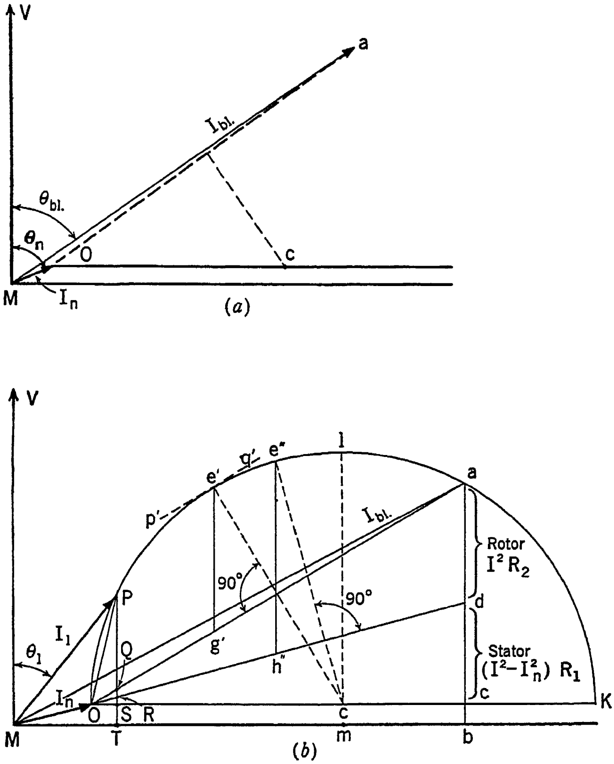


FIG. 174.

If the rotor-blocked current is I_{bl} and the stator resistance is R_1 ohms per phase, the stator copper loss on short circuit is $I_{\text{bl}}{}^2R_1$.

The power component of current needed to supply this loss is

$$\frac{(I_{\text{bl}}{}^2 - I_n{}^2)R_1}{V \text{ per phase}} = I_{cd}$$

Draw I_{cd} to the current scale and thus determine the position of d.

(*f*) Draw Od.

(*g*) The rated current of the motor per phase is I_1. Lay off MP equal to I_1, and draw the vertical line PT.

(*h*) With the correct scales the following values can now be determined:

No-load current $= MO$ (amperes per phase)
Stator current $= MP$ (amperes per phase)
Rotor current $= OP$ (amperes per phase in stator terms)
Power input to stator [2] $= TP$ (watts per phase)
"Constant losses" $= TS$ (watts per phase)
Stator "added" copper loss $= SR$ (watts per phase)
Rotor copper losses $= RQ$ (watts per phase)
Power transferred across air gap $= RP$ (watts per phase)
Useful output $= QP$ (watts per phase)
Power factor $= \cos\theta_1 = PT/MP$
Slip $= RQ/RP$ (as a fraction; no scale necessary)
Efficiency $= QP/TP$ (as a fraction)

The torque in synchronous watts is equal to RP in the watt scale. The torque scale in pound-feet equals the watt scale times

$$\frac{33{,}000}{746}\;\frac{1}{2\pi \times \text{synchronous rpm}}$$

By assuming different values of current I_1, the performance curves of the motor can be calculated over the entire load range. Such curves are shown in Fig. 175.

219. Maxima. The maximum pf at which the motor can operate will occur when the current vector I_1 is tangent to the circle. Inspection of the diagram shows that the maximum rotor pf occurs at no load.

The maximum power which the motor can take from the supply is indicated as ml.

The maximum power output will occur when the length QP is a maximum. To determine this position it is necessary to draw the line $p'q'$ tangent to the circle and parallel to Oa. A perpendicular bisector, erected on the line Oa, is a convenient means of locating this point of tangency. This bisector is shown as ce', and the maximum power is then $g'e'$. The current input for this condition is obviously Me'.

Since the torque is proportional to RP, the maximum torque can be found by drawing a tangent to the circle parallel to the line Od. This maximum torque is indicated as $h''e''$. If MP is the rated current, then RP represents the full-load torque, and the relative lengths of RP and $h''e''$ give the ratio of full-load to pull-out torque.

[2] The line TP can be read as watts by multiplying the amperes represented by TP by V. If the total input is desired, multiply by the number of phases.

The line Od is called the torque line since vertical distances from it to the circle represent the torque at the various slips. Then the starting torque will be proportional to the length da inasmuch as this represents

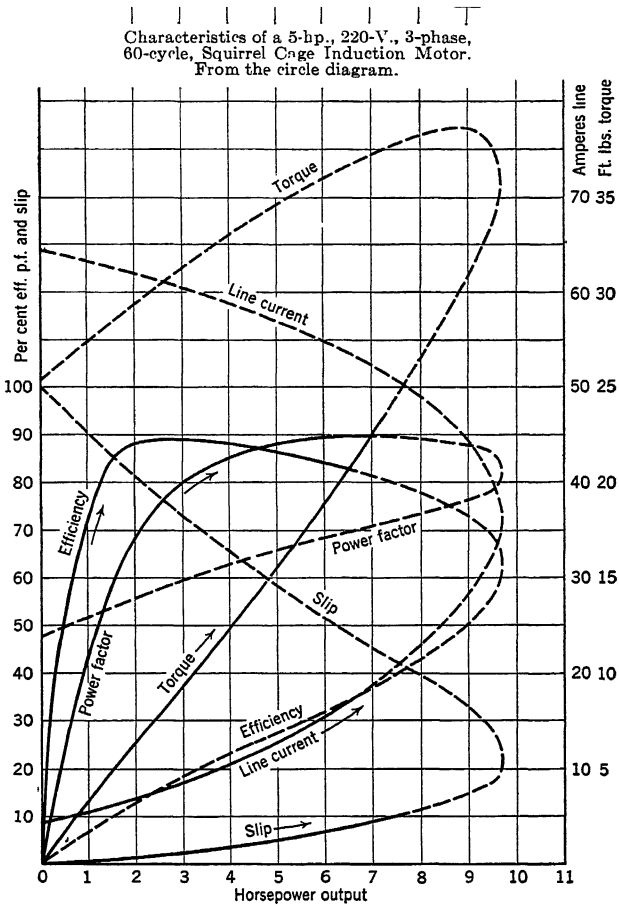


FIG. 175.

unity slip condition. If MP represents full-load stator current, the full-load torque is RP and the ratio of starting torque to full-load torque is da/RP.

220. Example of Characteristic from the Circle Diagram. By the same 5-hp, three-phase, 220-volt, 4-pole induction motor on which a load test was made, the following data on no-load and rotor-blocked tests were shown:

Y-connected stator Squirrel-cage rotor

$I_{\text{no-load}} = 4.35$ amperes per line

Power input (three phases) = 244 watts
$V_{\text{terminal}} = 220$
$I_{\text{blocked}} = 64.5$ amperes at 220 volts
$I_{\text{in-phase}}$, 220 volts applied = 31 amperes

To determine the circle diagram:

The pf angle of the no-load current is determined by

$$\frac{244}{3} = \text{input watts per Y leg}$$
$$= 81.3 \text{ watts}$$
$$\text{Voltage per Y leg} = \frac{220}{\sqrt{3}}$$
$$= 127$$
$$\cos \theta_n = \frac{81.3}{127 \times 4.35}$$
$$= 0.147$$

The rotor-blocked current at 220 volts across terminals or 127 volts per leg is 64.5 amperes. Of this, 31 amperes is in phase. Therefore the pf is

$$31 \div 64.5 = 0.480$$
$$\text{Input watts} = \sqrt{3} \times 220 \times 31$$
$$= 11{,}830 \text{ (total)}$$
$$= 3943 \text{ (per phase)}$$

The resistance of the stator is 0.545 ohm per phase.

The "added" loss which occurs in the stator at short circuit is corrected in the manner shown:

$$(I_{\text{bl}}^2 - I_n^2)R_1 = (64.5^2 - 4.35^2)0.545$$
$$= 2255 \text{ watts}$$

The point d at which the vertical line ba is divided can be determined from the ratio (see Fig. 174):

$$\frac{cd}{ba} = \frac{2255}{3943} = 0.572 \quad \left(\text{Some writers use the ratio } \frac{cd}{ca}\right)$$

Or, as shown under e of Article 218:

$$\frac{(I_{\text{bl}}^2 - I_n^2)R_1}{V \text{ per leg}} = \frac{2255}{127}$$
$$= 17.76 \text{ amperes}$$

Therefore the length cd represents 17.76 amperes with ba equal to 31 amperes.

Different input currents can now be assumed and the characteristic curves drawn from the diagram. The results are shown in Fig. 175.

221. Other Networks and Diagrams. In addition to the McAllister diagram as shown, many others are available, embodying various refinements or based on other equivalent networks. Several will be developed progressively.

If a circuit consists of a series resistance and reactance as shown in

Fig. 176*a*, it is obvious that as the resistance changes, the current vector will fall on the locus circle of Fig. 176*b*. The circle diameter will be

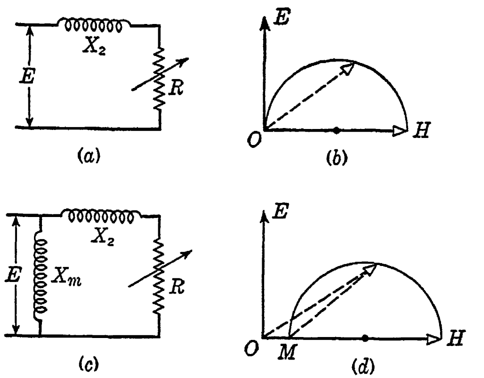


Fig. 176.

E/X_2. Then, as previously explained, the addition of a parallel reactance gives the circuit a constant out-of-phase current, resulting in a shift of the circle diagram as shown in Fig. 176*d*.

Now:

$$OM = \frac{E}{X_m} \qquad [293]$$

$$OH = E \div \frac{X_2 X_m}{X_2 + X_m} \qquad [294]$$

The expression $X_m/(X_2 + X_m)$ is a flux factor, found in the literature as K_s or F_s.

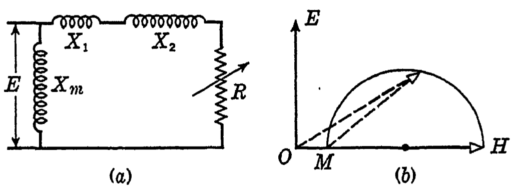


Fig. 177.

In the next step, shown in Fig. 177, no change has been made except the addition of the reactance X_1 to the circuit. Now:

$$OM = \frac{E}{X_m}$$

$$OH = E \div \frac{(X_1 + X_2)X_m}{X_1 + X_2 + X_m} \qquad [295]$$

$$MH = E \div (X_1 + X_2) \qquad [296]$$

In Fig. 178*a* the magnetizing reactance has been moved to a point midway between X_1 and X_2. Although the general appearance of the diagram is the same, now:

$$OM = \frac{E}{X_1 + X_m} = \frac{E}{X'_0} \quad [297]$$

$$OH = E \div (X_1 + K_s X_2) \quad [298]$$

$$MH = E \div \left(\frac{X_1 + K_s X_2}{K_r}\right) \quad [299]$$

$$\frac{MH}{OH} = K_r \quad [300]$$

Wherein K_s is $\dfrac{X_m}{X_2 + X_m}$, K_p is $\dfrac{X_m}{X_1 + X_m}$, and K_r is $K_s \times K_p$.

Note in these various cases that the currents will all vary directly with the voltage E_1 which fixes the diameters of the circles as well as other values.

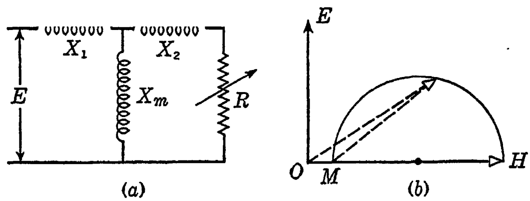


FIG. 178.

Suppose that stator impedances are added to the left of the circuits to which the applied voltage per phase is V. Figure 176 will become as shown in Fig. 179. If the current and pf at various loads on an induction motor, as well as other items of performance, are required from such a diagram, the circle diameter will change with each load. This results from the varying stator impedance drops, causing a change in E for constant applied voltage. As a practical working method, it is usual to assume E, draw the diagram, and determine V graphically. If V differs from the required value, the scale of currents on the diagram is modified by the ratio of V_{rated} to V_{read}.

In Fig. 178 only the stator resistance drop must be added to the circuit to complete the induction-motor equivalent circuit, as shown in Fig. 180. Now the voltage E, on which the circle diameter is fixed, differs from the applied voltage by the stator resistance drop. Again, with each load current assumed on a fixed circle locus, the different stator resistance drop results in a different value for applied voltage, V.

As this voltage is actually constant under the conditions of most problems, it is necessary to correct the scale of the diagram with each current assumed. This forms the LaTour diagram, utilized so effectively by Dr. W. J. Branson, whose papers have been cited previously.

The step bringing us to the McAllister diagram has already been shown in Fig. 173. This diagram is more convenient to use than that of

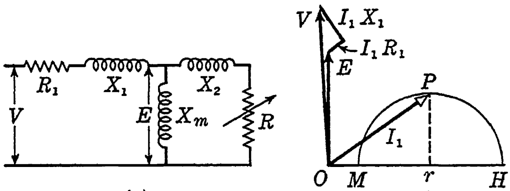


FIG. 179

Figs. 179 or 180, but, on smaller motors having appreciable stator resistance drops or in cases where the magnetizing current may be large (slow-speed motors), the approximations made may introduce wide discrepancies between actual and circle diagram values.

A number of rigorous methods are available for making the transformations from elementary to the final working diagrams. A convenient graphical method was prepared by Dr. McAllister and described in Behrend's book on the induction motor. The general method comes under the subject of conformal representation in the field of functions

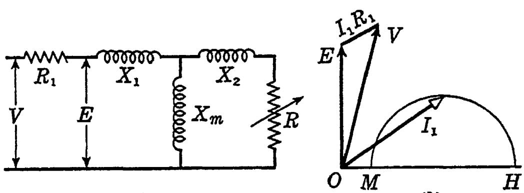


FIG. 180.

of a complex variable, and the process is called *inversion*. A more accurate approach usually results in a semicircle whose base is not on a horizontal line. In addition to the calculations shown for determining the circle diameter, it is possible to solve the equivalent network for three values of slip, say, zero, 20 per cent, and infinity, using these three values of current to fix the circle.[3]

[3] A. F. Puchstein, "Time-saving Method for Calculating Induction-motor Performance," Letters to the Editor, *Elec. Eng.*, May, 1940.

In each of these cases the consideration of core losses can be handled in several different ways. If it is assumed to take place in an equivalent resistance, connected across the machine terminals, it remains constant with load change. This may or may not agree with the physical facts. Other methods would connect this branch between R_1 and X_1 or between X_1 and X_2. The latter cases result in less calculated core loss with load increase. The net effect on the diagrams is to raise the diameter of the circle above the base line, or to lower the point O below the base line.

CHAPTER XXVI

HARMONICS IN THE AIR-GAP FLUX

222. Chapter Outline.

The Air-gap Flux.
  Harmonics in Space and Time.
  Rotation of Harmonics.
    Effect on Torque.
    Stability.
      Examples.
    Harmonics in the Rotor.

223. The Flux of the Air Gap. So far, we have considered the flux in the air gap as varying sinusoidally both in space and time. Actually, distortions of two kinds occur, so that space and time harmonics are found. Some of the space harmonics are due to the stator winding distribution, pitch, and slot opening; some are due to the same factors from the rotor. More accurately, they arise from the interaction of stator and rotor. To these must be added the effects of magnetic saturation.

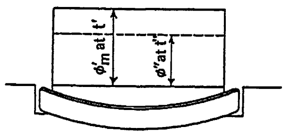


FIG. 181. Space and time harmonics. An alternating current flowing through this coil builds up a flux represented by the rectangle. It is possible for such a coil to give a flux varying sinusoidally with time. The space distribution of flux would never be a sine wave and would need to be represented, theoretically, by an infinite series of harmonics.

The voltage induced in the stator windings by the air-gap flux must be such as to be equal and opposite at all times to the impressed voltage. Except for the slight effect of the stator impedance drops, the flux wave must adapt its time variation to meet this condition. A sine wave of impressed voltage then results in a nearly sinusoidal variation in the flux wave with time, and the time harmonics are small.[1]

The flux distribution around the air gap will depend upon the distribution and pitch of the windings and the number and width of the slot openings. An increased number of slots and phases results in a nearer

[1] L. A. Doggett and E. R. Queer, "Induction Motor Operation with Non-sinusoidal Impressed Voltages," *Trans. A.I.E.E.*, p. 1217, October, 1929.

approach to a sine wave of flux distribution, but it could never be a perfect sine wave. Consequently, space harmonics occur.

Space and time harmonics are of interest because of their effect on the tooth losses and the torque. A little thought will show that time harmonics of the voltage wave will be of higher frequency, but of the same number of poles as the fundamental; space harmonics will be of fundamental frequency with a greater number of poles. Of the two, the latter are more likely to be troublesome, for the space distribution of the flux will have the effect upon the torque curve of giving it dips or hooks or even sub-synchronous speeds which may result in "crawling" at reduced speeds and sometimes excessive noise. Certain other factors, such as the relative number of stator and rotor teeth, slot openings, and skew, also add their effects, but to extents not yet known.

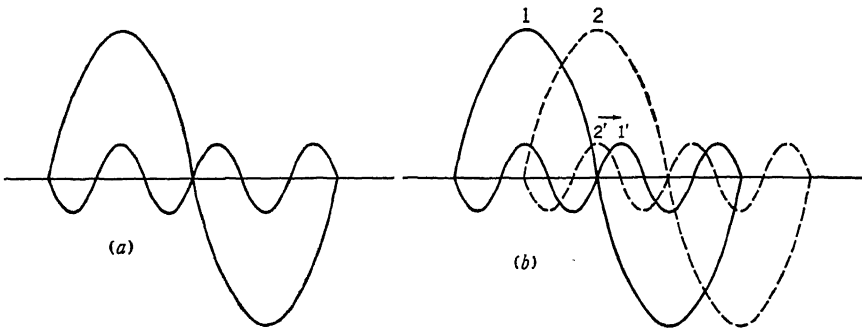


FIG. 182. Harmonics in two-phase motors showing the relative rotation of the fundamental and the third harmonic. (a) Assumes a field flux is of such a shape as to contain a pronounced third harmonic. (b) The flux waves of a two-phase motor with the third harmonics in each. Phase 1 leads phase 2 for the fundamentals. Phase 2′ leads 1′ for the third harmonics.

224. Possible Harmonics. All the odd harmonics can occur in the space distribution of the air-gap flux under the proper conditions. These harmonics are

$$h = 2nm \pm 1 \qquad [301]$$

where h = the order of the space harmonic

n = any assumed integer equal to or greater than 1,

m = the number of phases

Not all these space harmonics build up a rotating flux in the forward direction. This effect is shown in Fig. 182, in which a two-phase motor is considered. If the flux wave of this motor is of such shape as to contain a pronounced third harmonic, the phase sequence of the space har-

monics is shown to be opposite that of the fundamental. The effect on the torque-versus-speed curve is shown in Fig. 183. Those space-phase harmonics, expressed by the following formula, produced fields which move in the same direction as the fundamental.

$$h = 2nm + 1$$

In the two-phase motor they will be

$$2 \times 1 \times 2 + 1 = \text{fifth harmonic}$$

$$2 \times 2 \times 2 + 1 = \text{ninth harmonic}$$

The third and seventh harmonics will produce fields working against the fundamental or giving negative torque.

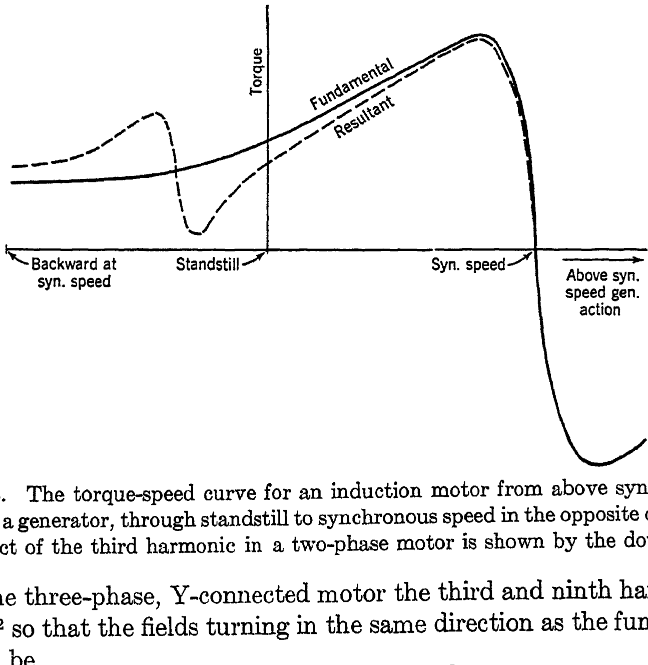


FIG. 183. The torque-speed curve for an induction motor from above synchronous speed as a generator, through standstill to synchronous speed in the opposite direction. The effect of the third harmonic in a two-phase motor is shown by the dotted line.

In the three-phase, Y-connected motor the third and ninth harmonics cancel [2] so that the fields turning in the same direction as the fundamental will be

$$2 \times 1 \times 3 + 1 = \text{seventh}$$

$$2 \times 2 \times 3 + 1 = \text{thirteenth}$$

[2] The "cancelation" of the third harmonics in this case results in a single-phase effect. That is, the third harmonics do not result in a rotating flux, but they do give pulsations to the flux wave. Their action is similar to that of the single-phase motor which has no rotating flux at standstill, but which develops a torque once the motor is running. Similarly the third harmonic in the three-phase, Y-connected motor has no effect upon the torque at standstill, i.e., the starting torque, but it can change the shape of the torque curve at low speeds.

With the third and ninth canceled, the fifth is the only harmonic to work against the fundamental. The direction of rotation caused by any of these harmonics can also be shown by drawing them as was done in Fig. 182. Because these space harmonics give the effect of a higher number of poles, the speed at which they rotate will be

$$\text{Rpm (harmonic)} = \frac{\text{synchronous rpm (fundamental)}}{2nm \pm 1} \qquad [302]$$

225. Effect upon the Rotor. The frequency of the rotor emf depends upon the slip. Since the hth harmonic has a synchronous speed of $1/h$ that of the fundamental, normal values of slip on a motor with reference to the fundamental become very high for the harmonics. The following relationship can be proved:

$$\text{The slip for any harmonic} = 1 \mp h(1 - s) \qquad [303]$$

where h = the order of the harmonic
s = the fundamental slip

Thus, in a case of a fifth harmonic in a two-phase motor, the sign of equation 301 will be positive, that of equation 303 will be negative, and for a normal slip of 10 per cent the slip of the fifth harmonic will be

$$1 - 5(0.90) \quad \text{or} \quad -350 \text{ per cent}$$

Remembering that the frequency of the rotor emf depends upon the slip, we can set up the following relationship between the rotor frequency due to the fundamental with a slip of s and the rotor frequency due to the hth harmonic.

$$\frac{\text{Rotor } f_{\text{harmonic}}}{\text{Rotor } f_{\text{fundamental}}} = \frac{1 \mp h(1 - s)}{s} \qquad [304]$$

If, for a certain value of slip, this ratio is not an integer, the emf set up in the rotor is not a true periodic function; the wave shape of the rotor emf is constantly changing because of the shifting of the fundamental and the harmonics.

226. Torque Curves.[3] **Crawling and Stability.** The torque required to accelerate a load connected to an induction motor may vary con-

[3] C. P. Steinmetz, "Theory and Calculation of Electrical Apparatus," McGraw-Hill Book Co.

E. E. Dreese, "Synchronous Motor Effects in Induction Machines," *Trans. A.I.E.E.*, p. 1033, July, 1930.

G. Kron, "Induction Motor Slot Combinations," *A.I.E.E. Paper*, 31–46.

A. M. Wahl and L. A. Kilgore, "Transient Starting Torques in Induction Motors," *Elec. Eng.*, November, 1940.

siderably for different cases. Only one, illustrated as curve 1 of Fig. 184, will be discussed here. If the motor torque developed follows the curve of fundamental torque as shown, the motor will accelerate to the point k as a final operating speed for the given load. On the other hand, any harmonic which causes a dip in the torque curve (dotted line) will cause the motor to "crawl" at the reduced speed indicated by g. A momentary reduction in load may permit the motor to accelerate to h, at which

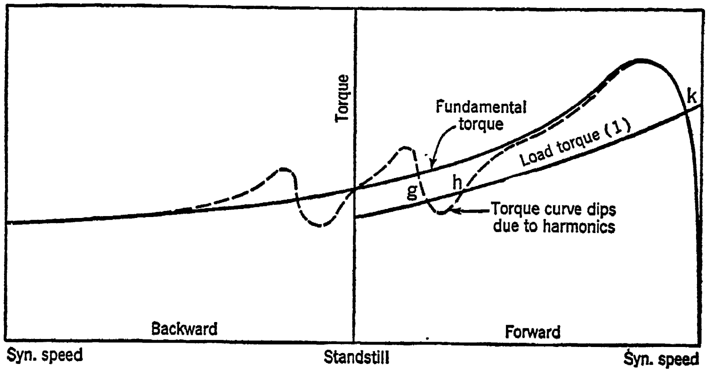


FIG. 184. The torque-speed curve for a three-phase induction motor.

developed and load torque are equal. Such operation, however, is unstable, and the motor, once it has reached the speed of h, may accelerate to the point k.

227. Eliminating the Disturbances. Unfortunately, no satisfactory method for predicting these disturbances with certainty is known at present so that it is still necessary to rely on trial and error. To quote from *Electrical Engineering:*

> Speed-torque curves of polyphase induction motors on test have never agreed with the smooth calculated characteristics. As a result of these unpredictable "hooks" or "cusps" in the speed-torque curve, some motors are found to crawl at unexpected speeds or are so noisy as to be useless for all practical purposes. Various explanations have been advanced as to the cause of these irregularities, but no satisfactory method of definitely pre-determining the speed-torque curve could be found.
>
> Certain factors, such as the harmonics in the revolving field caused by the distributed winding, or the relative number and shape of the stator and rotor slots, are known to influence the shape of the curve. However, dependence upon empirical rules for the selection of rotor slots always has been necessary. . . . As is to be expected, all these empirical rules not based on a knowledge of the disturbing phenomena often result in unpleasant surprises. Hence today manufacturers are led to the wasteful method of building a series of rotors with different slots whenever a new line of induction motors is required, to find by actual test which of the rotors is most satisfactory.[4]

[4] Articles by Gabriel Kron, P. H. Trickey, and R. D. Ball, "Irregularities in Speed-torque Curves of Induction Motors," *Elec. Eng.*, p. 936, December, 1931.

CHAPTER XXVII

WINDING RESISTANCE. LEAKAGE REACTANCE. ROTOR BAR SKEW

228. Chapter Outline.

Divers Topics on Induction Motors.
- Winding Resistances.
  - Stator.
  - Rotor.
- Discussion of Leakage Reactance Components.

Effect of Skewing Slots.

229. Calculation of Winding Resistances. The resistance of the stator winding per phase can readily be calculated from the conductor length, size, etc., by the usual formula, such as that given for alternators. In such calculations it is sometimes convenient to take the resistance of hot copper as 1 ohm per circular-mil inch. The resistance of a conductor in ohms is then equal to the length in inches divided by the area in circular mils.

The resistance of a squirrel-cage winding can be determined indirectly through test readings or calculated from measurements of conductor and end-ring dimensions. The next procedure, usually, is to obtain the resistance of the rotor in terms of the stator, using the ratio of transformation. As this entire process is likely to be mystifying to the novice, its details will be given briefly. This ratio for squirrel-cage rotors is

$$a = \frac{mZ_1k_pk_d}{N_2} \tag{305}$$

where m = the number of primary phases
Z_1 = the number of stator conductors per phase
k_p = the pitch factor of the stator winding
k_d = the distribution factor of the stator winding
N_2 = the number of bars in the squirrel cage

Next let L_2 = the length of the rotor bars in inches, taken between end rings
K_b = resistivity of bar material divided by resistivity of copper
A_2 = area of each bar in circular mils
I_2 = effective current in the rotor bars

If all the rotor bars were in series, the rotor resistance would be

$$R = \frac{N_2 L_2 K_b}{A_2} \text{ (ohms)} \qquad [306]$$

The loss in the bars would be

$$I_2^2 \frac{N_2 L_2 K_b}{A_2} \text{ (watts)} \qquad [307]$$

The result would be the same if the bars were connected in parallel, so the above formula expresses the copper loss in the rotor bars.

The Loss in the End Rings. To calculate this loss the assumption is made that the current in the rotor bars is sinusoidally distributed. Select some point a on the rotor circumference so that the ring currents flow away from it in opposite directions (Fig. 185). Then the average current in the bars between a and bb will be

$$\frac{2}{\pi} I_{2\max}$$

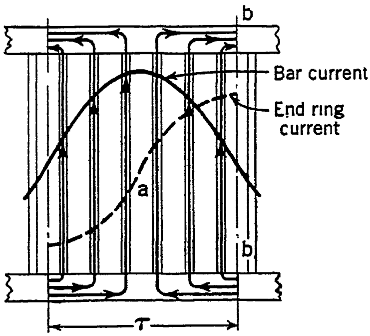


FIG. 185.

This current, multiplied by the number of bars in a half pole pitch, gives the maximum ring current.

$$I_{r_{\max}} = \frac{2}{\pi} I_{2_{\max}} \frac{N_2}{2P}$$

$$= \frac{I_{2_{\max}} N_2}{\pi P}$$

Or, in effective values,

$$I_r = \frac{I_2 N_2}{\pi P}$$

The resistance of one end ring is

$$R_r = \frac{\pi D_r K_r}{A_r} \text{ (ohms)} \qquad [308]$$

where D_r = the mean diameter of the end ring in inches
K_r = the resistivity of the ring material divided by the resistivity of copper
A_r = the area of the ring section in circular mils

The copper loss in the two rings is

$$\left(\frac{I_2N_2}{\pi P}\right)^2 \frac{2\pi D_rK_r}{A_r} \text{ (watts)}$$

The total copper loss in the rotor is the sum of the bar and end-ring losses:

$$I_2{}^2N_2{}^2\left(\frac{L_2K_b}{A_2N_2} + \frac{2D_rK_r}{\pi P^2A_r}\right) \text{ (watts)} \qquad [309]$$

Because of the similarity of this formula with the usual I^2R formula, it is obvious that the expression

$$N_2{}^2\left(\frac{L_2K_b}{A_2N_2} + \frac{2D_rK_r}{\pi P^2A_r}\right) \qquad [310]$$

represents the total resistance of the squirrel-cage rotor. As it is frequently more convenient to express this in terms of the stator resistance, it can be referred to the stator by multiplying by a^2. Then

$$R_2 \text{ (in stator terms)} = \left(\frac{L_2K_b}{A_2N_2} + \frac{2_rDK_r}{\pi P^2A_r}\right)(mZ_1k_pk_d)^2 \qquad [311]$$

Of this, $1/m$th is allotted to each phase.[1]

230. Example. A 20-hp, 220-volt, three-phase, 60-cycle, 4-pole induction motor shows the following design-sheet data:

	Stator	Rotor
Slots (number)	48	59
Slots (size)	0.300 by 1.45 in.	0.250 by 0.44 in.
Conductors per slot	8	1
Conductor size (circular mils)	20,800	74,500
Diameter	Bore 9.00 in.	8.948 in.
Axial core length (without $\frac{3}{8}$-in. duct)	4.72 in.	4.72 in.
Connection	Y	Squirrel-cage
Pitch	$\frac{5}{6}$	
Area of end ring		1.25 by 0.50 in.
Diameter of end ring (outside)		8.10 in.
Resistivity of end-ring material as compared with copper		1.82
Length of one conductor	15.1 in.	5.595 in.
Length between end ring		5.595 in.

[1] If the rotor bars are skewed, the resistance is increased and the ratio of transformation is changed. See Article 232.

Calculation of stator resistance:

$$8 \text{ conductors per slot, 48 slots} = 384 \text{ conductors}$$
$$\text{Conductors per phase} = 128$$
$$\text{Conductor length per phase} = 15.1 \text{ in.} \times 128$$
$$= 1935 \text{ in.}$$
$$\text{Ohms per phase} = \frac{1935}{20{,}800}$$
$$= 0.093 \text{ ohm (hot)}$$

Calculation of rotor resistance:

$$R_2 = N_2{}^2\left(\frac{L_2K_b}{A_2N_2} + \frac{2D_rK_r}{\pi P^2 A_r}\right)$$

and since $N_2 = 59$ bars
$L_2 = 5.595$ in. between rings
$K_b = 1$ (since the bars are of copper)
$A_2 = 74{,}500$ cir mils, bar area
$D_r = 7.60$ in., mean diameter of end ring
$K_r = 1.82$
$P = 4$ poles
$A_r = 1.25 \times 0.50 \times 1{,}273{,}000$ cir mils cross-sectional area

$$R_2 = \overline{59}^2\left(\frac{5.595 \times 1}{74{,}500 \times 59} + \frac{2 \times 7.60 \times 1.82}{\pi \times 4^2 \times 1.25 \times 0.50 \times 1{,}273{,}000}\right)$$
$$= \overline{59}^2(0.00000196)$$
$$= 0.00683 \text{ ohm}$$

To obtain the rotor resistance in terms of the stator,

$$R_2 \text{ in stator terms} = (0.00000196)(m_1Z_1k_pk_d)^2$$

and since $k_p = 0.966$ from the pitch
$k_d = 0.956$ from the slots per phase per pole
$Z_1 = 128$ conductors per phase
$m_1 = 3$ phases

then

$$R_2 = 0.2480 \text{ ohm}$$

This is the total resistance of all bars and the entire ring circumference. Of this value, one-third will be allotted to each phase so that the resistance of the rotor, per phase, in stator terms becomes

$$R_2 \text{ (per phase)} = 0.0826 \text{ ohm}$$

231. Leakage Reactance. The leakage reactance of an induction motor requires accurate formulas for its calculation. This is because of the importance of this reactance in determining the maximum torque, the starting current, the starting torque, and, in fact, the whole performance. In predicting the characteristics of an induction motor from design-sheet data by means of the circle diagram, the equivalent leakage

reactance determines the diameter of the circle and influences directly all the values read therefrom.

In general, the analysis of leakage reactance of the stator winding is similar to that of alternator windings except in some details. Methods

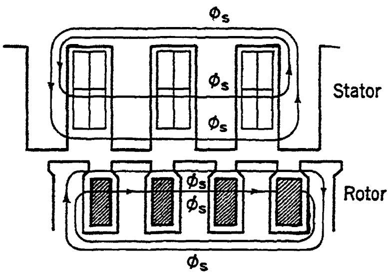


FIG. 186. Slot leakage around a phase belt.

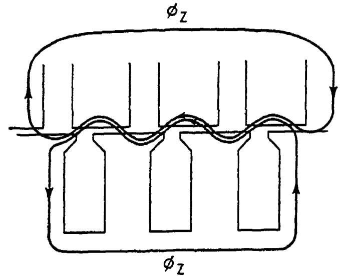


FIG. 187. Zigzag leakage.

of calculation will not be given here, but the various components of leakage flux will be discussed.

Slot Leakage Flux, ϕ_s. Figure 186 shows the slot leakage flux around one phase belt of the stator and a similar flux around the rotor conductors under it. A phase belt is made up of the adjacent conductors of one phase. The slot leakage is influenced greatly by the ratio of slot width to slot depth and by the general shape of the teeth.

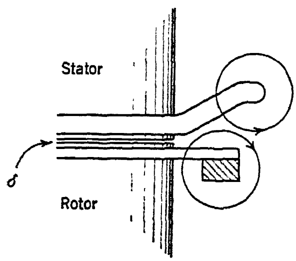


FIG. 188. End-connection leakage. Note that the projection of the bars of the squirrel cage beyond the rotor core would influence the leakage flux.

Zigzag Leakage Flux, ϕ_z. This flux zigzags from tooth to tooth across the air gap around a phase belt, as shown in Fig. 187. The comparative widths of tooth and slot in both stator and rotor and the length of air gap are the chief factors influencing its magnitude.

End-connection Leakage Flux,[2] ϕ_e. This flux is built up around the end connections of the stator winding over an air path.

[2] C. A. Adams, "The Leakage Reactance of Induction Motors," *Trans. Intern. Elec. Congr.*, Vol. 1, p. 706.

C. A. Adams, W. K. Cabot, and G. A. Irving, Jr., "Fractional Pitch Windings for Induction Motors," *Trans. A.I.E.E.*, Vol. 26, Part II, p. 1485.

R. E. Hellmund, "Zig-zag Leakage of Induction Motors," *Trans. A.I.E.E.*, Vol. 26, p. 1505.

Standard Handbook for Electrical Engineers, Sixth Edition, McGraw-Hill Book Co.

C. A. Adams, "The Design of Induction Motors: with Special Reference to Magnetic Leakage," *Trans. A.I.E.E.*, Vol. 24, p. 649.

Such a flux is also set up around the end connections of the rotor winding, or the end rings and projecting bars of a squirrel cage. The leakage flux on the end connections of the stator winding is affected, however, by the position of the end ring, and different correction factors have been applied for various constructions.

Belt or Differential Leakage Flux.[3] In a wound-rotor motor another element of leakage flux is present, caused by different distribution of the stator and rotor windings, or by the different positions which one phase belt of the rotor assumes with respect to one phase belt of the stator as the rotor turns. To understand this action several assumptions must be made.

The secondary winding is short-circuited and its resistance and local leakage reactances are neglected. The voltage induced in the secondary must then be zero, when the primary is excited. The current in the two windings will assume such values as to satisfy this condition. That is, the balancing component of secondary current is such that it neutralizes the mmf of the primary current. The resultant flux or mmf over a phase spread will then be zero, but, owing to the different distributions or positions of the stator and rotor windings, the flux will have positive and negative loops in the air gap of the same pole face. These positive and negative fluxes induce voltages in the parts of the secondary coils, but their directions are such as to neutralize and give no resultant secondary voltage. In the primary winding, however, these coils are in a position, with respect to the flux distribution, that will give a number of positive interlinkages and a resultant voltage drop in the primary. Hence the flux under these conditions actually links both primary and secondary, but induces a voltage in the primary only and behaves as a true leakage flux. This effect varies with different positions of the rotor.

It must be remembered that all of these are but components of the leakage flux, and though it is convenient to treat them as if they have an independent existence, they really occur as mere distortions in the main flux.

232. Skewed Slots. Many induction motors are built with skewed (or twisted) rotor bars. Skew reduces "cogging" and other torque defects and tends to eliminate motor noise. Indiscriminate skew, especially in machines with many poles, leads to a great reduction in short-circuit current, starting torque, and torque at high slips. The increase in skew angle has an effect similar in some respects to decrease in voltage.

[3] An excellent paper on this subject is "Transformer Ratio and Differential Leakage of Distributed Windings," by R. E. Hellmund and C. G. Veinott, *Trans. A.I.E.E.*, Vol. 49, p. 1043.

Skew increases the rotor resistance by increase in bar length, so that

$$R'_b = R_b\sqrt{1 + \left(\frac{\tau\alpha}{L_c \times 180}\right)} \qquad [312]$$

where τ = the pole pitch in inches
L_c = the core length in inches
R_b = bar resistance without skew
R_r = end ring resistance
R_2 = secondary resistance or $R'_b + R_r$
α = the angle of skew in degrees
$= \dfrac{\text{slot pitches of skew}}{\text{total rotor slots}} \times \dfrac{\text{poles}}{2} \times 360°$

The above formula indicates that the rotor resistance is increased by the skew, and hence the effect on the performance of an induction motor (so far as this one change is concerned) is exactly the same as that which would be brought about by redesigning the motor for a slightly higher rotor resistance.

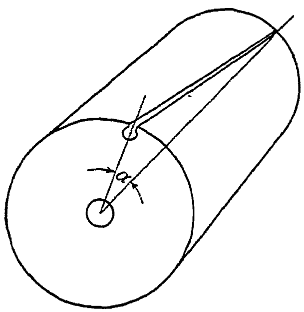


FIG. 189. Skew angle of a 2-pole motor.

In practice it is noticed that the maximum torque of the motor, the torque developed at any speed, as well as the no-load characteristics, are all modified by rotor-bar skew. These effects are due to a change in mutual linkages between stator and rotor, differential leakage, and a possible axial zigzag leakage (because stator and rotor slot mmf's are no longer in complete phase opposition), for all of which no adequate analysis is available.

Starting torque and starting current are greatly affected by skew, both for the reasons given in the above paragraph as well as for the change in rotor resistance.

The above effects may or may not be detrimental, but the beneficial effects of skew occur from reducing the influence of space harmonics in the air-gap flux. This can be shown as follows:

Suppose an elementary d-c generator armature winding consisted of 1 coil. If this was skewed so that both sides of the coil had an equal projection under like poles, no emf would be generated to cause current to flow.

It has already been pointed out that space harmonics in the gap flux have the effect of building up multiple poles, i.e., the fifth harmonic

results in 10 poles in a 2-pole motor, or n times the number of fundamental poles for the nth harmonic. Therefore, if a rotor bar is skewed so that one end of the bar is under a north harmonic pole, and the other end is under a north harmonic pole also, no current of that harmonic will flow in the bar. Although skewing does not remove the space ripple in the flux wave, it may eliminate a rotor current of that harmonic upon which it can react to produce noise, or a dip in the speed-torque curve.

233. Effects of Skew Analyzed. To determine the effect of skew, analytically, upon the short-circuit (or starting) current, use can be made of the two Kirchhoff law equations for coupled circuits. They are given in Article 131 for the usual case, and for skewed slots they would be written

$$(L'_1D + r_1)i_1 + kMDi_2 = V \sin wt \qquad [313]$$

$$kMDi_1 + (L'_2D + r'_2)i_2 = 0 \qquad [314]$$

The terms are defined in the article previously cited, but note now that:

L'_1 = the total inductance of the primary with secondary open, including the effect of the leakage flux with skew

L'_2 = the total inductance of the secondary with primary open, including the effect of the secondary leakage flux with skew

r'_2 = the skewed rotor resistance

k = the skew factor

$= \left(\sin \frac{\alpha}{2}\right) \div \frac{\alpha}{2}$ (using α in radians)

For any skew, equations 313 and 314 may be solved for primary and secondary short-circuit current I'_1 and I'_2, respectively.

$$I'_1 = \frac{V}{Z'_s + \dfrac{k^2X_m{}^2}{Z''_s}} \qquad [315]$$

$$I'_2 = \frac{kX_m}{Z''_s} I'_1 \qquad [316]$$

Using r'_2/s in place of r'_2 would enable the performance to be obtained at any value of slip. Here we are interested in the effect of skew on the short-circuit currents, and so the ratio of primary locked

current with skew to that without skew will be designated as r. Then:

$$r = \frac{I'_1}{I_1} = \frac{Z'_e + \dfrac{X_m^2}{Z''_e}}{Z'_s + \dfrac{k^2 X_m^2}{Z''_s}} \qquad [317]$$

wherein:

$$Z'_s = r_1 + j(X_m + x_1) \quad \text{(skewed)} \qquad [318]$$

$$Z''_s = r'_2 + j(X_m + x_2) \quad \text{(skewed)} \qquad [319]$$

The terms Z'_e and Z''_e are also defined by equations 120 and 121.

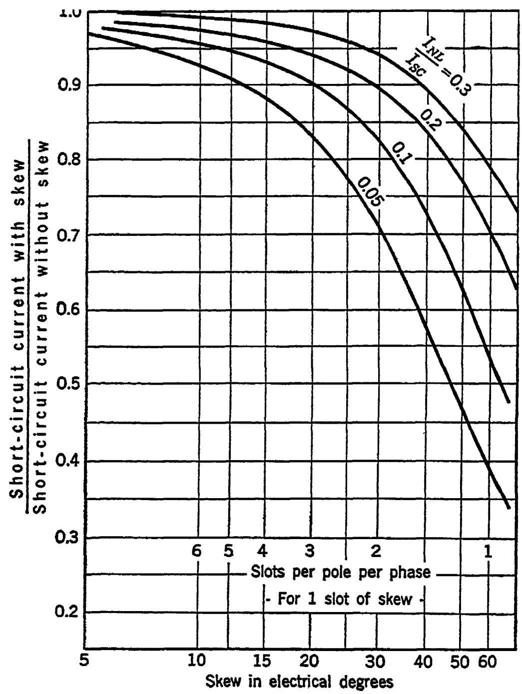


FIG. 190. Effect of skew on short-circuit current. I_{NL} is the no-load current and I_{SC} is the short-circuit current. [Dr. M. Liwschitz, *Electrical Machines, Calculation and Design,* Vol. III, p. 215. (In German.) Used by permission of the author.]

Figure 190 illustrates this effect, indicating that machines with a high ratio of no-load to locked current are less sensitive to skew than others. That is, machines with many poles are less affected for a given value of α.

Table X indicates the comparison between machines with and without skew, assuming that air-gap flux is the same in each case.

TABLE X

	Zero Skew $\alpha = 0$	Skew Angle $= \alpha$ Radians
Slip	s	s
Secondary induced emf	sE_2	ksE_2
Secondary leakage reactance	X_2	$X'_2 > X_2$
Secondary resistance	R_2	$R'_2 > R_2$
Secondary current	$I_2 = \dfrac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$	$\dfrac{ksE_2}{\sqrt{R'^2_2 + (sX'_2)^2}}$
Torque produced *	$K_2 I_2 \phi_m \cos(\phi, I_2)$ $= K_2 \phi_m \dfrac{sE_2 R_2}{R_2^2 + (sX_2)^2}$	$K_2 \phi_m \dfrac{k^2 sE_2 R'_2}{R'^2_2 + (sX'_2)^2}$
Primary current required to balance the rotor mmf	I_b	kI_b
Ratio of transformation	$a_1 = \dfrac{m_1 N_1 k_{w_1}}{m_2 N_2 k_{w_2}}$	$\dfrac{a_1}{k}$

* K_2 can be evaluated by comparison with the torque equation of Articles 200, 204, and 206.

CHAPTER XXVIII

COMMERCIAL STANDARDS. STARTING METHODS

234. Chapter Outline.

Ratings.
Standards.
  Maximum and Starting Torques.
  Starting Currents.
  Frame Sizes.
Starting Methods.
  Wound-rotor.
  Deep-slot Rotor.
  Double-cage Rotor.
Autotransformers As "Compensators."
Y-delta Starting.

235. Standards and Ratings. Various standardizing bodies, such as the National Electrical Manufacturers Association, the American Institute of Electrical Engineers, and the American Standards Association, have drawn up regulations and recommendations concerning the forms, frame dimensions, tests, and performance of polyphase induction motors. The literature is too detailed and voluminous to consider here, but a brief outline of some of the information is of interest.

The most common form of polyphase induction motors generally available are classed as *general purpose*. They are defined as 220 hp or less, at speeds of 450 rpm or more, for continuous operation, and are offered in standard horsepower ratings without restriction to a particular classification.

Squirrel-cage induction motors are further classified into such groups as (1) normal torque, normal starting current; (2) normal torque, low starting current; (3) high torque, low starting current, etc.

Ratings. Standard ratings are sizes of $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{2}$, 2, 3, 5, $7\frac{1}{2}$, 10, 15, 20, 25, 30, 40, 50, 60, 75, 100, 125, and 150 hp in speeds (for 60 cycles) corresponding to 2, 4, 6, 8, 10, 12, 14, and 16 poles.

In attempting to standardize, some difficulty is encountered in "defining" a motor. The usual understanding is that if a motor is rated at a

certain horsepower it will be capable of carrying a continuous rating without exceeding the specified temperature rise. To this is usually added a *service factor* of 1.15, meaning that a load of 115 per cent can be carried continuously without exceeding a safe temperature rise.

Suppose we consider a 10-hp 3435-rpm motor. Full-load torque is 15.3 lb-ft. Let us assume that this motor had a maximum torque of 300 per cent of full load (45.9 lb-ft) and, by reason of good design and ventilation, had a full-load temperature rise of only 30 C. A second motor, also rated at 10 hp, had a maximum torque of 150 per cent of full load (22.9 lb-ft) and a full-load temperature rise of 38 C. Both motors might be labeled 10 hp, 40 C rise, yet it is obvious that the first motor had much more reserve capacity, whereas in the second motor a reduction in line voltage along with a momentary overload might cause the motor to stall, with excessive current. The purchaser of the first motor obtains more for his money.

As a protection to purchasers, the N.E.M.A. has set up minimum values of torque, both maximum and starting, for general-purpose motors.

The minimum value of maximum or breakdown torque is expected to be at least 200 per cent of rated full-load torque, except in a few special cases.

Starting-torque limitations should not be less than the following for 25- and 60-cycle squirrel-cage motors:

2 and 4 poles	150 per cent of full-load torque
6 poles	130 per cent of full-load torque
8 poles	125 per cent of full-load torque
10 poles	120 per cent of full-load torque
12 poles	115 per cent of full-load torque
14 poles	110 per cent of full-load torque
16 poles	105 per cent of full-load torque

236. Standards for Starting Amperes. Power companies are interested in the amount of starting current that motors of various ratings require, since this starting current may cause troublesome voltage fluctuations on their supply lines. Because of this possibility, local regulations are frequently set up. The N.E.M.A. is now coordinating these values, evolving a table of recommended starting kilovolt-amperes for motors as a function of horsepower and speed. Thus, general-purpose motors of any rating are expected to have starting kilovolt-amperes within certain limits or "bands," the maximum value of which will not result in starting currents exceeding the regulations for some of the larger power companies.[1]

[1] At the present printing these values have not been adapted as a standard and are not available for publication, 1942.

237. Frame Sizes. Standardization. Until a decade ago, a manufacturer of some device that required a motor drive, might find, on attempting to purchase identically rated motors from another source, that the available motors had entirely different dimensions from those of his initial supplier. A score of 50-hp motors from as many different manufacturers might bear no resemblance to each other in shaft size, mounting dimensions, or height of shaft above the base. Thus a user

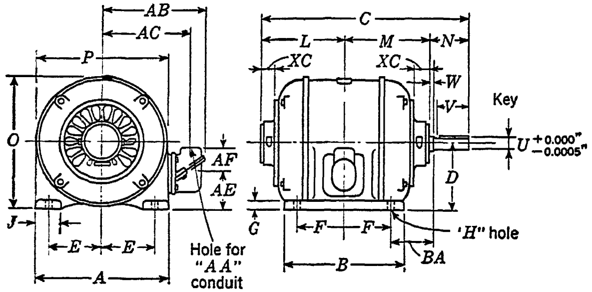


FIG. 191. Typical drawing for a NEMA motor. Only the dimensions *A*, *B*, *D*, *E*, *F*, *BA*, *H*, *N–W*, *U*, and *V* would be fixed by standardization.

of motors was penalized by not having alternate sources of supply for interchangeable motors.

To remedy this chaotic condition, the N.E.M.A. standardized important frame dimensions, such as height of shaft above base, shaft-extension sizes, and relation of pulley position to base-mounting holes. These frames are listed as 203, 204, 224, 225, 254, 284, 324, 326, 364, 365, 404, 405, 444, 445, 504, and 505. The first two digits of the frame number, divided by 4, equals the mounting height in inches, i.e., center of shaft above the base.

Not only are the sizes standardized but also the horsepower ratings, which are built in each frame for different speeds at 25 and 60 cycles, are standardized. Thus frame 284, for example, will have a shaft height of 7 in. (28 divided by 4) and a diameter of approximately 14 in. In this frame are specified ratings as follows:

Open Type, 60 Cycles	40 C Rise
10 hp, 2 poles	$1\frac{1}{2}$ hp, 12 poles
$7\frac{1}{2}$ hp, 4 poles	1 hp, 14 poles
5 hp, 6 poles	$\frac{3}{4}$ hp, 14 poles
3 hp, 8 poles	$\frac{3}{4}$ hp, 16 poles
2 hp, 10 poles	

Totally Enclosed, 60 Cycles	55 C Rise
3 hp, 4 poles	2 hp, 8 poles

Open Type, 60 Cycles	Wound Rotor 40 C Rise
$7\frac{1}{2}$ hp, 4 poles	3 hp, 8 poles
5 hp, 6 poles	
Open Type, 25 Cycles	40 C Rise
5 hp, 2 poles	$1\frac{1}{2}$ hp, 6 poles
3 hp, 4 poles	

The above values are given only as typical examples of the standardization program; in a similar manner all ratings are related to frame sizes from $\frac{1}{2}$ to 150 hp. Standardization in the N.E.M.A. dimensioned frames has been carried out in large single-phase and d-c motors as well.

238. The Importance of Rotor Resistance. In selecting polyphase induction motors for specific duties, attention must be paid to such items as degree of enclosure, duty cycle, speed, and many other factors. Included here are such considerations as starting torque and starting current. On this basis alone, the motor type may have to be modified from plain squirrel-cage to deep slot, double-cage or wound rotor. This subject will be investigated briefly.

It has been shown that the squirrel-cage induction motor, when considered at standstill as a simple, equivalent series impedance would have a starting current as follows:

$$I = \frac{V}{Z_e} \qquad [320]$$

wherein applied volts and equivalent impedance are phase values. Numerically:

$$Z_e \approx \sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2} \qquad [321]$$

For a $7\frac{1}{2}$-hp motor, consider the following constants:

220 volts 4 poles three-phase 60 cycles

$R_1 = 0.3175$ ohm at 20 C

$X_1 + X_2 = 0.928$ ohm

$R_2 = 0.148$ ohm at 20 C

Then $Z_e = 1.038$ ohms and the starting current is 122.5 amperes.

The starting torque in synchronous watts is 6670.

$$T_{\text{lb-ft}} = \frac{7.04 \times 6670}{1800} \quad \text{or} \quad 26.0 \qquad [322]$$

The full-load torque at 4 per cent slip can be calculated as follows:

$$\text{Full-load torque} = \frac{\text{horsepower} \times 5500}{\text{synchronous rpm}} = \frac{7.5 \times 5500}{1800} \quad \text{or} \quad 23 \text{ lb-ft}$$

Note in Article 235 that the starting torque should be at least 150 per cent of full-load torque or 34.5 lb-ft. Suppose the rotor resistance is increased from 0.148 to 0.297 ohm. Now Z_e equals 1.11 ohms, and the starting current is 114.3 amperes per line. The starting torque is 45.2 lb-ft.

The point is that reduced starting current and increased starting torque can be obtained by using larger values of rotor resistance. The stator winding resistance and the total leakage reactance are kept the same, as they would otherwise modify the maximum torque. On this basis, rotor resistance should be high.

Suppose we consider *running* conditions. The first rotor (0.148 ohm) displayed a full-load slip of 0.023 at 75 C. The second rotor gave a slip of 0.046 at full load and 75 C, and a rotor copper loss of almost twice the former value. Either high slip or increased losses with attendant heating might be objectionable and we can now see the practical limits of increase on rotor resistance. Hence every ordinary squirrel-cage motor is a compromise between high starting torque and current requirements versus full-load slip and copper loss. The ideal case would involve a variable resistance in the rotor which would be high at start and low under load. This is actually provided when necessary, leading us to three other motor types: *wound rotor*, *deep bar*, and *double cage*.

Wound Rotor. This will be described briefly in Article 242, wherein the variation in rotor resistance is employed as a means for speed control. Aside from that application, the wound rotor offers the advantage of enabling external resistance to be connected in series with the leads (brought out through slip rings), thereby offering a large value of R_2 for starting. The heat generated at starting is thus, in part, kept out of the motor. The maximum possible starting torque is equal to the breakdown value. Once in operation, these resistances are short-circuited, reducing R_2 for efficient load operation to a value limited by winding resistance alone. This method is effective, but comparatively more costly than the squirrel-cage designs.

Deep Bar. At standstill, the rotor frequency equals the frequency impressed on the stator; at full load, it is only that of slip frequency. This change in rotor frequency is used to bring about an apparent change in cage resistance by the skin effect in special rotor bars. That is, by using deep, narrow bars, the higher frequency at starting crowds the current to the upper surface; all the copper is not used effectively. When running, the entire bar section is useful, reducing the effective value of the cage resistance.

To consider a specific case:

$7\frac{1}{2}$ hp 4 poles 60 cycle three-phase 220 volts

Rotor bars are 0.156 in. by 0.875 in. The rotor resistance at 20 C is 0.291 ohm, but at standstill this increases to an effective value of 0.613 ohm.

$$R_1 = 0.30 \text{ ohm}$$

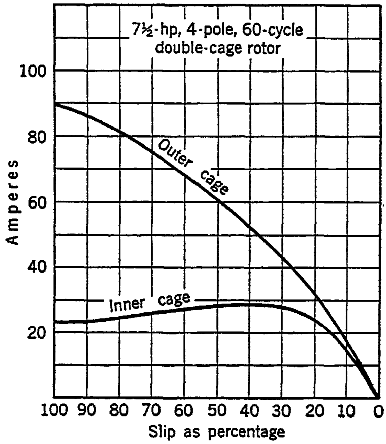
$$X_1 + X_2 = 1.18 \text{ ohms}$$

Starting performance:

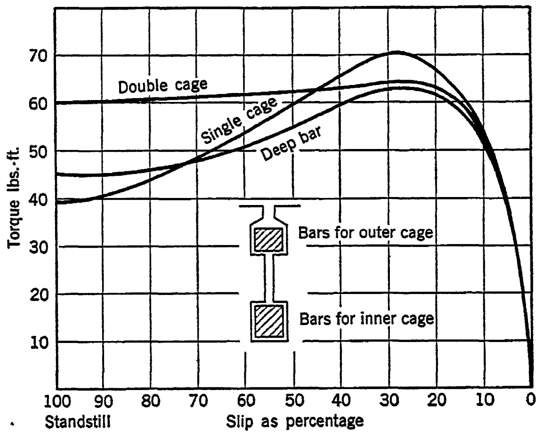
$$Z_e = \sqrt{(1.18)^2 + (0.30 + 0.613)^2} \quad \text{or} \quad 1.49 \text{ ohms}$$

$$I = 85.1 \text{ amperes, starting current}$$

$$\text{Starting torque} = 13{,}300 \text{ synchronous watts or } 52 \text{ lb-ft}$$



(a)



(b)

FIG. 192. Double-cage rotors. (a) Division of currents between the cages. (b) Comparison of characteristics, single-cage, deep-slot, and double-cage rotors.

Comparison of these figures indicates that an unusually high starting torque with low starting current is obtained, yet the full-load slip is only 0.045 with comparable copper losses.

Double Cage. Mention has already been made of this type of motor, utilizing two cages on the rotor, one below the other. The outer cage is

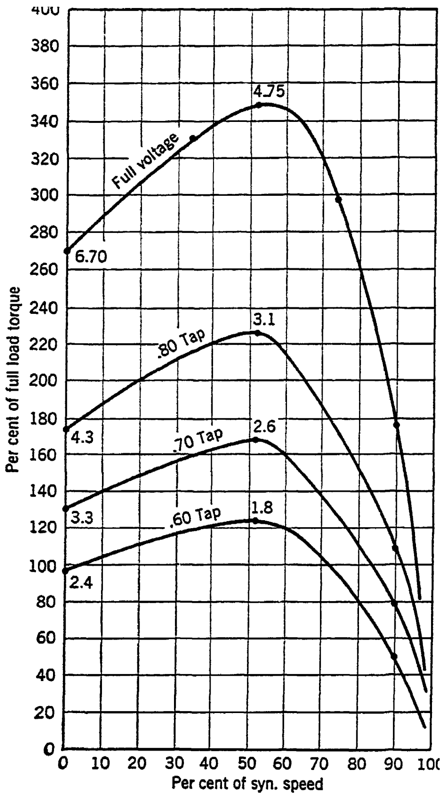


FIG. 193. Torque-speed curves for an induction motor starting at various voltages. The starting transformers are equipped with 0.8, 0.7, and 0.6 voltage taps. The figures indicate the line current. Thus at full voltage the line current at starting is 6.7 times rated current; at maximum torque the current reduces to 4.75 times normal.

of fairly high resistance, giving high starting torque with comparatively low amperes. Being deeply embedded in the iron of the rotor core, the inner cage displays high reactance so that at the instant of starting, with line frequency in the rotor, this cage takes very little current. As the speed picks up and the rotor frequency reduces, more and more current flows through the inner cage until, under running conditions, both cages

are active, in parallel. A calculated division of current is shown in Fig. 192*a* for a $7\frac{1}{2}$-hp, 4-pole design.[2]

239. External Starting Devices. Since the induction motor is a simple impedance at the instant of starting, any reduction in applied voltage brings about a proportionate reduction in starting amperes. Under certain conditions, especially on large motors, reduced voltage starting

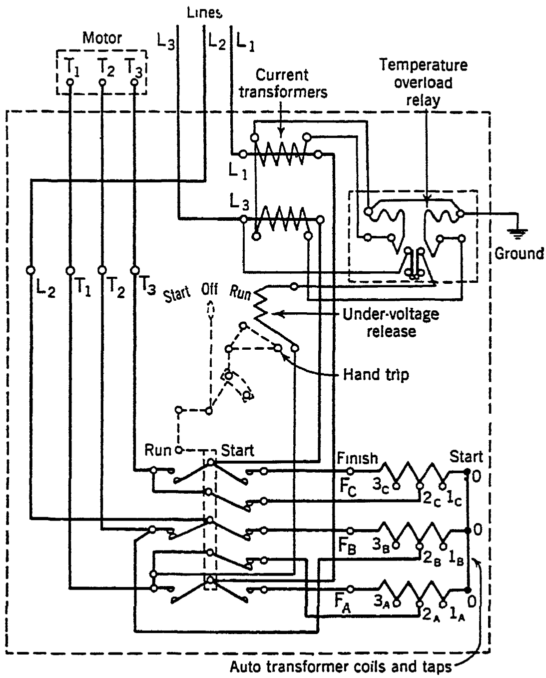


FIG. 194. Connections for a "hand-starting compensator." (*General Electric Co.*)

is necessary. Reduced voltage is usually obtained by two autotransformers, connected in open Δ for three-phase motors. Half voltage would reduce the motor starting current to half value. If the motor starting current on full voltage were six times normal, half-voltage starting would give three times normal current. By transformer ratio, the *line* current would then be 1.5 times the full-load amperes.

Since starting torque varies as the square of the voltage, this reduction in supply would give only one-quarter of the normal starting torque. As such a reduction is often undesirable; higher voltage taps are provided on commercial starting transformers. They are of the manually

[2] See the text by Punga and Raydt, previously cited.

operated or automatic type. A connection diagram of the "hand-starting compensator" is shown in Fig. 194. After the motor comes up to speed, the switch is thrown over, applying full voltage to the motor and removing the transformer from the line.

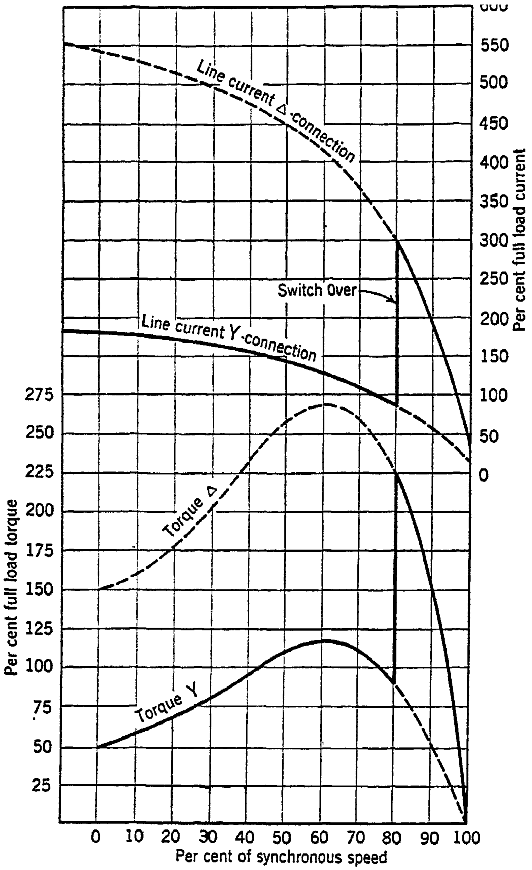


FIG. 195. Y–Δ starting. The solid lines show the current on switching over from Y to Δ when the motor has reached 80 per cent of synchronous speed.

Y-delta Starting. If the stator windings of the motor are normally connected in Δ, reconnection to Y reduces the voltage on each phase at starting and results in less current. This can be accomplished by a double-throw switch without other expensive equipment. Typical current and torque curves for this starting arrangement are shown in Fig. 195.

CHAPTER XXIX

SPEED CONTROL

240. Chapter Outline.

Speed Control of Induction Motors.
- Pole Change.
  - Two or More Independent Windings.
  - Regrouped Coils.
- Resistance in the Rotor Circuit.
- Electromotive Force in the Rotor Circuit.
  - Frequency Converter Systems.
  - Brush-shift or Schrage Motor.
- Concatenation.

241. Methods: Pole Change. The induction motor is essentially a constant-speed device, inasmuch as its speed results from a rotating flux varied only by the number of stator poles and the frequency of supply. Speed control is one of the requirements of motors for many industrial applications; to achieve this speed control in induction motors a number of different methods involving various principles have been utilized.

Examination of the speed formula for the induction motor shows that a wide choice of speed is possible by changing the number of poles.

$$\text{Synchronous rpm} = \frac{f \times 120}{\text{poles}} \qquad [323]$$

The number of poles can be changed by the use of two, or more rarely three or four, independent windings on the stator, or by regrouping the parts of one winding.[1] For three-phase machines, three leads are required for each speed.

A stator winding for 8 poles and 900 rpm, when reconnected by means of a drum controller to 4 poles and 1800 rpm, would have a wide variation in speed but no nicety of adjustment. Such a motor is known as a *multispeed* motor. If the rotor winding is squirrel cage, it will change its number of rotor poles automatically with the pole change in the stator.

[1] Creedy, in "Theory and Design of Electric Machines" (Isaac Pitman and Sons), shows how as many as six different speeds can be obtained.

Such a regrouping of coils must be done judiciously, for doubling the number of poles by using the same coils reconnected means that each coil will have twice its previous pitch, and the motor may require a different voltage for satisfactory operation.

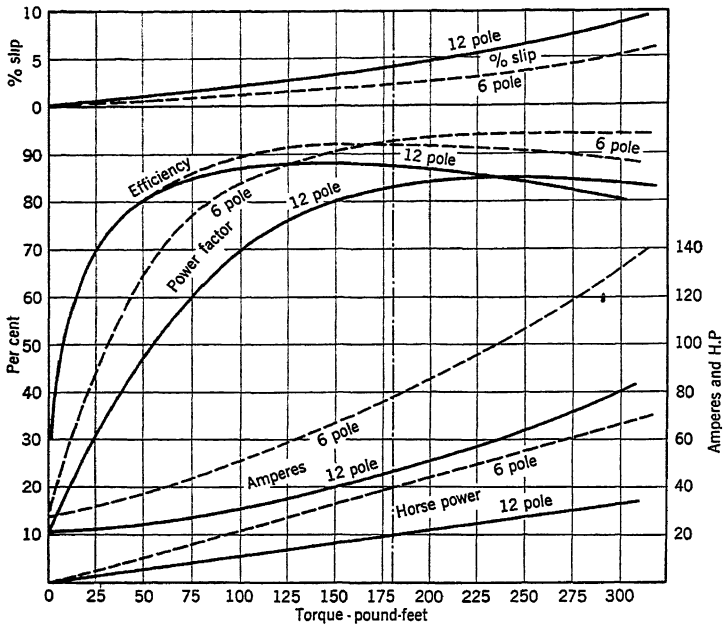


FIG. 196. Characteristics of a pole-changing induction motor. Three-phase, 60-cycle, constant-torque type, rated at 40/20 hp, 1160/570 rpm.

TABLE XI

6 POLES TAKEN AS NORMAL CONNECTION

Poles	6	8	10	12
Synchronous rpm	1200	900	720	600
Coil throw	1–7	1–7	1–7	1–7
Electrical degrees	120°	160°	200°	240°
Pitch factor	0.866	0.99	0.99	0.866
Voltage multiplier from speed change	1.00	0.75	0.60	0.50
Voltage multiplier from pitch	1.00	1.14	1.14	1.00
Operating voltage	440	375	300	220
Horsepower as percentage of 6-pole rating	100	86	68	50

Consider a motor with the following rating:

440 volts Three-phase 60 cycles 54 stator slots 54 coils

Double-layer, Y-connected winding Coil throw: slots 1 to 7

One effect of pole change is shown in Table XI.

242. Speed Variation by Resistance in the Rotor Circuit. In starting wound-rotor induction motors it is customary to connect extra resistance between the rotor slip rings to increase the starting torque and reduce the starting current. If this resistance (or part of it) is left in the circuit under running conditions, the speed will be reduced, the slip at a given torque increasing directly with the rotor resistance.

One equation for torque previously given, in synchronous watts per phase:

$$T = \left(\frac{sR_2V^2}{(R_1s + R_2)^2 + s^2(X_1 + X_2)^2}\right) \qquad [324]$$

By solving this equation for slip it will be found that the slip will vary directly as the rotor resistance, the impressed voltage (or mutual flux) and the internal torque being assumed constant.

The power output per phase of the rotor is

$$P_0 = I_2{}^2R_2\left(\frac{1 - s}{s}\right) \text{ watts} \qquad [325]$$

$$= \frac{I_2{}^2R_2}{s} - I_2{}^2R_2$$

If the copper losses are assumed to be the only losses occurring in the rotor, the power transferred across the air gap to each phase of the rotor will be

$$P_{\text{air gap}} = \frac{I_2{}^2R_2}{s} \qquad [326]$$

Now considering the rotor as a unit, the electrical rotor efficiency will be rotor output divided by rotor input, or:

$$\text{Electrical rotor efficiency as a decimal} = \frac{\dfrac{I_2{}^2R_2}{s} - I_2{}^2R_2}{\dfrac{I_2{}^2R_2}{s}} = 1 - s \qquad [327]$$

This shows that when the slip is increased, say, 25 per cent, by a proportionate increase in rotor resistance, the rotor efficiency will be

$$100 - 25 = 75 \text{ per cent}$$

The rotor behaves in this respect like a slipping friction clutch.

Inasmuch as friction and windage losses are always present, the total efficiency of the motor probably would be only 65 or 70 per cent, depending upon its size.

As the load is reduced on such a motor, the rotor current will decrease and the resistance will be comparatively less effective. This results in a drooping speed characteristic as shown in Fig. 197. Such a motor is said to have a "series" characteristic.

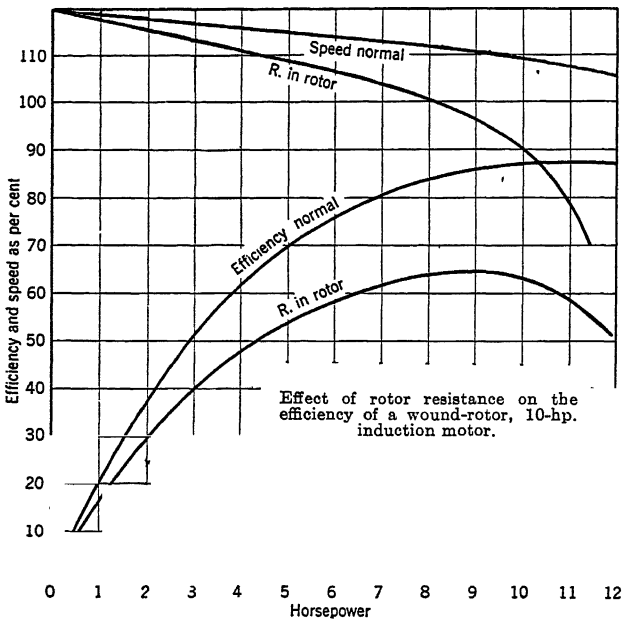


FIG. 197.

243. Summary. Speed reduction by additional resistance in the rotor circuit of the polyphase induction motor results in:

(*a*) Reduced rotor efficiency and consequently reduced motor efficiency.

(*b*) Poor speed regulation with drooping speed characteristic.

(*c*) A variation in the slip at which maximum torque occurs without change in the value of torque.

244. Speed Control by Electromotive Force in the Rotor Circuit. The fact, previously deduced mathematically, that the speed decreases when a resistance is added to the rotor circuit of an induction motor can also be explained on the basis of the emf induced in a phase winding of the rotor. If a motor load requires a constant torque, it is necessary that

the air-gap flux and the rotor current and its pf remain constant. The flux will be assumed constant with a constant applied emf. When extra resistance is added to the rotor circuit, it is necessary that I_2 remain at its previous value, and for it to do so the induced voltage must be increased. This induced voltage is E_2s, and it can increase only by an increase in slip since E_2 is fixed by the applied voltage and the transformation ratio. With the rotor current constant, the stator current will be constant and the power input to the motor will be constant. Horsepower output depends upon torque and speed. Torque is constant under the conditions assumed here, and so horsepower output will vary directly as the speed with no corresponding change in motor input. The result is that the extra resistance added to the rotor circuit must absorb the difference between electrical input and the mechanical output.

Speed control by rotor resistance is merely one example of a principle applied under various guises to achieve variable speed in induction motors. That principle can be outlined as follows:

Consider a wound-rotor induction motor with a constant output under all conditions (stator losses are neglected):

(*a*) With blocked rotor and an external resistance added to the rotor circuit all the input is absorbed by rotor I^2R loss (neglecting other losses) and the motor becomes a transformer. Rotor frequency is f and the voltage is E.

(*b*) At half speed (achieved by the correct value of R_2) one-half of the motor input is utilized in mechanical output at a fixed torque, and the other half in electrical output. Rotor frequency is $\frac{1}{2}f$ and the voltage is $\frac{1}{2}E$.

(*c*) At synchronous speed (assumed resistance of the rotor is zero) the entire motor input appears as mechanical power on the shaft with the same value of torque as in 244*b*. Rotor voltage is zero, and the frequency is zero.

(*d*) At $1\frac{1}{2}$ times synchronous speed, the motor input appears as mechanical power on the shaft. But the shaft power, due to $\frac{3}{2}$ speed, is one-half more than the input, and the extra power at constant torque must come from *power supplied* to the rotor terminals instead of absorbed by the rotor circuit. The power input to the rotor must come from an external source of voltage, $-\frac{1}{2}E$ and frequency $\frac{1}{2}f$.

From the above outline it will be seen that an induction motor can be made to operate at constant torque and at any speed above or below synchronism by the addition of positive or negative power to the rotor circuit at the correct slip frequency and voltage. A resistance in the rotor circuit can merely absorb power, and hence can be used for control below synchronism only.

By recalling the transformer type of diagram applied to the induction motor, it will be seen that the rotor current must be balanced by a stator component I_b, equal and opposite to I_2. Hence the primary pf can be affected by the phase position of the emf introduced into the rotor circuit. This fact is made use of for pf improvement of induction motors.

So far no consideration has been given the fact that the emf added to the rotor circuit must be at slip frequency. A number of methods have been developed whereby such emf's at the proper frequency can be supplied, and these systems have been utilized widely in variable-speed drive for steel mill motors. One such system will be discussed briefly here. The student is referred to the Bibliography for more detailed analyses.

245. Frequency Converter System. This method for constant-torque output at variable speed utilizes a frequency converter on the same

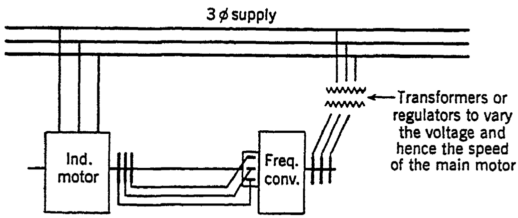


FIG. 198. Schematic layout of the frequency converter system for constant-torque speed control.

shaft as the main induction motor. The frequency converter is similar to a synchronous converter except that it has no winding on the stator. The rotor winding has the usual arrangement of commutator on one end and slip rings on the other. The brushes on the commutator are spaced to collect polyphase currents with three brush arms per pole pair for three-phase operation. As shown in Fig. 198, the slip rings are connected to the same source of supply as used for driving the main motor.

The action of the frequency converter can be explained as follows: Consider that its rotor is turning at synchronous speed with line frequency applied to its slip rings. The magnetic field built up by the rotor sweeps around the rotor periphery at synchronous speed. The rotor itself is turning at synchronous speed in the opposite direction, and hence the air-gap flux is stationary in space, giving the effect of a d-c field in a dynamo. The voltage collected from the commutator under this condition is unidirectional, with a definite relationship between its magnitude and that of the alternating voltage applied, as in the ordinary synchronous converter.

Next assume that the rotor is turning at three-fourths synchronous speed. The flux still sweeps past the rotor at synchronous speed, but since the rotor speed is $\frac{3}{4}s$, the rotation of the flux in space will be at $\frac{1}{4}s$, in a backward direction. Voltage collected at the commutator brushes will be at a frequency of $\frac{1}{4}f$. That is, on the frequency converter:

Frequency on slip rings		Frequency due to rotor speed		Frequency from commutator
	$-$		$=$	
$\frac{4}{4}f$		$\frac{3}{4}f$		$\frac{1}{4}f$

If the rotor of the main motor is to turn at $\frac{3}{4}s$, the frequency of the emf applied at its slip rings will be $\frac{1}{4}f$; and we can see that the necessary agreement exists between main motor and frequency converter speeds and frequency so that they can be interconnected electrically and mechanically.

To reduce the speed of this outfit it is necessary to vary the taps of the transformers shown in Fig. 198 so as to increase the voltage applied to the converter. This increases the voltage applied to the slip rings of the main motor and causes it to reduce in speed until its rotor induced voltage E_2s, minus the applied voltage from the converter, is sufficient to force the necessary torque current through its rotor circuit.

As these sets are usually applied to slow-speed operation and the frequency converter is directly connected, its size, weight, and cost are high.

246. The Brush-shift Motor. All the methods of speed control just described require auxiliary apparatus. Similar principles have been incorporated in a motor which, in a sense, employs its auxiliary equipment on the same frame and rotor. This is the Schrage [2] or brush-shift motor which is described below.

A schematic diagram of connections for this motor is shown in Fig. 199. The power is supplied to the rotor circuit through slip rings, and the stator winding is the secondary, each phase of which is independent. An additional winding similar to that of a d-c armature is placed in the same rotor slots and on top of the primary winding in order to reduce the commutation reactance voltage. Two sets of brushes are arranged around the commutator periphery on independent brush yokes, and the voltage collected from the brushes is introduced into the secondary circuit as shown.

[2] So called for its inventor, K. H. Schrage, of Sweden. As developed in the United States by the General Electric Co., it is known as the brush-shift, adjustable-speed motor, or by its trade designation BTA. See:

A. G. Conrad, F. Zweig, and J. G. Clarke, "Theory of the Brush-shifting A-C. Motor," *Elec. Eng.*, August, 1941.

The air-gap flux is set up by the primary (rotor) winding and is practically constant over the rated load range due to the constancy of the applied voltage and frequency. At synchronous speed, with the rotor revolving forward in one direction, the mmf built up by its windings will revolve at synchronous speed in the opposite direction and will be stationary in space. With the rotor running below synchronous speed the

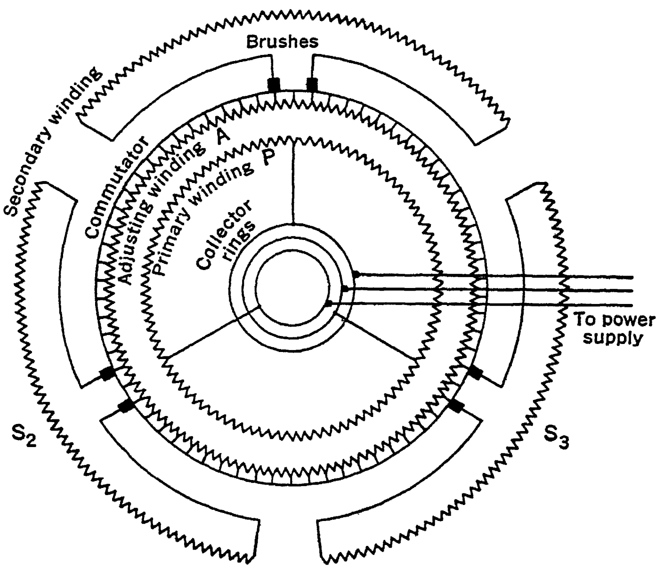


FIG. 199.

air-gap flux will revolve backwards in space and will determine the frequency of the emf induced between the brushes bearing on the commutator, to which is connected the second rotor winding (called the *adjusting winding*). Since the frequency of the voltage built up in the adjusting winding is proportional to the slip, it is suitable for connection to the secondary windings which also carry current at slip frequency.

The voltage between two brushes connected to the end of a secondary winding depends upon the number of segments between them. Because the air-gap flux is practically constant, the voltage built up per conductor of the adjusting winding is constant. This voltage will obviously be zero when the brushes are both on the same commutator segment or are on segments of the same potential. Separating the brushes a like amount for each phase until they are 180 electrical degrees apart on the commutator will give the maximum counter emf in each secondary

winding. Movement of the brushes, relative to each other, will then include more or less of the adjusting winding in the secondary at slip frequency and will cause a change in the speed. The combined effect of the secondary induced voltage and the added voltage of the adjusting winding permits just enough current to flow in the secondary to give the correct torque demanded by the load and losses. That is:

$$\frac{E_2s - E_{(\text{adjusting winding})}}{Z_2} = I_2 \qquad [328]$$

If two brushes are brought together on the same commutator segment and are then shifted past each other in the opposite direction, the phase position of E_{aw} will be reversed, and large currents will flow in the secondary. These will accelerate the motor above synchronism. Stable speed will be reached when the sum of E_2s and E_{aw} results in sufficient torque current to handle the load and losses at the new speed. The frequency of emf's in the secondary and the adjusting winding will again be equal, and the action agrees with those principles outlined in Article 244, in which operation can occur above synchronous speed when power is added to the secondary circuit.

247. Power-factor Improvement. Suppose that the two brushes connected to the ends of one secondary winding are separated by, say, six commutator segments. Then movement of this pair of brushes relative to their position in space (with simultaneous movement of the other brushes) will not change the voltage if the brushes are still separated by six segments. The phase position of the voltage will be changed, however, as will also the phase position of the effective secondary voltage. Since the secondary current lags a definite angle behind the effective secondary voltage the phase position of the current will change. Considering the ordinary transformer diagram of the induction motor it will readily be observed that any change in the position of I_2 will change the position of the balancing primary current component and will result in a new primary pf angle.

248. Characteristics. The brush-shift motor differs from the wound-rotor induction motor, in which speed reduction is brought about through added secondary resistance, in that it displays practically constant-speed characteristic with varying load. Since speed control by added rotor resistance gives a "series characteristic," this motor has been distinguished by its "shunt characteristic." Furthermore a 50 per cent reduction in speed results in only a slight reduction in efficiency. In general the efficiency is lower than that of the squirrel-cage induction motor, but higher than the wound-rotor type, with external resistance used for speed control.

The pf at high speed and full load is from 0.95 to unity; at low speed, full load, from 0.65 to 0.70. At a slip of 50 per cent, either positive or negative, the slip energy will then be 50 per cent of the power developed. If the adjusting winding is built with a capacity of 50 per cent of the stator winding capacity, a speed range of 3 to 1, 50 to 150 per cent of synchronous rpm, will then be possible.

The starting torque of this motor is 150 to 250 per cent of full-load torque, with rated voltage applied and the brushes spread to their low-speed position. Under these conditions, the starting current is 125 to 175 per cent of full-load, full-speed line current.

Typical name-plate data on a small motor of this type are as follows:

220 Volts	Three-phase	60 Cycles	
Horsepower, 5	Amperes, 15.5	Speed, 1650	40 C rise
Horsepower, 2.5	Amperes, 12.0	Speed, 825	40 C rise
Horsepower, 1.67	Amperes, 11.0	Speed, 550	50 C rise

249. Concatenation. If two wound-rotor induction motors are arranged so that the stator of one is connected to an external source and its rotor output is connected to the stator of the other, the second motor will operate at the slip frequency and voltage of the first. The first machine acts, in a sense, as a frequency converter, changing part of its energy into mechanical output and part, through the slip rings, into

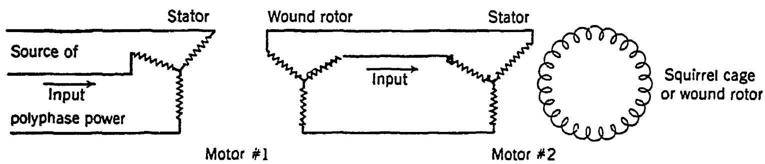


Fig. 200. Motors in concatenation.

electrical energy to the second motor. The second motor could be of the squirrel-cage type or with wound rotor and external resistance. The slip frequency of the second motor is the difference between its applied frequency (which is the slip frequency of motor 1) and its frequency of rotation.

If the two motors have the same number of poles and are mechanically coupled, they will operate at practically half speed. Let the subscripts 1 and 2 refer to the respective motors.

The impressed frequency on the first motor is f_1.

The impressed frequency on the second motor is f_2.

$$f_2 = f_1 s_1 \tag{329}$$

The synchronous speed of motor 2 is

$$\text{rpm} = \frac{120 f_2}{p_2} \tag{330}$$

$$= \frac{120 f_1 s_1}{p_2} \tag{331}$$

Its speed at any slip is

$$\text{rpm} = \frac{120 f_1 s_1}{p_2} (1 - s_2) \tag{332}$$

The synchronous speed of motor 1 is

$$\text{rpm} = \frac{120 f_1}{p_1} \tag{333}$$

Its speed at any slip is

$$\text{rpm} = \frac{120 f_1}{p_1} (1 - s_1) \tag{334}$$

With the motor shafts coupled, the speeds must be equal, hence:

$$\frac{120 f_1}{p_1} (1 - s_1) = \frac{120 f_1 s_1}{p_2} (1 - s_2) \tag{335}$$

and

$$s_1 = \frac{p_2}{p_1 + p_2 - s_2 p_1} \tag{336}$$

If the rotor of the second motor is a squirrel cage or is operating with short-circuited slip rings so that the slip is small, s_2 may be neglected for the approximate relations:

$$s_1 \approx \frac{p_2}{p_1 + p_2} \tag{337}$$

then

$$\text{rpm} \approx \frac{120 f}{p_1 + p_2} \tag{338}$$

To sum up: The speeds obtained in a concatenated system are naturally a function of the frequency and the numbers of poles.

If the two motors have an equal number of poles, the speeds available are the rated speed, operating the two in parallel, and the half speed when operating in cascade.

If a different number of poles is used on each motor, three speeds are available (*a*) by using motor 1 alone, (*b*) by using motor 2 alone, or (*c*) by using both in cascade. Only in the last case is the full torque of the setup available.

The performance of a pair of concatenated induction motors at any speed may be calculated by the equivalent-circuit method; i.e., the apparent impedance of the second motor at its slip s_2 is added to the secondary impedance of the first motor at its slip s_1, and the network calculation repeated for the first motor.

For a description and an analysis of a type of motor used for multispeed work by means of "internal" concatenation, the reader is referred to Creedy's work, previously cited.

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CHAPTER XXX

THE ASYNCHRONOUS GENERATOR

250. Chapter Outline.

The Asynchronous Generator.
  Theory of Operation.
    The Vector Diagram.
    Characteristics.
      Obtained from Load Run.
      Obtained from Circle Diagram.

251. Vector Diagram. When the terminals of an induction motor are connected to the proper supply lines, the current drawn from the source is conveniently divided up into two components: one to supply the necessary magnetization, and the second or load component which varies with the load. If such a motor is mechanically connected to a prime mover, operating so that the motor runs at synchronous speed, the motor continues to draw its exciting current from the lines; the rotor current is zero and no balancing component is necessary in the stator for any demagnetizing effect of the rotor.

If the prime mover is speeded up so that the motor runs above synchronism, i.e., with a *negative slip*, the motor becomes a generator and forces electric power back into its connecting lines. This power varies with the speed above synchronism.

Consider the case of the flux built up in the air gap of an induction motor rotating in a given direction and with the rotor revolving at a subsynchronous speed. If the rotor speed is then raised above synchronism, the rotation of the air-gap flux will not change in direction, but the relative direction in which the rotor conductors are cutting the flux is reversed. Consequently the emf's and currents built up in the rotor are reversed from their usual directions in motor action.

Except for the magnetizing ampere turns, the ampere turns of the stator must balance those of the rotor; hence, with negative slip, the balancing component of the stator windings is reversed with reference to its position during normal motor action. Such a reversal in current direction represents an electric power output rather than input, and explains the generator action.

252. Excitation and Power Factor. The air-gap flux is assumed to remain constant with constant voltage and to be unchanged by the variation in the speed of the rotor. Hence, when running as a generator, the machine must be supplied with its usual magnetizing current I_ϕ. The reversal of the voltages due to generator action makes this magnetizing current *lead* the output voltage. This current cannot come from the induction generator itself and must come from a synchronous generator. Hence the induction generator must always be operated in parallel with an ordinary alternator to supply a common load; the removal

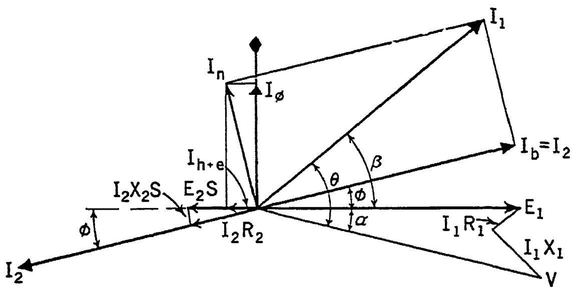


FIG. 201.

of the alternator from the line reduces the excitation of the induction machine to zero and its generator action ceases.

Figure 201 shows the vector diagram for an induction generator. The pf angle between the induced voltage and current is fixed by the constants of the rotor and by the slip.

$$\cos \phi = \frac{R_2}{\sqrt{R_2^2 + X_2^2 s^2}} \tag{339}$$

The angle β is fixed by the relative values of the no-load current and load component. The angle α is fixed by the primary impedance drops. The resulting pf angle θ between the line current and the terminal voltage is thus fixed by the machine constants rather than the load. It will be noted that the generator can supply a load with leading current only; its pf will vary slightly with the load, but the current will always lead. If it is connected to a load which requires a lagging current, it is necessary that the synchronous machine, connected in parallel with the induction generator, operate at a low-lagging pf to neutralize the induction generator lead.

Another explanation is as follows: The exciting current leads the voltage of the induction generator, but lags the voltage of the synchronous generator on account of the phase relationships existing between the two voltages with reference to their connecting leads. Hence, with the

lagging exciting current coming from the synchronous machine, the pf at which that machine operates is reduced.

This fact militates against the use of induction generators for loads which are already of low-lagging pf.

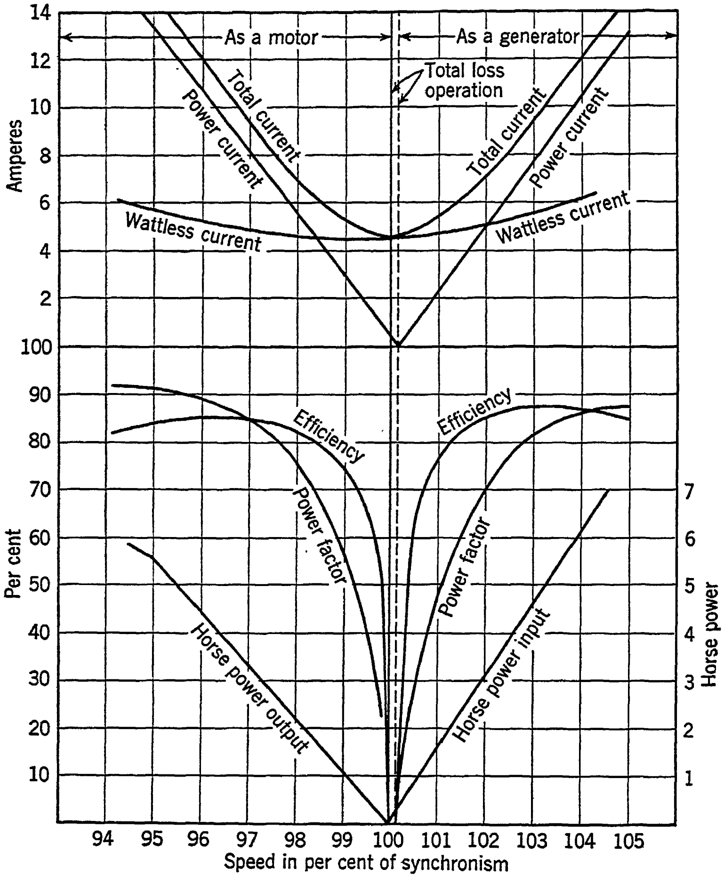


Fig. 202. Characteristics of a 5-hp, 220-volt, three-phase, 60-cycle, 4-pole induction motor, as motor and generator.

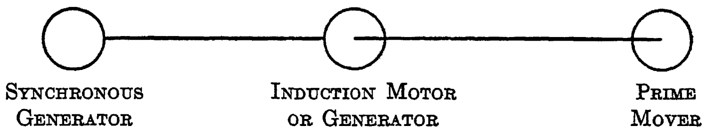
253. Frequency and Load. In dealing with the induction motor we found that the rotor frequency, regardless of its value as determined by the slip, reacted against the stator at stator frequency. In the same manner, a change in the speed or slip of the induction generator makes no change in the frequency of its output, which is fixed by the frequency of the magnetizing current coming from the synchronous apparatus connected in parallel. A change in slip, however, does change the output.

Increased speeds above synchronism increase the rate at which the rotor conductors cut the air-gap flux and hence increase the rotor induced voltage and current. Any increase in rotor current requires a greater balancing component I_b in the stator. This balancing component is the output current and represents the value put out to the load. The power output of the induction generator is then controlled by its slip. Usually 3 to 5 per cent negative slip is necessary to make the generator deliver rated kilovolt-amperes.

Running above synchronism, the generator supplies its own core loss. At small values of slip, the entire generator action may be necessary to supply these losses; increased negative slip then results in useful output.

254. Summary. The transition from induction motor to induction generator action can be summarized, as in Table XII.

TABLE XII



Slip	Mechanical Losses from	Core Losses from	Magnetization Current from	Copper Losses	Action
Positive	Generator	Synchronous generator	Synchronous generator	From generator	Motor
Zero	Prime mover	Synchronous generator	Synchronous generator	Practically zero	
Small negative	Prime mover	Induction generator and prime mover	Synchronous generator	Practically zero	Generator zero output
Normal negative	Prime mover	Induction generator and prime mover	Synchronous generator	From induction generator	Generator

The frequency of the induction generator is determined by that of the synchronous generator in parallel with it.

The pf is determined by the machine constants and not entirely by the connected load.

The voltage is determined by the synchronous generator; the reduction of this voltage by short circuit on the connecting leads between the two machines makes all generating action cease in the induction generator. Its field in case of short circuit does not collapse instantly, and it is capable of feeding large currents for a short time.

The output is determined by the speed; as stated before, a small negative slip of about 5 per cent will cause the induction generator to give rated output.

255. Characteristics from Load Run. The characteristics of an induction generator can be obtained by loading it down in parallel with a synchronous generator and by reading watts, amperes, and volts with various values of slip. The procedure is identical with that for load tests on the induction motor, as shown in Article 214, and will be illustrated by the calculation of one point on the characteristic curves.

Induction generator: 220 volts, three-phase, 60 cycles. Synchronous speed, 1800 rpm; resistance of the stator per phase at 75 C equals 0.545 ohm.

Observed data, as a generator, at one value of slip:

$$V = 220 \qquad W \text{ (output)} = 3910$$

$$I = 12 \text{ amperes per line} \qquad \text{Slip} = 72 \text{ rpm}$$

Constant losses from a no-load run as a motor equal 213 watts (see Article 211), of which 65 watts represent friction and windage.

Calculations:

$$\cos\theta = \frac{W}{\sqrt{3}VI} \quad \text{or} \quad 0.857 \qquad [340]$$

Stator copper loss:

$$3I^2R_1 = 3 \times 12^2 \times 0.545 \quad \text{or} \quad 235.5 \text{ watts}$$

By the American Standards for Rotating Machinery:

$$\text{Rotor } I^2R \text{ loss} = s \text{ (measured output + stator } I^2R \text{ loss + core loss)}$$

$$\text{Slip} = \frac{-72}{1800} \times 100 \quad \text{or} \quad -4.0 \text{ per cent}$$

$$\text{Rotor } I^2R \text{ loss} = 0.04\,[3910 + 235.5 + (213 - 65)] \qquad [341]$$

$$= 171.7 \text{ watts}$$

The sum of output and losses is then

$$3910 + 235.5 + 213 + 171.7 = 4530.2 \text{ watts, input}$$

The efficiency is then

$$\frac{3910}{4530.2} \times 100 = 86.4 \text{ per cent}$$

Similar calculations are made with other values of output so that the characteristic curves as shown in Fig. 202 can be plotted. The curves as a motor, shown on the same axes, were plotted from a similar load run as described in Article 214. When the "constants" are known, the performance at any given slip $(-s)$ may be found by solving the network in a manner similar to that previously described for the motor.

256. Applications. The induction generator has had a relatively limited application. Some use has been made of it in comparatively isolated plants operating from hydraulic turbines,[1] and paralleled with a network supplied also by synchronous generators. One advantage of this type of generator, frequently quoted, is the lessened short-circuit risk; when a short circuit occurs on the terminals of an induction generator, its excitation is removed and generator action ceases. Tests and analyses have shown,[2] however, that the initial short-circuit current of such a generator has exactly the same magnitude as that of synchronous machines with the same constants. The initial short-circuit current is limited only by the leakage reactance, but it rapidly decays to zero. That is, there is no sustained short-circuit current.

Several railway electrifications, employing induction-motor drives in the locomotive, make use of generator action for regenerative braking.

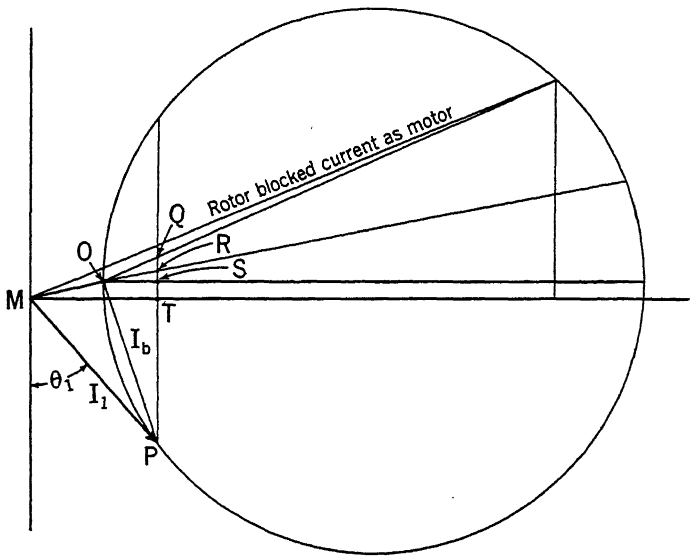


FIG. 203.

257. The Circle Diagram. The approximate circle diagram explained in Chapter XXV can be applied with fair accuracy to the determination of induction-generator characteristics by completing the circle of current

[1] J. G. Tarboux, "Electric Power Equipment," First Edition, p. 97, McGraw-Hill Book Co.

C. P. Steinmetz, *Trans. A.I.E.E.*, Vol. 37, p. 985, 1918.

[2] R. E. Doherty and E. T. Williamson, *Trans. A.I.E.E.*, Vol. 40, p. 509, 1921.

locus. The lower semicircle represents generator action. Such a diagram is shown in Fig. 203, plotted in values per phase.

OM = the no-load current in amperes
MP = the stator current in amperes
OP = the rotor current in stator terms
QP = the input in watts, to the watt scale
ST = friction, windage, and core loss to the watt scale
RS = stator copper loss to the watt scale
QR = the rotor copper loss to the watt scale
TP = useful output in watts
$\cos \theta_1$ = stator pf
TP/QP = efficiency
QR/RP = slip

SINGLE-PHASE INDUCTION MOTORS

CHAPTER XXXI

CONSTRUCTION. STARTING METHODS AND CHARACTERISTICS

258. Chapter Outline.

The Single-phase Induction Motor.
- Two Theories of Single-phase Motor Action.
- Starting Methods.
- Construction.
- Characteristics.
  - Shaded-pole.
  - Split-phase.
  - Capacitor-start.
  - Repulsion-start.

259. Introduction. The single-phase induction motor displays less satisfactory operating characteristics than the polyphase machine, but it has achieved, nevertheless, a wide field of usefulness on applications where a polyphase supply is not available. Advances in design have made the single-phase motor quite satisfactory in its smaller sizes so that millions of them are in use in the commercial and domestic field.[1] The chief disadvantages of the single-phase induction motor, in its simplest form, are its lack of starting torque and its reduced pf and

[1] Single-phase induction motors of various types are covered in some detail in this text for several reasons. They present a rather complex but interesting collection of theoretical analyses; they are of great commercial and industrial importance. On the latter point the commercial activities in various fields is very illuminating. The Department of Commerce Census of Manufacturers for 1937 shows that for the year in question sales were as follows:

Fractional-horsepower (under $\frac{1}{20}$) 11,665,867 motors, valued at $9,639,472.
Fractional-horsepower ($\frac{1}{20}$ to 1), split-phase, capacitor, and repulsion-induction 5,039,662 motors, valued approximately at $48,425,000.
Integral-horsepower (over 200 hp) polyphase induction, 889 motors, valued at $2,941,722.
Integral-horsepower (over 200 hp) synchronous, 919 motors, valued at $3,307,087.

efficiency. Scores of devices have been developed for starting methods. Many of these are commercially undesirable and are not used at present.

When a single-phase stator winding is connected to a source of alternating current, the resultant field built up alternates along one axis alone. Such a pulsating field cannot produce rotation directly. Once the rotor is turning, however, there is produced a cross-flux which is in both space and time quadrature with the main axis flux. These are the two conditions necessary to produce a rotating field. Hence the single-phase motor will continue in rotation as long as the load torque is not excessive. The above concepts are included in what is known as the *cross-field theory* for explaining single-phase motor action.

A different explanation which leads to the same results is based on what is known as the *double-revolving field theory.* This makes use of the idea that any alternating uniaxial quantity can be represented by two oppositely rotating vectors of half magnitude. That is, a uniaxial flux with sinusoidal variation, expressed as $\phi = \phi_m \cos 2\pi ft$, is equivalent to two fluxes rotating in opposite directions, each with a magnitude of $\frac{1}{2}\phi_m$ and an angular velocity of $2\pi f$.[2]

When the rotor turns, the torque produced by the forward flux is greater than that of the backward flux even if the fluxes are equal. But actually the rotation of the rotor increases the forward flux above, and reduces the backward flux below, the half value. Hence the action is not too greatly inferior to that of the polyphase motor.

It is unfortunate that, for a motor of fairly simple construction and operation, both the theories needed to explain its action should be as complex as these are in their theoretically exact form. Both will be given in the chapters which follow.

[2] Note that this is the same idea used in considering armature reaction in single-phase alternators.

Euler's expression for the cosine of an angle is significant in expressing the decomposition of the pulsating field. His equation is, in general terms,

$$\cos \theta = \frac{\epsilon^{j\theta} + \epsilon^{-j\theta}}{2}$$

The term $\epsilon^{j\theta}$ indicates a vector rotated counterclockwise, through an angle θ; $\epsilon^{-j\theta}$ represents such a rotation clockwise. Then a flux represented by the term $\phi = \phi_m \cos 2\pi ft$ can be expressed by Euler's equation:

$$\phi_m \cos 2\pi ft = \frac{\phi_m}{2} (\epsilon^{j2\pi ft} + \epsilon^{-j2\pi ft})$$

The right-hand member represents two oppositely rotating vectors of half magnitude.

260. Common Starting Arrangements. Because the single-phase induction motor has no inherent starting torque, various methods have been developed for starting. Only four of these methods are at present of industrial importance. They will be described briefly.

Shaded Pole. This represents the smallest and simplest type of induction motor. It is built with salient poles on the field and concentrated windings. Around one portion of each pole is wrapped a copper strap, forming a closed circuit. This "shading coil" acts to delay the flux passing through it in time, so that the flux lags in phase behind that in the unshaded part. The combined action gives a sweeping action, magnetically, across the face of the pole, resulting in more or less of a revolving flux. This supplies the starting torque. The shading coil influences the running characteristics also. A schematic arrangement is shown in Fig. 204. No adequate methods of analysis are available, but efforts have been made by Kron, Trickey, and the present writers.[3]

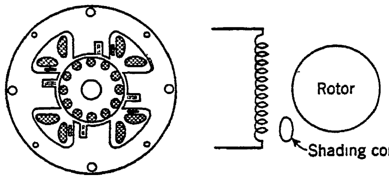


FIG. 204. Representations of the shaded-pole motor.

The lack of any centrifugal switch and the use of concentrated windings reduces the cost of such a design. Starting torque is always comparatively small and limits the field of application of these motors. On ratings of about 1/100 hp the starting torque in ounce-feet per ampere of starting current is about 0.15; this may increase to about 0.4 for 1/20 hp ratings. These are for 2- and 4-pole motors. Six-pole ratings may run as high as 0.65 ounce-feet per ampere of starting current. These ratios are all figured on 110 volts applied. Usual horsepower ratings are from 1/200 to 1/20. Reversal of rotation can be obtained only by the use of shading coils on the opposite edges of the poles, with the means for opening and closing each separately.

Split Phase. From the standpoint of starting torque and commercial sizes, the split-phase motor stands second in the order of motor sizes.

Figure 206 shows a parallel circuit in which one branch (M) has a relatively high reactance and low resistance. This represents the main or running winding of the motor. A second stator winding, displaced in space by 90 electrical degrees, is represented on this circuit by a branch (S), having a higher resistance and a lower reactance.

Such motors are designed so that I_S and I_M are about 30° out of phase in time, as shown in the vector diagram of Fig. 206*b*. Considering that these two vectors represent the respective stator winding currents of a split-phase induction motor, it can be seen that the single-phase supply results in currents and fluxes displaced in space and time, and so yield a

[3] P. H. Trickey, "An Analysis of the Shaded-pole Motor," *Elec. Eng.*, September, 1936.

A. F. Puchstein and T. C. Lloyd, "Capacitor Motors with Windings Not in Quadrature," *Elect. Eng.*, November, 1935.

motor torque. The arrangement forms an imperfect two-phase motor, which, because of the difference in current magnitudes and in unlike circuit constants, results in a rotating field which is not uniform in either time or space, but is sufficient for starting the motor.

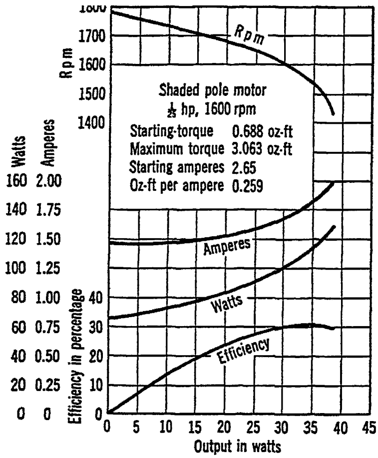


FIG. 205. Typical performance curves.

Continued operation on both windings results in rapid overheating and inefficient, noisy performance. The usual design calls for a centrifugal switch on the rotor, in series with and automatically disconnecting the starting winding at about 70 per cent of synchronous speed. Excessive overload of the motor may reduce its speed below the closing value of the centrifugal switch, reconnecting it so that the motor operates

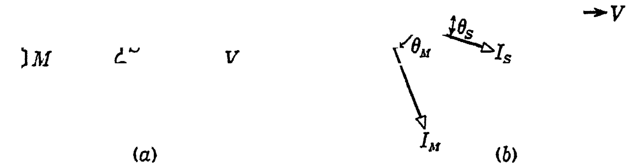


FIG. 206. The split-phase motor.

with the start winding on. The added torque usually speeds the motor slightly, giving a "seesaw" action with the switch operating on and off. Start winding burnouts usually result.

Starting torque in ounce-feet per ampere is considerably higher than in the shaded-pole motor. Values as high as 0.8 to 0.9 are not uncommon

(on a 110-volt basis). Commercial sizes are most commonly from about 1/30 to 1/3 hp in 60-cycle speeds of 3450, 1725, 1140, or 840 rpm.

The starting torque in ounce-feet can be calculated:[4]

$$T \approx \frac{225.4}{\text{synchronous rpm}} I_M I_S \frac{r_2}{a} \sin \theta_t \qquad [342]$$

$$a = \frac{\text{effective conductors of main winding}}{\text{effective conductors of start winding}} \qquad [343]$$

$$a \approx \sqrt{\frac{X_M}{X_S}} \qquad [344]$$

X_M = total leakage reactance, main winding terms
X_S = total leakage reactance, start winding terms
θ_t = the time-phase angle between main and start winding currents
r_2 = the resistance of the rotor in main winding terms

Capacitor Motors. The past ten years have seen the rapid development and widespread use of a new type of single-phase induction motor known as the capacitor or condenser start motor. The method was first proposed by Steinmetz. Although the principle on which the motor operates was understood about forty years ago, the motor had to await the development of a cheap, sturdy condenser of reduced bulk, which was required as an auxiliary device,[5] before it could be made commercially feasible.

The chief advantages of this type of motor lie in its unusually high starting torque (without excessive starting current), the versatility of its application, and its mechanical simplicity.

The motor is very similar to the ordinary split-phase type, having a squirrel-cage rotor winding and two stator windings displaced 90 electrical degrees in space. One stator winding, the main winding, is fixed by the running performance required. The second auxiliary winding, with a capacitor in series, is designed to result in a displacement in time between the current of the two windings. As shown in Fig. 207, the condenser enables the auxiliary winding current to lead the main winding current by a much more favorable angle than could be obtained by the use of the resistance and leakage reactance of the windings alone. This auxiliary winding is opened by a centrifugal switch at about 70 per cent of the final speed.

[4] A. F. Puchstein and T. C. Lloyd, "Starting Torque of Split-phase Motors," *Product Eng.*, June, 1937 and February, 1938.

[5] T. C. Lloyd, "Capacitator Motors," *Product Eng.*, November and December, 1939.

The total starting current is the vector sum of both I_S and I_M. Note in Figs. 206 and 207 that, for the same respective values, the vector sum of the currents of the capacitator motor will be less than for the ordinary

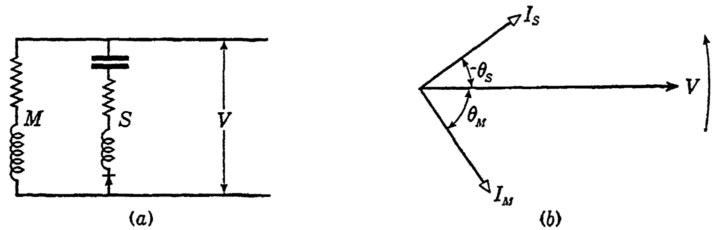


FIG. 207. The split-phase motor of the condenser or capacitor type.

split-phase machine. Note, too, in the starting torque formula that the torque is a function of the sine of the angle between I_S and I_M. This may approach unity with the capacitor motor, thereby resulting in large values of starting torque, with lower starting current. As can be expected, the ounce-feet per ampere of starting current are relatively high,

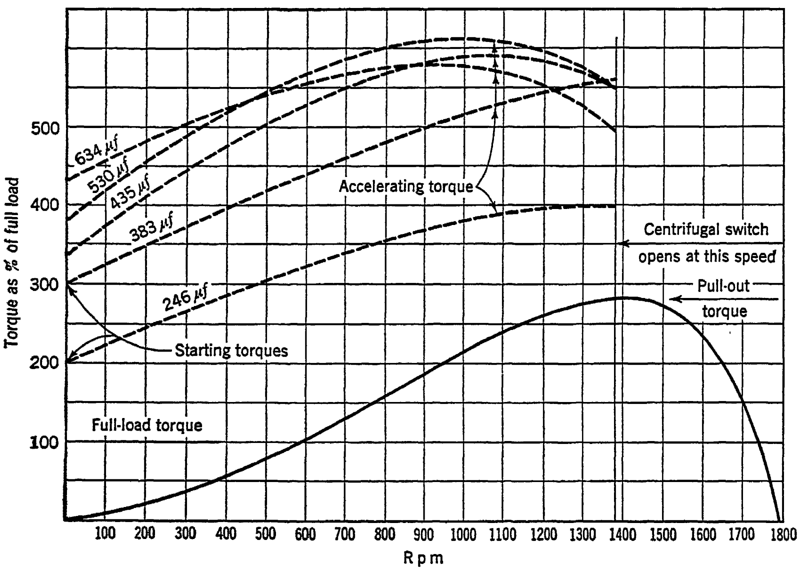


FIG. 208. Speed-torque curves for a ¾ hp, 110-volt, 60-cycle, 4-pole, capacitor-start, induction-run motor. The effect of various condenser capacities on starting and "pull-up" torque is indicated.

about 3 to 3.5 on a 110-volt basis. Such values are much better than can be obtained from split-phase motors although they are below repulsion-start induction-motor starting performance.

The formula for starting torque is exactly the same as that given for the split-phase motor. In actual calculation, the current in the start winding involves the reactance of the capacitor as well as the impedance of the winding. This enters also into the calculation of the sine of the time angle between I_S and I_M.

Usual ratings are $\frac{1}{8}$ to 1 hp although many manufacturers build this type of motor to 10-hp outputs.[6]

261. Capacitors for Starting Duty. Delay in the development of the capacitor-start motor was occasioned by a fact proved by all early experimental work in this field and borne out many times since: To obtain sufficient starting torque, large condensers of high microfarad capacitance, and hence high cost, were required. The ordinary type of foil-paper, oil-immersed condensers available were several times as large as the motor and were entirely too expensive for practical use. Their lifetime and reliability were uncertain. The development of the electrolytic condenser changed this picture completely, although improvements in the oil-insulated type have also been of commercial importance.

By continued research in the electrolytic field, condenser manufacturers now put 80 μf of this type in a container which would formerly hold only 4. However, electrolytic condensers have distinctive properties of their own, which require that they be applied intelligently. The chief drawback is their lack of continuous service rating. This point must be stressed. An electrolytic type of condenser used for starting duty is usually guaranteed for not more than 20 periods of operation per hour. Each period should not continue for more than 3 sec. For many applications, as a starting device, this limitation is acceptable. Furthermore, like every other type, the electrolytic condenser has a maximum permissible voltage rating. In this case the voltage depends upon the puncture voltage of the oxide film on the aluminum sheets which make up the active electrodes. If the condensers are exposed to voltages slightly higher, for short periods only, the damage may be self-healing. Otherwise, too much excess voltage ruins the condenser.

Guarantees are usually for 25 per cent overvoltage for commercial capacitors that are built for 110 volts, and for 10 per cent overvoltage for commercial capacitors rated at higher voltages.

Several years ago, manufacturers changed their rating methods from a

[6] For a formula for starting torque, see the following papers and Mr. Boothby's discussion:

Benjamin F. Bailey, "The Condenser Motor," *Trans. A.I.E.E.*, Vol. 48, April, 1929.

H. C. Specht, "The Fundamental Theory of the Capacitor Motor," *Trans. A.I.E.E.*, Vol. 48, April, 1929.

nominal value, with limits, to a double rating. Thus, instead of rating, say, 80 μf, the standard is 72–87 μf.

To give some idea of the usual capacitance of condensers required for general purpose, capacitor-start motors, the sizes, ratings, and dimensions are shown in Table XIII. Shown in the last column of this table

TABLE XIII

CAPACITOR START MOTORS

(Electrolytic Condensers)

Hp	Rpm	Typical Condenser Capacity*	Typical Dimensions of Round Can	Approximate Dimensions of Oil Condenser
		μf	Inches	Inches
$\frac{1}{8}$	3450 1725 1140	72–87	$1\frac{3}{8}$ diam $2\frac{3}{4}$ long	5 x 5 x 5
$\frac{1}{6}$	3450 1725 1140	88–106	$1\frac{3}{8}$ diam $3\frac{1}{4}$ long	5 x 5 x 6
$\frac{1}{4}$	3450 1725 1140	124–149	$1\frac{3}{8}$ diam $3\frac{1}{4}$ long	5 x 5 x 8
$\frac{1}{3}$	3450 1725 1140	158–191	$1\frac{3}{8}$ diam $4\frac{1}{4}$ long	5 x 5 x 11
$\frac{1}{2}$	3450 1725 1140	233–281	2 diam $3\frac{1}{8}$ long	5 x 5 x 16
$\frac{3}{4}$	3450 1725	341–412	2 diam $4\frac{1}{8}$ long	5 x 5 x 22
1	3450	341–412	2 diam $4\frac{1}{8}$ long	5 x 5 x 22

* As the condenser influences the starting torque it is important to note that for the above values, starting torque as percentage of full load torque would be about as follows:

3450 rpm	350 to 400%
1725 rpm	400 to 475%
1140 rpm	285 to 390%

are the approximate dimensions of the condensers if the oil-insulated type has to be used. Comparative bulk strikingly illustrates the difference between the two types and the importance of the electrolytic condenser developments.

262. Repulsion-start, Induction-run Motors. This type of motor has long been the largest single-phase induction motor in ratings and starting

torques. It has been manufactured in large quantities, but is gradually being replaced by the capacitor-start motor. Theoretically, the starting torque per ampere is higher on this type than on any other, being 5 to 7 oz-ft per amp at 110 volts.

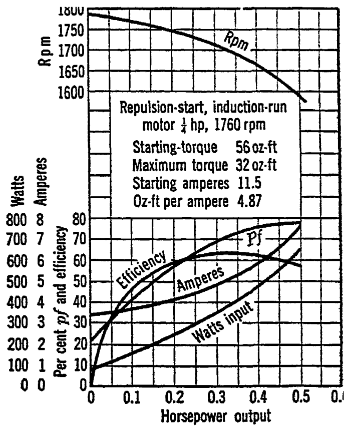


FIG. 209. Typical performance characteristics of a repulsion-start, induction-run motor.

No detailed explanation of the theory will be given here, for it will be covered in the section Repulsion Motors.

The construction and operation of this type of motor are as follows: The armature of the repulsion-start motor is similar to that of the d-c machine, except that the brushes are short-circuited. A centrifugal mechanism causes the commutator bars to be short-circuited about two-thirds of final speed, after which the armature acts like an ordinary squirrel-cage rotor. The same mechanism may lift the brushes to reduce

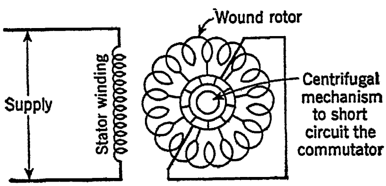


FIG. 210. Schematic arrangement of repulstion-start, induction-run motor.

wear and noise during operation. The cheaper alternate design, called the brush-riding type, does not lift the brushes when final speed is reached.

Reversal of this type of motor is accomplished by a shifting of the brushes. An alternative method involves supplying two identical stator

windings, so displaced from each other that the use of one or the other is equivalent to a brush shift.

Usual ratings of repulsion-start, induction-run motors are $\frac{1}{8}$ to 10 hp. A few larger ratings to about 25 hp are occasionally demanded.

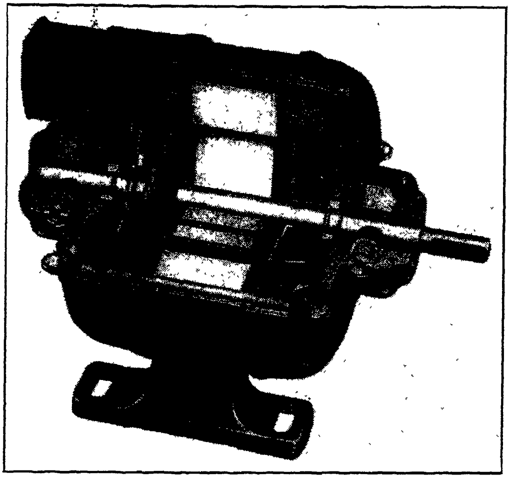


FIG. 211. Cut-away view of a split-phase motor, showing the main winding, the start winding (of smaller wire size), and one type of centrifugal switch. (*Robbins & Myers, Inc.*)

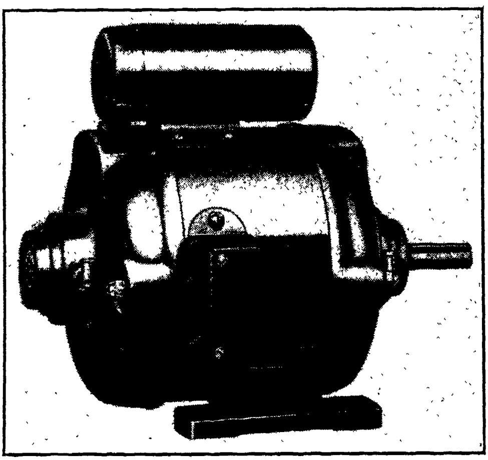


FIG. 212. Capacitor motor. This may be capacitor-start, induction-run, or capacitor start and run. Both electrolytic and oil-type condensers are available in cans as shown. (*Robbins & Myers, Inc.*)

263. Standards. Fractional horsepower motors are not standardized on frame sizes as are the "integral" ratings. Standards have been established for minimum performance values, shaft dimensions, and ratings,

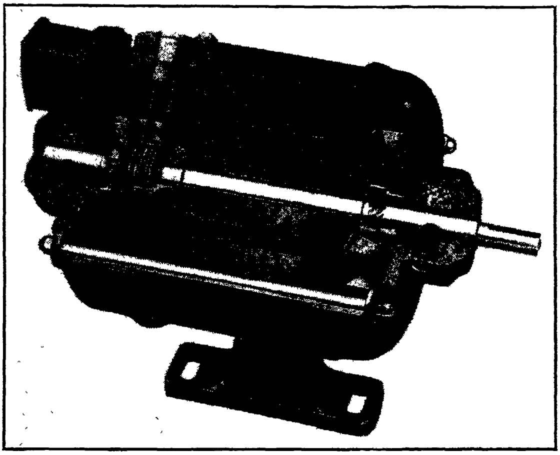


FIG. 213. Cut-away view of a repulsion-start, induction-run motor. Note the wound armature and the commutator. At about two-thirds of rated speed the centrifugal mechanism pushes a copper disk against the end of the commutator, effectively short-circuiting the winding. In this type the brushes are not lifted but continue to bear on the commutator surface. (*Robbins & Myers, Inc.*)

but there are no standard N.E.M.A. frames below size 203 (approximately 9.5-in. diameter).

Standard voltages are 115 and 230 volts.

Standard ratings are $\frac{1}{20}$, $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$ hp.

Rated locked rotor-torque, pull-up torque and breakdown torque of 60-cycle, single-phase fractional horsepower motors, expressed as a percentage of full-load torque, shall not be less than the values of Tables XIV, XV, and XVI which follow:

TABLE XIV

Hp Rating	Locked Rotor	Pull-up	Breakdown
		60 cycles	1725 rpm
$\frac{1}{8}$	150	150	200
$\frac{1}{6}$	150	150	200
$\frac{1}{4}$	90	90	185
			1140 rpm
$\frac{1}{8}$	125	125	175
$\frac{1}{6}$	125	125	175
$\frac{1}{4}$	75	75	175

TABLE XV

CAPACITOR-START AND CAPACITOR-MOTOR HIGH TORQUE

Hp Rating	Locked Rotor	Pull-up	Breakdown
		60 cycles	1725 rpm
$\frac{1}{8}$	350	200	200
$\frac{1}{6}$	350	200	200
$\frac{1}{4}$	350	200	200
$\frac{1}{3}$	325	200	200
$\frac{1}{2}$	300	200	200
$\frac{3}{4}$	275	200	200
			1140 rpm
$\frac{1}{8}$	300	185	185
$\frac{1}{6}$	300	185	185
$\frac{1}{4}$	300	185	185
$\frac{1}{3}$	300	185	185
$\frac{1}{2}$	300	185	185

The pull-up torque of a motor is the minimum torque developed by the motor during the period of acceleration from rest to full speed with rated voltage applied at rated frequency.[7]

TABLE XVI

REPULSION-START INDUCTION

Hp Rating	Locked Rotor	Pull-up	Breakdown
		60 cycles	1725 rpm
$\frac{1}{8}$	350	200	200
$\frac{1}{6}$	350	200	200
$\frac{1}{4}$	350	200	200
$\frac{1}{3}$	350	200	200
$\frac{1}{2}$	350	200	200
$\frac{3}{4}$	350	200	200
			1140 rpm
$\frac{1}{8}$	300	150	185
$\frac{1}{6}$	300	150	185
$\frac{1}{4}$	300	150	185
$\frac{1}{3}$	300	150	185
$\frac{1}{2}$	300	150	185

[7] The above definitions and tables are taken from the N.E.M.A. Motor and Generator Standards, May, 1938.

CHAPTER XXXII

DOUBLE-REVOLVING FIELD THEORY. EQUIVALENT CIRCUIT

264. Chapter Outline.

Analysis by the Double-revolving Field Theory.
The Equivalent Circuit.
Example.
Calculation of Torque.

265. The Double-revolving Field Theory. It has been pointed out previously that two principal theories are used for the explanation of single-phase motor action. Each of these has its advantages and its field of usefulness. They ultimately lead to identical results. The double-revolving field theory will be treated here. It is probably no simpler in application than the cross-field theory, but, when the revolving field theory of the polyphase motor is understood, the fundamental concepts are more readily grasped. They rest on the idea that the gap flux in the single-phase motor may be resolved into two rotating fields of unequal magnitude, revolving in opposite directions. Assuming for the moment that each of these two fields is of half amplitude, the speed-torque curve for each is shown in Fig. 214. The $Y-Y$ axis represents the condition of zero speed. The dotted line shows the combined effect, which is zero at standstill, of the two opposite torques. Now if the armature is started in one direction, the slip decreases with respect to one field, the torque of that field increases, and that of the other decreases. This results in acceleration in the direction in which the armature was started. If the load torque is T', the motor reaches a stable speed at the value of slip S'. The torque of the forward field is T_f, that of the backward field is T_b, and the net torque is the difference or T'. Note that at zero load, or rather zero torque, the speed would not be synchronous as is theoretically true for polyphase induction motors. The above explanation holds equally well for operation in either direction. The motor runs in whichever direction its starting method demands.

To apply the double-revolving field theory quantitatively, it is necessary to determine the value of each of the two fields at any given slip. When the two fields are known, the torque produced by each can then

be calculated by the same formulas used for polyphase motors, and the difference used as the net torque.

The equations and diagrams are built up by assuming that the actual stator and rotor of the single-phase motor are replaced by two stators and rotors, each equal to the original machine with the stators in series and the rotors on the same shaft. It is assumed that one rotor turns with its magnetic field and the other turns against its magnetic field.

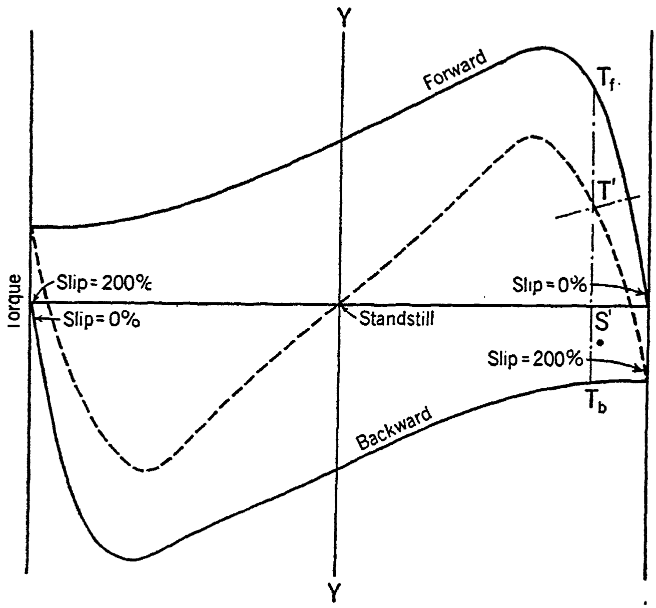


FIG. 214. An approximate representation of the torque, due to each component of the double-revolving field, and the resultant.

Each stator has the same primary winding as the actual machine, but the primary local impedance is counted only once.

266. Equivalent Circuit. One equivalent network for the single-phase induction motor based on this theory is shown in Fig. 215. The motor is made up of a stator primary winding and of two imaginary rotors. The stator impedance contains the usual values of R_1 and X_1. The impedance of each rotor is calculated so that r_2 *and* x_2 *are each half of the actual rotor values in stator terms, as used in transformer or cross-field theory.* In the first equivalent circuit to be considered here, iron loss will be neglected. Hence the exciting branch will be made up of exciting reactance only. *One-half of the total magnetizing reactance will be assigned to each rotor also.* This point must be stressed: If r_2, x_2 and x_m are

measured or calculated by any usual method, the values which appear on the diagram are half values. To help in this distinction, half values will be shown by small r's and x's. Full values of constants as used in the cross-field theory will be indicated by capitals.

As the first rotor operates at a slip of s, the second rotor has a slip of $2 - s$. Hence, assuming values for slip, the performance calculation is based on the solution of the circuit for the component currents.

The forward torque in synchronous watts, following the reasoning which results from polyphase induction motor analyses, is

$$T_f = I_3^2 \frac{r_2}{s} \text{ (synchronous watts)} \quad [345]$$

The backward torque is

$$T_b = I_5^2 \frac{r_2}{2 - s} \text{ (synchronous watts)} \quad [346]$$

The resultant torque is the difference between these two.

$$T = T_f - T_b \quad [347]$$

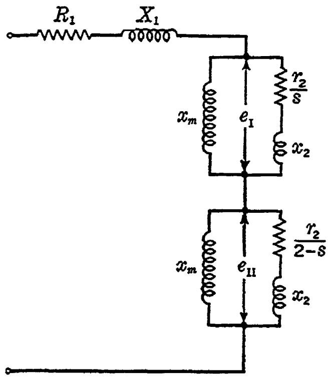


FIG. 215. An equivalent circuit for the single-phase motor, double-field theory. Core loss is neglected.

The procedure requires the determination of the impedances of the two imaginary rotors. It will be illustrated by a motor rated as follows:

Motor: $\frac{1}{4}$ hp, 4-pole, 60-cycle, 110-volt.
$R_1 = 1.86$. $X_1 = 2.56$. Friction and windage, 13.5 watts.
Actual $R_2 = 3.56$; so use 1.78 ohms for each rotor.
Actual $X_2 = 2.56$; so use 1.28 ohms for each rotor.
Actual $X_m = 53.5$; so use 26.7 ohms for each rotor.

The performance will be calculated for a slip of 0.05.

Impedance of the "forward" rotor:

$$Z_{\mathrm{I}} = \frac{jx_m\left(\frac{r_2}{s} + jx_2\right)}{\frac{r_2}{s} + j(x_2 + x_m)} \quad [348]$$

In rationalizing this fraction and in using $x_0 = x_m + x_2$, this equation will become

$$Z_{\mathrm{I}} = x_m \frac{\frac{r_2}{s} x_m + j\left[\left(\frac{r_2}{s}\right)^2 + x_2 x_0\right]}{\left(\frac{r_2}{s}\right)^2 + x_0^2} \quad [349]$$

Numerically:

$$Z_{\mathrm{I}} = 26.7 \frac{\frac{1.78}{0.05} \cdot 26.7 + j\left[\left(\frac{1.78}{0.05}\right)^2 + 1.28 \times 27.98\right]}{\left(\frac{1.78}{0.05}\right)^2 + (27.98)^2} \qquad [350]$$

$$= 12.4 + j17.15 \quad \text{or} \quad 21.15 \text{ ohms}$$

In a similar manner, for the second rotor:

$$Z_{\mathrm{II}} = \frac{jx_m\left(\frac{r_2}{2-s} + jx_2\right)}{\frac{r_2}{2-s} + j(x_2 + x_m)} = x_m \frac{\frac{r_2}{2-s}x_m + j\left[\left(\frac{r_2}{2-s}\right)^2 + x_2x_0\right]}{\left(\frac{r_2}{2-s}\right)^2 + x_0^2} \qquad [351]$$

For a slip of 0.05, $2 - s$ is 1.95, and $r_2/2 - s$ becomes 0.91. Supplying numerical values,

$$Z_{\mathrm{II}} = 0.84 + j1.26 \quad \text{or} \quad 1.51 \text{ ohms} \qquad [352]$$

The total impedance of the circuit is the sum

$$Z_T = Z_1 + Z_{\mathrm{I}} + Z_{\mathrm{II}} \qquad [353]$$

$$= 15.10 + j20.97 \quad \text{or} \quad 25.85 \text{ ohms}$$

The motor current at this slip is

$$I_1 = \frac{110}{25.85} \quad \text{or} \quad 4.27 \text{ amperes} \qquad [354]$$

The pf:

$$\frac{R_{\mathrm{total}}}{Z_T} = \frac{15.10}{25.85} \quad \text{or} \quad 0.585 \qquad [355]$$

To find the "branch" currents:

$$I_1Z_{\mathrm{I}} = e_{\mathrm{I}} = 4.27 \times 21.15 \quad \text{or} \quad 90.4 \text{ volts} \qquad [356]$$

$$I_1Z_{\mathrm{II}} = e_{\mathrm{II}} = 4.27 \times 1.51 \quad \text{or} \quad 6.44 \text{ volts} \qquad [357]$$

$$Z_3 = \sqrt{\left(\frac{r_2}{s}\right)^2 + x_2^2} \quad \text{or} \quad 35.7 \text{ ohms} \qquad [358]$$

$$I_3 = \frac{e_{\mathrm{I}}}{Z_3} \quad \text{or} \quad 2.53 \text{ amperes} \qquad [359]$$

$$Z_5 = \sqrt{\left(\frac{r_2}{2-s}\right)^2 + x_2^2} \quad \text{or} \quad 1.57 \text{ ohms} \qquad [360]$$

$$I_5 = \frac{e_{\mathrm{II}}}{Z_5} \quad \text{or} \quad 4.1 \text{ amperes} \qquad [361]$$

The torques:

$$T_f = I_3^2 \frac{r_2}{s} \quad \text{or} \quad 228 \text{ synchronous watts} \qquad [362]$$

$$T_b = I_5^2 \frac{r_2}{2 - s} \quad \text{or} \quad 15.3 \text{ synchronous watts} \qquad [363]$$

$$T_{\text{net}} = 228 - 15.3 \quad \text{or} \quad 212.7 \text{ synchronous watts}$$

$$\text{Output in watts} = \text{synchronous watts} \times (1 - s) \qquad [364]$$

$$= 212.7 \times 0.95 \quad \text{or} \quad 202 \text{ watts}$$

Minus friction and windage loss, for net output:

$$202.0 - 13.5 = 188.5 \text{ watts} \quad \text{or} \quad 0.252 \text{ hp}$$

The efficiency is then

$$\frac{\text{Output}}{\text{Input}} = \frac{188.5}{110 \times 4.27 \times 0.585} \quad \text{or} \quad 0.68 \qquad [365]$$

A check on calculations can be obtained by determining the copper losses in both "forward" and "backward" rotors, adding them to the

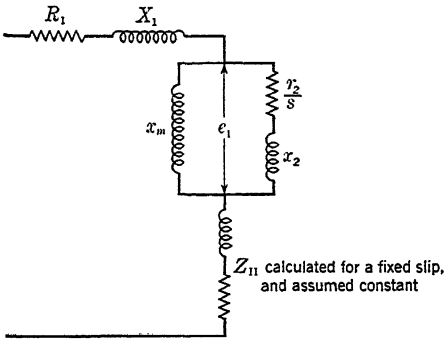


FIG. 216.

other losses to obtain the difference between input and output as given above.

Note in these calculations that the voltage across the backward rotor at operating speed is not high, nor will a small change in speed make a great difference in the value of $2 - s$ and the impedance Z_{II}. For that reason, to simplify calculations, little error is introduced (except at higher slips) by calculating Z_{II} for a slip such as 3 or 5 per cent, and then using Z_{II} as a constant for any other speed in the normal operating range at which motor performance is desired. This circuit is shown in Fig. 216.

267. Consideration of Core Losses. The simplest way to treat core losses is to lump them into one equivalent resistance connected across the motor terminals as shown in Fig. 217. This results in an in-phase addition to the current and a constancy for these losses at all loads. The connection of the equivalent resistance beyond the stator impedance, as shown in dotted lines, causes a reduction in losses with the counter emf which also reduces with the load increase. This assumption may or may not agree with physical facts.

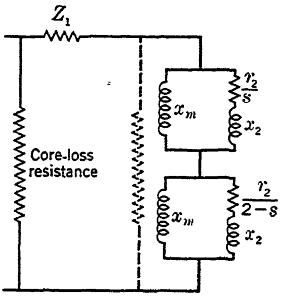


FIG. 217.

Following the method set up for transformers or polyphase induction motors, the equivalent resistance for considering core losses would be grouped with the magnetizing reactance, either in parallel or by replacement with a new equivalent series impedance. This is shown in Fig. 218*a* and *b*. In attempting to estimate an equivalent resistance for a given core loss, one must consider that the voltage across the second rotor is usually very low and that most of the iron loss takes place in motor I. Otherwise the core-loss watts, obtained when the running performance is calculated, will be abnormally small. This point can best be illustrated by an example.

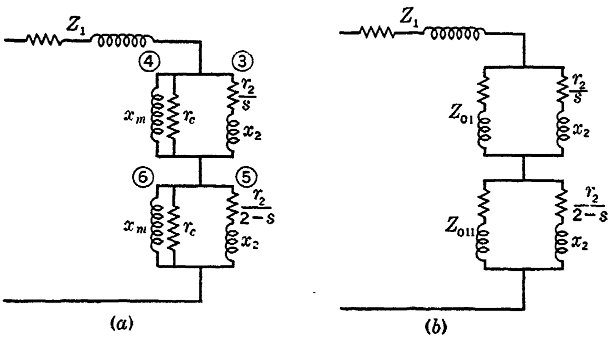


FIG. 218.

Example. Assume that the motor just considered has a core loss of 36 watts. The equivalent resistance will be determined by assuming a voltage of 85 to 95 per cent of the applied voltage across the first "motor." In this case, say, 92 volts.

Then the core-loss current is

$$I_c = \frac{36 \text{ watts}}{92 \text{ volts}} \quad \text{or} \quad 0.392 \text{ ampere}$$

$$r_c = \frac{92}{0.392} \quad \text{or} \quad 235 \text{ ohms}$$

This value would be used on both motors, as shown in Fig. 218*a*. Or, replacing by an equivalent series impedance, one obtains

$$z_{0\mathrm{I}} = z_{0\mathrm{II}} = \frac{1}{\dfrac{1}{r_c} + \dfrac{1}{jx_m}} \tag{366}$$

$$= \frac{1}{\dfrac{1}{235} + \dfrac{1}{j26.7}} \quad \text{or} \quad 3.01 + j26.5 \text{ ohms}$$

268. Example. Core Losses Considered. The $\frac{1}{4}$-hp motor, used previously as an example, will be recalculated, considering the core losses as shown on the circuit of Fig. 218. The assumed slip is again 5 per cent.

(1) Conductance of the core loss branch $= \dfrac{1}{r_c} = \dfrac{1}{235}$ or 0.00426

(2) Susceptance of the magnetizing branch $= \dfrac{-j}{x_m} = \dfrac{-j}{26.7}$ or $-j0.0374$

(3) Admittance of branch 3:

$$\frac{\dfrac{r_2}{s} - jx_2}{\left(\dfrac{r_2}{s}\right)^2 + x_2^2} = \frac{35.6 - j1.28}{1264} \quad \text{or} \quad 0.028 - j0.00101 \tag{367}$$

(4) Admittance of "motor" I = sum of items 1, 2, and 3:

$$Y_{\mathrm{I}} = 0.03226 - j0.03841 \text{ mho}$$

(5) Impedance of "motor" I = $12.96 + j15.20$ or 19.9 ohms

(6) Admittance of branch 5:

$$\frac{\dfrac{r_2}{2-s} - jx_2}{\left(\dfrac{r_2}{2-s}\right)^2 + x_2^2} = \frac{0.91 - j1.28}{2.469} \quad \text{or} \quad 0.369 - j0.517 \tag{368}$$

(7) Admittance of "motor" II = sum of items 1, 2, and 6:

$$y_{\mathrm{II}} = 0.3733 - j0.555 \text{ mho}$$

(8) Impedance of "motor" II = $0.836 + j1.242$ or 1.50 ohms

(9) Impedance of entire motor:

$$Z = Z_1 + Z_{\mathrm{I}} + Z_{\mathrm{II}} = 15.66 + j19.00 \quad \text{or} \quad 24.7 \text{ ohms} \tag{369}$$

(10) Primary current:

$$I_1 = \frac{V}{Z_T} = \frac{110}{24.7} \quad \text{or} \quad 4.46 \text{ amperes}$$

(11) Power factor:

$$\cos\theta = \frac{R_T}{Z_T} = \frac{15.66}{24.7} \quad \text{or} \quad 0.635$$

(12) Voltage across "motor" I:

$$e_{\mathrm{I}} = I_1 Z_{\mathrm{I}} = 4.46 \times 19.9 \quad \text{or} \quad 88.8 \text{ volts}$$

(13) Core-loss current in "motor" I:

$$I_c = \frac{e_{\mathrm{I}}}{r_c} = \frac{88.8}{235} \quad \text{or} \quad 0.378 \text{ ampere}$$

(14) Rotor current I_3 (see item 3):

$$I_3 = \frac{88.8}{\sqrt{1264}} \quad \text{or} \quad 2.5 \text{ amperes}$$

(15) Core loss in "motor" I:

$$I_c^2 \times r_c = 0.378^2 \times 235 = 33.6 \text{ watts}$$

(16) Copper loss in "motor" I:

$$I_3^2 \times r_2 = 2.5^2 \times 1.78 \quad \text{or} \quad 11.0 \text{ watts}$$

(17) Forward torque, "motor" I:

$$T_f = I_3^2 \times \frac{r_2}{s} = 2.5^2 \times 35.6 \quad \text{or} \quad 222 \text{ synchronous watts} \qquad [370]$$

(18) Voltage across "motor" II:

$$e_{\mathrm{II}} = I_1 Z_{\mathrm{II}} = 4.46 \times 1.50 \quad \text{or} \quad 6.69 \text{ volts}$$

(19) Core-loss current in "motor" II:

$$I''_c = \frac{e_{\mathrm{II}}}{r_c} = \frac{6.69}{235} \quad \text{or} \quad 0.0284 \text{ ampere}$$

(20) Rotor current I_5 (see item 6):

$$I_5 = \frac{6.69}{\sqrt{2.47}} \quad \text{or} \quad 4.25 \text{ amperes}$$

(21) Core loss in "motor" II:

$$I''^2_c \times r_c = 0.0284^2 \times 235 \quad \text{or} \quad 1.90 \text{ watts} \qquad [371]$$

(22) Total core loss = item 15 + item 21 or 35.5 watts, versus 36 watts which was assumed as the no-load value in fixing r_c.

(23) Copper loss in "motor" II:

$$I_5^2 \times r_2 = 4.25^2 \times 1.78 \quad \text{or} \quad 32.2 \text{ watts}$$

(24) Backward torque, "motor" II:

$$T_b = I_5^2 \times \frac{r_2}{2 - s} = 4.25^2 \times 0.913 \quad \text{or} \quad 16.5 \text{ synchronous watts} \qquad [372]$$

(25) Torque produced $= T = T_f - T_b$ or 205.5 synchronous watts

(26) Watts converted = synchronous watts $\times$ $(1 - s)$

$$= 205.5 \times 0.95 \quad \text{or} \quad 195 \text{ watts}$$

(27) Net watt output = item 26 − (F and W)

$$= 195.0 - 13.5 \quad \text{or} \quad 181.5 \text{ watts } (0.243 \text{ hp})$$

(28) $$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{181.5}{110 \times 4.46 \times 0.635} \quad \text{or} \quad 0.578$$

(29) Losses from input − output = 314 − 181.5 or 132.5 watts

(30) Sum of individual losses, as a check:

$I_1^2R_1 = 4.46^2 \times 1.86$	or	37.0 watts
Rotor copper (item 16)	=	11.0
Rotor copper (item 23)	=	32.2
Core loss (item 22)	=	35.5
F and W	=	13.5
Total losses		129.2 versus 132.5

269. Torque from Flux Calculations. The forward and backward torques can be calculated from the component fluxes, using the equation derived for polyphase motors. In pound-inches:

$$T = 2.22 \times 10^{-8} \times k_d k_p C I_2 \phi P \cos (\theta_{I_2}{}^{\phi}) \qquad [373]$$

$k_d k_p$ = the winding factors
ϕ = the effective flux per pole
C = the series conductors on the stator
I_2 = the effective rotor current in stator terms

The equivalent circuit must be solved in part and, in addition, other data must be known on the motor. The procedure will be illustrated briefly by the motor just used, which displays the following additional values:

Maximum flux per pole at 110 volts = 118,000 lines
Effective flux per pole at 110 volts = 83,500 lines
Series conductors on the stator = 872
Winding factor, $k_p k_d = 0.8$

Calculations will be made for both forward and backward fields.

(*a*) Voltage across "motor" I = 88.8 (see item 12)
(*b*) Voltage across "motor" II = 6.69 (see item 18)
(*c*) Effective forward flux, by proportion:

$$\frac{88.8}{110} \times 83{,}500 \quad \text{or} \quad 67{,}600 \text{ lines}$$

(*d*) Effective backward flux, by proportion:

$$\frac{6.69}{110} \times 83{,}500 \quad \text{or} \quad 5070 \text{ lines}$$

(*e*) Forward rotor current $I_3 = 2.5$ (see item 14)
(*f*) Backward rotor current $I_5 = 4.25$ (see item 20)
(*g*) Cosine of angle between ϕ and I_3 (forward):

$$\frac{r_2}{s} \div Z_3 \approx 1.0 \qquad [374]$$

(*h*) Cosine of angle between ϕ and I_5:

$$\frac{r_2}{2 - s} \div Z_5 = \frac{0.913}{1.57} \quad \text{or} \quad 0.582 \qquad [375]$$

Forward torque, by equation 373:

$$T = 2.22 \times 10^{-8} \times 0.8 \times 872 \times 2.5 \times 67{,}600 \times 4 \times 1.0$$
$$= 10.5 \text{ lb-in.} \quad \text{or} \quad 14.0 \text{ oz-ft}$$

Backward torque by equation 373:

$$T = 2.22 \times 10^{-8} \times 0.8 \times 872 \times 4.25 \times 5070 \times 4 \times 0.582$$
$$= 0.777 \text{ lb-in.} \quad \text{or} \quad 1.03 \text{ oz-ft}$$

Effective or net torque developed = $14 - 1.03$ or 12.97 oz-ft

Since (112.5 $\times$ synchronous watts/(synchronous rpm) = ounce feet of torque, it follows that 12.97 oz-ft corresponds to 207 synchronous watts. This compares with 205.5 synchronous watts (item 25) by the previous analysis.

CHAPTER XXXIII

CROSS-FIELD THEORY

270. Chapter Outline.[1]

Introductory Statement of Relationships.
Equations for Stator and Both Rotor Axes.
Equations of Current and Torque.
Torque.
No-load Conditions.
Examples.

271. Introduction. Before going into the details of the cross-field theory in an exact manner, a brief outline of the action, along with some fundamental ideas, will be given.

When a winding is supplied by an alternating emf so that a mutual flux is set up, such a flux crosses the air gap and links both the supply (primary) and the secondary circuits, inducing an emf in each. In this analysis, this type of emf will carry the subscript T, indicating that it arises from transformer action.

When the rotor revolves through the air-gap flux, its conductors cut the flux lines, generating an emf by speed action. This emf will be identified by the subscript S, indicating its origin.

Rotation of a conductor through a flux produce S times the voltage produced by pulsation of that flux; where

$$S = \frac{\text{actual rotor rpm}}{\text{synchronous rpm}} \tag{376}$$

The counter emf built up by transformer action is numerically equal to the product of magnetizing reactance X_m and magnetizing current I_ϕ.

$$E_T = I_\phi X_m \tag{377}$$

Combining these statements, numerically,

$$E_S = SI_\phi X_m \tag{378}$$

[1] For a circle diagram based on the cross-field theory see:
W. J. Branson, "Single-phase Induction Motors," *Trans. A.I.E.E.*, Vol. 31, Part II, p. 1749, 1912.

The single-phase induction motor winding is distributed in the stator slots, somewhat as shown in Fig. 219*a* for the 2-pole design. When this winding is excited by an alternating voltage, two components of flux result. These are (1) the mutual flux, crossing the air gap and linking the rotor bars as well as the primary or stator turns, and (2) the primary leakage flux, which links only the stator turns. The mutual flux component induces a stator counter emf and a rotor emf. The two are equal

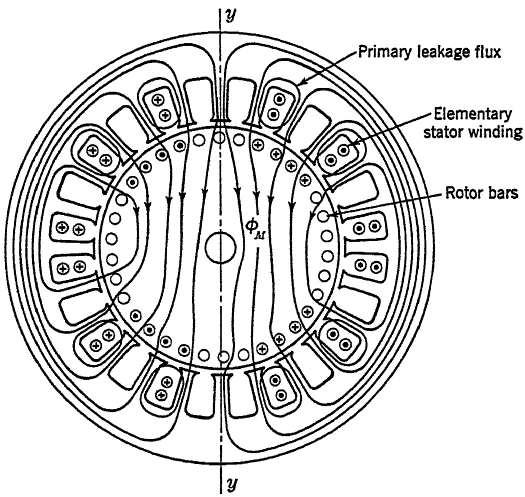


FIG. 219*a*. The stator winding when excited by an alternating emf produces a mutual flux ϕ_M and a stator leakage flux. The mutual flux pulsates along a vertical axis.

and opposite, on the assumption that the ratio of transformation between stator and rotor is unity, or that rotor quantities are expressed in stator terms. The primary leakage flux component gives the stator winding a leakage reactance drop. The production of this flux requires a magnetizing component of stator winding current I_ϕ. The mutual flux pulsates (as shown in the diagram) along a vertical axis; hence it will be called the y- or main-axis flux, ϕ_M.

As the rotor turns, its bars cut through the main-axis flux, generating a voltage by speed action. Since the rotor is a symmetrically closed circuit, the voltage results in a current, circulating through the rotor winding. This current, together with the rotor winding turns, gives the rotor an mmf, and results in a flux system, *originating in the rotor.*

A rigorous derivation will be given later, but now we will accept the fact that this flux is in space quadrature with the main winding flux. This is the cross-flux ϕ_C, from which the theory takes its name. It is

shown in Fig. 219*b*. Note that it originates in speed action and hence disappears at standstill.

To identify the various components of flux, voltage, and current, a rather complex system of subscripts will be used. Mention has been made of S and T as subscripts indicating the primary cause of a voltage as speed or transformer action, respectively. With two flux systems we will show whether the emf originates from the main- or the cross-axis flux by the second subscript M or C, respectively. If the emf is

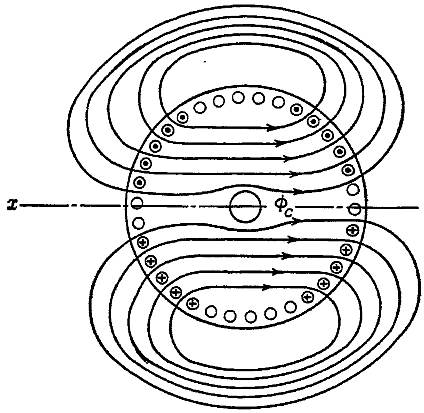


FIG. 219*b*. Rotation of the rotor bars through the main axis flux generates speed voltages in the bars, causing a current to flow and producing the cross flux ϕ_C as shown.

instrumental in producing a current and an mmf which affect the winding of the main (or y) axis or the cross (or x) axis, the third subscript will so indicate by y or x, respectively.

To apply this system: The main winding flux induces an emf in the rotor which will be designated E_{TMy}. It is produced by transformer action (T) of the main winding flux (M), and the final resulting current reacts on the winding of the main or y axis (y), as in an ordinary transformer.

As the rotor bars cut ϕ_M, a speed voltage is built up as previously described. This voltage will be designated E_{SMx}, indicating speed action (S), main-axis flux (M), and the final current and mmf are active on the winding of the cross or x axis (x).

272. Further Voltage Components. We now have two flux systems. The main-axis flux ϕ_M originates from the stator winding. The cross-axis flux ϕ_C originates from a current in the rotor. They are in space quadrature.

Both fluxes pulsate and induce two rotor emf's by transformer action. The first, E_{TMy}, has already been mentioned. The pulsation of the

cross-axis flux through the rotor bars induces the voltage E_{TCx} (transformer action, cross-axis flux with the resulting current active in the x or cross axis).

The rotor bars cut through both flux components and generate two speed voltages. The voltage caused by cutting the main-axis flux E_{SMx} has already been mentioned. In addition, the speed-action voltage E_{SCy} will be considered. Its position or phase is such that it influences the current flowing in the y axis.

A proper grouping of these four voltages, with the impedance drops of the rotor winding, enables the rotor currents to be determined. The current components are identified by the rotor axis along which they produce an mmf.

These components of current and voltage are, of course, fictitious, and so are chosen only for the purpose of analysis. The task is to locate each in space and time, to develop their quantitative expressions, and to show how their interaction produces motor torque.

273. Fundamental Relationships. The ideas previously presented will now be considered in a more detailed fashion. The single-phase induction

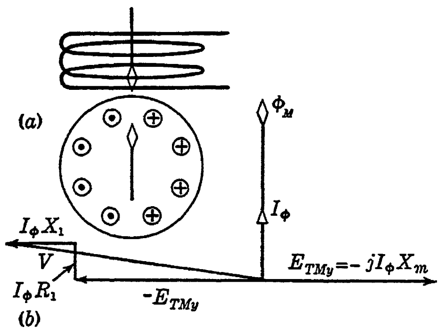


FIG. 220. The induced rotor emf is E_{TMy}. The main-axis flux ϕ_M also induces the stator winding counter emf $(-E_{TMy})$, assumed equal and opposite to the rotor value on the assumption that stator to rotor transformation is unity. Only the magnetizing current I_ϕ is considered. This produces stator winding resistance and leakage reactance drops, which together with the counter emf "use up" the applied voltage.

motor with a squirrel-cage rotor can be represented schematically as shown in Fig. 220*a*. Although the stator winding is actually distributed, it is equivalent electromagnetically, to the concentrated stator coil. Exciting the stator with an alternating emf causes the alternating flux along the main axis. This will be designated ϕ_M.

The voltage induced in the rotor winding is E_{TMy}, and from transformer fundamentals it is in lagging quadrature with the flux causing it. This will be equal and opposite to the counter emf induced in the stator

winding. These facts enable us to draw the partial vector diagram of Fig. 220*b*. The core-loss component of the magnetizing current is

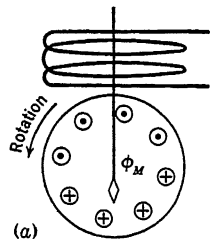


FIG. 221*a*. As the rotor bars turn, emf's are built up in them as shown.

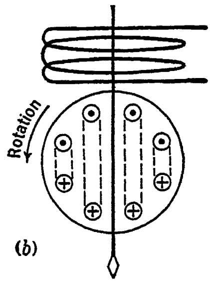


FIG. 221*b*. Because of the symmetrical nature of the rotor winding, we can consider that each conductor above the horizontal axis is connected with one below, giving an equivalent winding, electromagnetically, to that indicated by the dotted lines. The magnetic axis of such a winding is horizontal.

neglected. The stator winding is assumed to have the usual resistance and leakage reactance drops, $I_\phi R_1$ and $I_\phi X_1$, respectively.

Rotation of the rotor bars through ϕ_M produces a generated emf, E_{SMx}. As shown in Fig. 221, those bars above the horizontal axis will

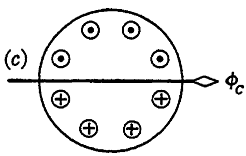


FIG. 221*c*. The rotor currents resulting from the emf's of Fig. 221*b* build up a rotor mmf and flux system which pulsates along the x-axis. This is the cross-axis flux in space quadrature with the main-axis flux.

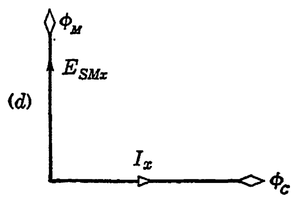


FIG. 221*d*. The speed voltage originating this series of reactions is E_{SMx}, which will be a maximum at the same *time* that ϕ_M is a maximum. The rotor winding is largely reactive; hence the current I_x lags practically 90° in *time* behind the voltage causing it. When I_x is a maximum, so is the flux ϕ_C which it produces. This neglects the core loss (or hysteretic) angle between I_x and ϕ_C.

have an emf in one direction; those below the horizontal axis will have a reversed voltage. The resulting currents will react along a horizontal

axis. Electrically, these bars can be considered as being connected, as shown by the dotted lines of Fig. 221*b*.

At the instant when ϕ_M is a maximum, the voltage generated will be a maximum, and therefore ϕ_M and E_{SMx} are in *time* phase. Whether the voltage is in time phase or phase opposition depends upon the direction of rotation. A speed voltage will be considered positive and drawn in a positive direction, if it ultimately results in a positive flux. For the conditions shown, both E_{SMx} and ϕ_C are positive.[2]

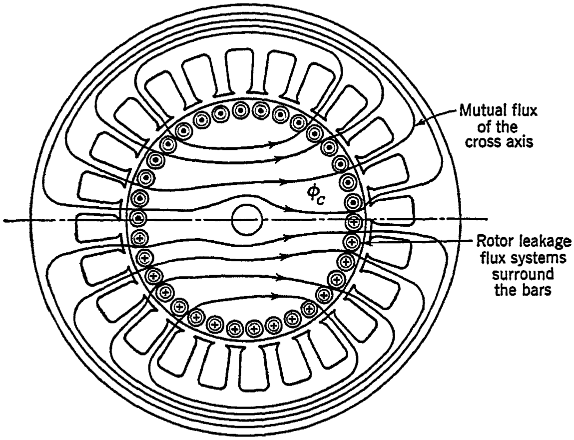


FIG. 222*a*. The cross-axis flux in a typical 2-pole magnetic circuit. Note that the only stator winding is one having its magnetic axis vertical, and hence there is no stator winding upon which this mmf or flux can react. For convenience the flux system is divided into two components. One crosses the gaps and has practically the same type of path as the mutual flux of the main axis. The other component surrounds the rotor conductors, giving the winding a rotor leakage reactance.

E_{SMx} will cause a current to flow through the rotor bars, which act like turns centered on the horizontal axis. The resulting mmf has no stator winding upon which it can react.

Hence the cross axis of the rotor acts like a reactance coil; the flux ϕ_C gives the winding a large reactance. The current in a reactance coil lags behind its voltage by nearly 90° in time.

We can see, therefore, that E_{SMx} is in time phase with ϕ_M, and I_x will lag its voltage by nearly 90°. These relationships are shown in Fig. 221*d*. The flux of the cross axis will be a maximum when the current

[2] The novice is usually confused by the directions given these speed voltages. If the statement given above is accepted at this point, the final Kirchhoff law equations and the analysis of torque which follow will indicate more conclusively the basis for the viewpoint.

causing it is a maximum, and hence we can draw ϕ_C in time phase with I_x.

Keep in mind that the axis of the flux ϕ_C is horizontal. That of the main flux ϕ_M is vertical. These two flux systems are in *space* quadrature. Check again with the time diagram of Fig. 221*d* and see that they are in time quadrature as well. We have produced two flux systems in the air gap of the single-phase induction motor, in time and space quadrature, somewhat similar to those which would result in a two-phase motor.

Numerically:

$$E_{SMx} = SE_{TMy} \qquad [379]$$

Both voltages originate from the same flux, but, when a conductor cuts a pulsating flux at a speed of S times synchronous, it generates a voltage only S times as great as the voltage caused by the flux pulsation.

The manner in which ϕ_C is produced will be examined further.

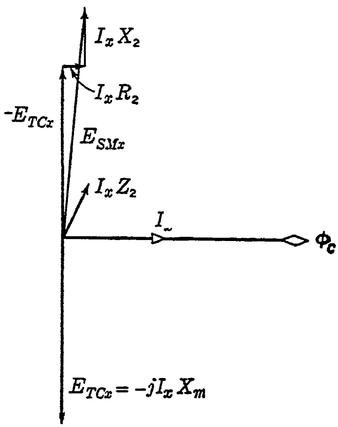


FIG. 222*b*. The cross-axis current and voltage relationships. The speed voltage E_{SMx} is the "applied" voltage to a reactance coil (rotor winding) for which I_x is the magnetizing current producing ϕ_C. As in the usual transformer diagram the voltage is "used up" in overcoming the drop caused by the mutual flux ($-E_{TCx}$) and the local impedance drop I_xR_2 and I_xX_2. Compare with Fig. 220*b*.

Note the cross-axis flux system of Fig. 222*a* in which the flux paths are shown in more detail. Part of this flux crosses the two air gaps, passing through the stator laminations. The permeance of this path is the same as that for the mutual flux of the main axis. Hence, since the fictitious rotor winding turns of the two axes are equal, the reactance of the rotor windings are practically the same in each axis and the mutual flux of the cross axis induces a rotor emf, I_xX_m. Compare this with the main-axis counter emf, $I_\phi X_m$. Here all the cross-axis current is magnetizing.

Numerically, the voltage, induced in the rotor by transformer action of the flux ϕ_C and previously identified as E_{TCx}, is equal to I_xX_m. In addition, the current I_x, flowing through the rotor conductors, builds up a small leakage flux system ϕ_x which gives a leakage reactance drop. The rotor windings also possess a resistance R_2. These elements can all be combined as follows:

Consider that the speed voltage E_{SMx} is the voltage "applied" to a reactance coil. The voltage is used up in overcoming (1) the reactance

drop of the mutual reactance, (2) the drop caused by the leakage reactance of the winding, and (3) the resistance drop of the winding.

Without making use of any relationships which have not already been explained, consult the vectors drawn in Fig. 222*b*. The location of

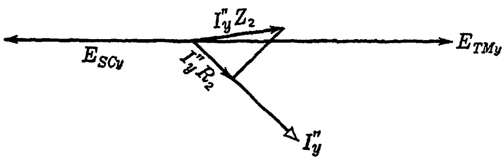


FIG. 223*a*. The voltage induced in the rotor by transformer action E_{TMy} is opposed by a speed emf E_{SCy}, caused by cutting the cross-axis flux. The difference is used up in overcoming the rotor impedance drop $I''_y Z_2$. The main-axis rotor current takes up the magnitude and position necessary to satify this relationship.

these vectors follows from the position of E_{SMx}, taken from 221*d*. Note the similarity between this figure and that of 220*b*.

This whole system of vectors (Fig. 222*b*) arises from speed. As the speed changes, the diagram "expands" or "contracts" accordingly. At zero speed, all these vectors reduce to zero. No energy required by this axis (and there are cross-axis rotor copper and core losses present) can come directly from the stator winding because there is no magnetic link between them. But, since these cross-axis losses result from the generated voltage E_{SMx}, the cross axis forms a generator load on the motor, exerting a negative torque which must be overcome. Neither this component of torque nor the useful motor torque has been explained as yet.

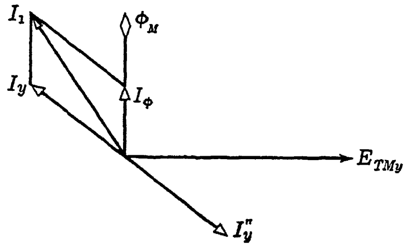


FIG. 223*b*. The rotor load current I''_y together with the rotor winding produces a demagnetizing mmf. This is identical with the similar effects in the ordinary transformer secondary. The primary draws a balancing component which together with the magnetizing current $I\phi$ makes up the total stator winding current I_1.

Only one main component of voltage still remains for discussion. This is the voltage caused by the speed action from the cross-axis flux. E_{SCy} will be in time phase with, or in phase opposition to, ϕ_C since they will reach their respective maxima at the same instant in time. For reasons to be demonstrated, E_{SCy} will be a maximum in the negative sense when ϕ_C is positive.

The position of E_{SCy} is such that it combines vectorially with E_{TMy} to produce a current (I''_y) active in the y-axis of the rotor. In a sense,

E_{SCy}, depending as it does on speed action, is a generated counter emf which together with the rotor impedance drop of the y-axis "uses up" the voltage "applied" to the rotor, i.e., E_{TMy}. See Fig. 223a.

Vectorially:

$$E_{SCy} + I''_y Z_2 = E_{TMy} \qquad [380]$$

It is this relationship which fixes the value and position of the rotor current component I''_y.

Since E_{SCy} results from speed action on the cross-axis flux, it will be S times as large as the transformer voltage caused by the same flux.

Numerically:

$$E_{SCy} = SE_{TCx} \qquad [381]$$

Since I''_y is active in the y-axis, the mmf that it produces is similar to that of any transformer secondary, in phase opposition to that mmf built up by the primary or stator winding. The rotor current I''_y, therefore, if acting alone would produce a flux in phase opposition to ϕ_M. Such a flux would be negative. Therefore, by the definition previously given for positive and negative speed voltages, E_{SCy} is in phase opposition to the flux ϕ_C which causes it, and the direction chosen for this voltage is correct.

The rotor current I''_y requires a balancing component in the primary, equal and opposite (for an assumed ratio, stator to rotor, of unity). This stator current will be designated I_y.

$$I_y = -I''_y \qquad [382]$$

Together with the magnetizing component of stator current, they combine to form the total current taken by the motor. See Fig. 223b.

Vectorially:

$$I_y + I_\phi = I_1 \qquad [383]$$

By examining Figs. 220b, 222b, and 223a and b it will be seen that the relative positions of all vectors have been established. Their exact positions will be given later, but first a summary of the diagrams is in order.

274. Drawing Approximate Vector Diagrams. At this point the reader is doubtless aware that to master the cross-field theory he must have a clear mental picture of various current and voltage components and their approximate positions. As an exercise it is suggested that the student draw the diagrams in the following order:

First, draw the ordinary transformer diagram of Fig. 224a. This is the stator diagram.

Reproduce E_{TMy}, which is the voltage induced in the main axis of the rotor by transformer action. Draw I''_y and I''_yZ_2. E_{SCy} is approximately equal and opposite to E_{TMy}, although actually as vectors

$$E_{SCy} + I''_yZ_2 = E_{TMy} \tag{384}$$

The diagram is shown in 224*b*.

Reproduce E_{SCy}, and opposite it lay out ϕ_C. Below synchronous speed, ϕ_C is always less than ϕ_M. On ϕ_C lay out another ordinary trans-

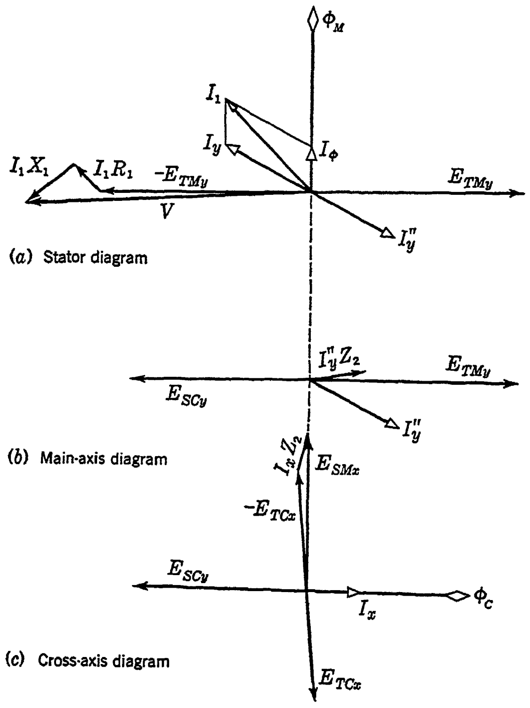


FIG. 224. Vector diagrams showing current and voltage components. As drawn, the diagrams do not "close" properly because of the approximations made. They are intended as a means to help to visualize vector directions.

former diagram, in which I_x is the magnetizing current and E_{TCx} is the counter emf. This counter emf, together with the rotor impedance drop of the cross axis, "uses up" the "applied" rotor voltage E_{SMx}. (The speed voltage E_{SMx} is in phase with the main flux which causes it, because it results, in the final analysis, in a flux in the positive direction.) The third component part of the diagram is shown in Fig. 224*c*.

275. Torque and Power. Considerations up to this point have fixed the general locations of the vectors whose exact positions will be deter-

mined by a simultaneous solution of all equations governing them. But, given the general locations of the currents, voltage, and flux vectors, an understanding of the operation of the single-phase induction motor by the cross-field theory can now follow.

Torque in a motor results from the interaction of the rotor current and the air-gap flux. But to obtain useful torque it is necessary that these factors maintain the correct positions in space and time. To illustrate this, consider a 2-pole d-c motor. With the brushes located on the neutral axis, the mmf system built up by the armature current (armature reaction) is in space quadrature with the main field flux. If the brushes are shifted 90°, the armature will still carry current, but the reaction of the current in the conductors and the air-gap flux is such as to produce torque in opposite directions on the two sides of the armature. When this happens, the armature currents and the air-gap flux are in *space* phase.

From this we can state the general principle that, to produce torque, the air-gap flux and the flux system built up by the armature currents must be out of space phase, being most effective in space quadrature.

Suppose a shunt motor had both its armature and its field excited by alternating current. The field supply is shifted in phase until the flux it produces reaches its maximum at the instant the armature current is zero. Now, even though the brushes are on neutral so that the correct relationships of *space* phase are maintained, no torque and power result because the air-gap flux and armature current are in *time* quadrature.

To produce the maximum possible useful torque it is therefore necessary for the air-gap flux and the flux system built up by the armature current to be in space quadrature, but in time phase.

Examining this theory as built up so far, we can see that the component of rotor current I''_y (effective on the main axis) and the cross-flux ϕ_C, satisfy these requirements. But only the component of I''_y which projects on ϕ_C on the time diagram (Fig. 224*b*) will be useful.

The vectors ϕ_M and I_x will satisfy the space-phase relationship for producing torque, but as drawn on Fig. 224*b* they have no projection on each other on the time diagram. Hence, they represent no torque. When the effect of iron losses is considered and the equations are solved for locating these vectors accurately, we will find that I_x has a small projection on the flux vector.

The torque of the single-phase induction motor could also be anticipated from the fact that there are two flux systems in the air gap which are almost exactly in space and time quadrature. It is unfortunate that these two fluxes differ in magnitude at various speeds. At synchronous speed the trace of the resultant flux vector is a circle. Below synchro-

nous speed the value of ϕ_C decreases with respect to ϕ_M and the trace becomes an ellipse with a vertical main axis. Above synchronous speed ϕ_C will be greater than ϕ_y and the trace of the resultant flux vector will be an ellipse with its major axis horizontal. See Fig. 225.

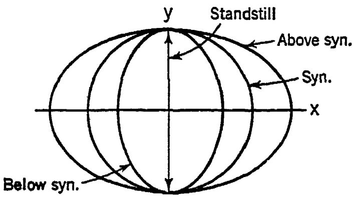


Fig. 225*a*. Loci of flux vector.

The inherent torque characteristic of a single-phase induction motor is a pulsating one, varying in magnitude for various positions in the rotation. This is unfortunate as it means that these pulsations may set up vibrations in the motor or its mechanical load, and result in noisy operation. Resilient mountings to damp out such effects are often required on motor-driven domestic appliances.

A clearer conception of the variable torque developed can be obtained from the instantaneous diagrams of Fig. 225*c* and the curves of *b*. From these diagrams it will be seen that the rotor-exciting current I_x, reacting with ϕ_M, produces alternately motor and generator torque—motor torque from *a* to *b*, generator torque from *b* to *d*, etc.

Also, the diagrams show that the rotor power current I''_y, reacting with ϕ_c produces periods of both motor torque and generator torque—motor torque from *a* to *b*, generator torque from *b* to *c*, and motor torque

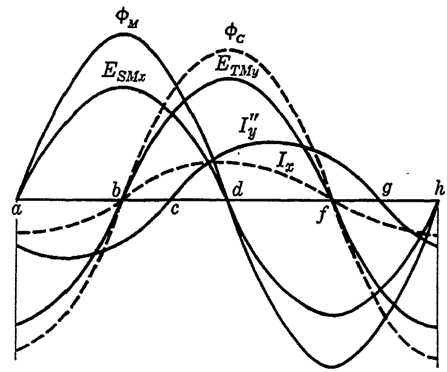


FIG. 225*b*. Voltage, current, and flux components. E_{TCx} would be approximately 180° from E_{SMx}, and E_{SCy} would be approximately 180° from E_{TMy}.

from *c* to *f*. As the motor is more heavily loaded, I''_y, in addition to increasing in magnitude, shifts its phase nearer to E_{TMy}; thus the period in which generator torque is developed is decreased with a corresponding increase in the period in which motor torque is developed. The dif-

ference of the total motor and generator torques throughout a cycle is the net or useful motor torque.

At no load the current I''_y lags E_{TMy} nearly 90°, so that point c nearly coincides with point d. As can be seen from the instantaneous torque diagrams, the period of generator torque production now almost equals that for the motor torque. This shows also that the torque goes through two cycles of variation from motor to generator torque during one cycle of the flux variation.

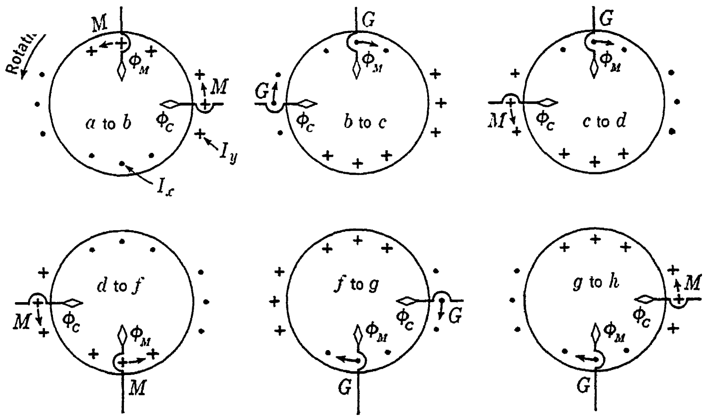


FIG. 225c. Development of torque from instant to instant.

276. Final Equations. Up to this point five components of current have been considered. Of the two components of rotor current, I''_y and I_x, the latter has no stator winding upon which it can react. The mmf produced by I''_y tends to demagnetize the main-axis primary and hence the stator or primary winding will draw a balancing component from its source to maintain the mutual flux. Considering that the rotor values are in stator terms, or that the ratio of transformation is unity, this balancing component I_y becomes

$$I_y = -I''_y \qquad [385]$$

We know that the total current taken by the stator winding equals the vector sum of the components,

$$I_1 = I_y + I_\phi \qquad [386]$$

or

$$I_\phi = I_1 - I_y$$

Also, as previously stated, a counter emf by transformer action is numerically equal to the product of magnetizing current and magnetizing reactance.

Hence

$$I_\phi X_m = E_{TMy} \tag{387}$$

As vectors:

$$-jX_m(I_1 - I_y) = E_{TMy} \tag{388}$$

The fundamental Kirchhoff law equation for the stator is

$$V = I_1 Z_1 + jX_m(I_1 - I_y) \tag{389}$$

This is the first of the simultaneous equations which will have to be solved to determine the total current I_1 and the rotor currents I''_y and I_x.

For the cross-axis (see Figs. 222*b* or 224*c*):

The speed voltage E_{SMx} will be S times as large as the transformer voltage built up by the same flux.

Numerically:

$$E_{SMx} = SE_{TMy}$$

As vectors:

$$E_{SMx} = SX_m(I_1 - I_y) \tag{390}$$

$$E_{TCx} = -jX_m I_x \tag{391}$$

$$SX_m(I_1 - I_y) - jX_m I_x = I_x Z_2 \tag{392}$$

We are now going to take the viewpoint, following Arnold and West, that speed voltages result from cutting not only the mutual flux (Steinmetz) but the rotor leakage fluxes as well. Then a speed voltage from the leakage flux would be S times as large as the leakage-reactance drop caused by that flux. The sign of the speed voltage from such a source will be the same as the sign of the speed voltages from the mutual flux.

Now, referring to equation 392, we will add to the speed voltage by the term $SI''_y X_2$ or $-SI_y X_2$. Then as a final equation for the cross-axis:

$$I_x Z_2 = SX_m(I_1 - I_y) - jX_m I_x - SI_y X_2 \tag{393}$$

This vector diagram is given in Fig. 226*b*.

For the main axis (see Figs. 223*a* and 224*b*):

$$E_{TMy} = -jX_m(I_1 - I_y) \tag{394}$$

$$E_{SCy} = -SX_m I_x \tag{395}$$

Then:

$$-jX_m(I_1 - I_y) - SX_m I_x = I''_y Z_2 \tag{396}$$

Reversing I''_y to obtain the reflection of the rotor current in the stator:

$$-jX_m(I_1 - I_y) - SX_m I_x = -I_y Z_2 \tag{397}$$

The speed voltage arising from the leakage flux is SI_xX_2, which takes its sign from the corresponding speed voltage of the mutual cross-axis flux. Then for the final equation of the main axis:

$$I_yZ_2 = jX_m(I_1 - I_y) + SI_x(X_m + X_2) \qquad [398]$$

The vector diagram corresponding to this condition is shown in Fig. 226c.

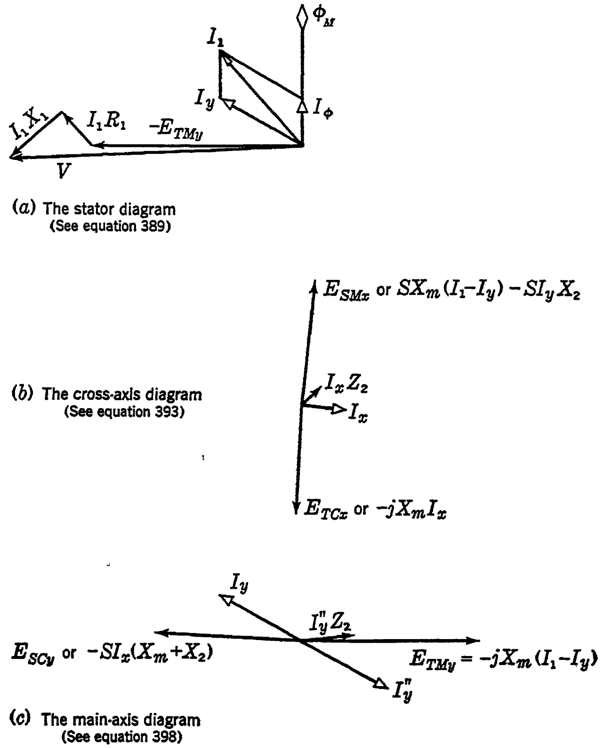


FIG. 226. Vector diagrams showing the quantities used in the final, simultaneous equations.

277. Equations for Current.[3] Although the analyses and explanations previously given may acquaint the reader with the action of the single-

[3] For calculating performance in practice, it is convenient to select a range of motor constants sufficient to cover all likely commercial requirements and to express them in terms of ratios. With the aid of circle diagrams or equations such as are given here, auxiliary curves can be plotted from which significant values can be obtained for calculating performance. This greatly reduces the labor required. Such curves have been given by the present writers in "Single-phase Induction Motor Performance," *Elect. Eng.*, October, 1937, and by H. M. Himebrook, in *Elect. Eng.*, February, 1941.

phase induction motor, based on the cross-field theory, it is assumed that a method of predetermining the performance for a given speed is ultimately desired. To obtain items of performance, it is first necessary to calculate the current. Expressions for current will be set up, based on the simultaneous solution of equations 389, 393, and 398. These are

$$I_1 = -V \frac{-R_2^2 + (1 - S^2)X_0^2 - j2R_2X_0}{U + jW} \qquad [399]$$

$$I_y = -VX_m \frac{(1 - S^2)X_0 - jR_2}{U + jW} \qquad [400]$$

$$I_x = +V \frac{SX_mR_2}{U + jW} \qquad [401]$$

wherein

$$U = -R_1R_2^2 + 2R_2X_1X_0 + R_2X_m(X_0 + X_2) + (1 - S^2)R_1X_0^2 \qquad [402]$$

$$W = -R_2^2X_1 - 2R_1R_2X_0 - R_2^2X_m + (1 - S^2)(X_1X_0^2 + X_2X_mX_0) \qquad [403]$$

The above values are calculated from the applied voltage. It is sometimes useful to obtain expressions for rotor currents in terms of the counter emf. In this case the equations for rotor currents can be solved simultaneously, giving results as shown ($E = E_{TMy}$).

$$I_y = -E \frac{R_2 + jX_0(1 - S^2)}{R_2^2 - X_2(1 - S^2)X_0 + j(R_2X_0 + R_2X_2)} \qquad [404]$$

$$I_x = E \frac{+jR_2S}{R_2^2 - X_2(1 - S^2)X_0 + j(R_2X_0 + R_2X_2)} \qquad [405]$$

In the equations set up for the Kirchhoff law relationships in both the stator and rotor, no provision has been made for the iron losses. That is, the currents have no components which represent iron losses. It will be recalled that the cross-axis of the rotor is the region in which the cross-axis flux is located, and hence the core loss of that axis should appear as an added load on the cross-axis "circuit." In addition, the main-axis core loss can be considered as being supplied from the stator. For convenience, when the entire core loss is known either by calculation or test, the assumption can be made that it is divided equally between the two axes (more usually 55 per cent to the main, and 45 per cent to the cross-axis). By so doing, the term jX_m can be replaced by $Z_m = X_m(\sin\psi + j\cos\psi)$ where ψ is the electrical angle of hysteretic advance. This replacement should be made in the three fundamental equations

where jX_m occurs only in the transformer voltages and not in the speed voltages. Simultaneous solution of these equations results in new expressions for primary and rotor currents, with core loss considered.

In the first edition of this text, after an analysis similar to that of Arnold, the core loss was considered by an impedance:

$$Z_0 = \frac{1}{\dfrac{1}{R_F} + \dfrac{1}{jX_m}} \qquad [406]$$

wherein R_F = the equivalent resistance in which one-half of the total core loss would occur

By this viewpoint, the expressions for rotor currents in the two axes in terms of counter emf become

$$I_y = E\frac{(Z_0 + Z_2)(1 - S^2) + R_2S^2}{(Z_0 + Z_2)(Z_2 - jS^2X_2) + jR_2X_2S^2} \qquad [407]$$

$$I_x = E\frac{-jR_2S}{(Z_0 + Z_2)(Z_2 - jS^2X_2) + jR_2X_2S^2} \qquad [408]$$

278. Calculation Procedure. At this point several methods of procedure are possible in order to determine the operating characteristics of a single-phase motor at any desired speed. The procedures will be illustrated by a numerical example.

A 4-pole, 60-cycle, 110-volt, single-phase, $\frac{1}{4}$-hp induction motor displays the following values:

$R_1 = 1.86$	$R_2 = 3.56$	$X_1 = X_2 = 2.56$
$X_0 = 55.8$	$X_m = 53.5$	$F_e = 36$ watts (core loss)
Friction and windage = 13.5 watts		

The complete performance will be calculated at a slip of 5 per cent, obtaining first the three currents.

From equation 402:

$$U = 12{,}725$$

From equation 403:

$$W = 108$$

From equation 399:

$$I_1 = -110\,\frac{-(3.56)^2 + (0.1)55.8^2 - j2 \times 3.56 \times 55.8}{12{,}725 + j108}$$

$$I_1 = -2.54 + j3.45$$

$$I_1 = 4.28 \text{ amperes}$$

From equation 400:

$$I_y = -110 \times 53.5 \frac{(0.1)55.8 - j3.56}{12{,}725 + j108}$$

$$I_y = -2.54 + j1.64$$

$$I_y = 3.06 \text{ amperes}$$

From equation 401:

$$I_x = 110 \frac{0.95 \times 53.5 \times 3.56}{12{,}725 + j108}$$

$$= 1.565 - j0.0133$$

$$I_x = 1.565 \text{ amperes}$$

To obtain the counter emf:

$$I_\phi = I_1 - I_y = (-2.54 + j3.45) - (-2.54 + j1.64)$$

$$= j1.81$$

$$-jX_m(I_\phi) = (-j53.5)(j1.81) \quad \text{or} \quad 96.9 \text{ volts}$$

The rotor input:

$$E \times I''_y \times \cos\theta_2 \text{ watts} \tag{409}$$

Since E, the counter emf, is the voltage of reference, the in-phase portion of I'_y is simply the real component or 2.54 amperes. Hence, for rotor input:

$$96.9 \times 2.54 = 245 \text{ watts}$$

The rotor input converted to mechanical form:

$$\text{Rotor input} - (I''_y)^2R_2 - I_x{}^2R_2 \tag{410}$$

$$245 - 3.06^2 \times 3.56 - 1.565^2 \times 3.56 \quad \text{or} \quad 203 \text{ watts}$$

The core loss in the cross-axis is not being considered, and hence the net output is

$$\text{Mechanical power} - \text{friction and windage}$$

$$203 - 13.5 = 189.5 \text{ watts} \quad \text{or} \quad 0.254 \text{ hp}$$

The input to the stator can be obtained in several ways.

$$\text{Input} = V \times I_y \times \cos(\text{angle } V \text{ and } I_y) \tag{411}$$

It will be necessary to obtain the expression for applied voltage.

From equation 389:

$$\begin{aligned} V &= I_1 Z_1 + jX_m(I_1 - I_y) \\ &= (-2.54 + j3.45)(1.86 + j2.56) - 96.9 \\ &= -13.59 - j0.07 - 96.9 \\ &= -110.49 - j0.07 \text{ volts} \end{aligned}$$

Note that this calculation does not result in exactly 110 volts, owing to cumulative errors in calculation. The out-of-phase component is so small in this case that E and V are practically in phase. Hence the angle between V and I_y equals the angle between E and I_y, and so:

$$\text{Input} = -110 \times (-2.54) \quad \text{or} \quad 279 \text{ watts}$$

As a check, the stator copper loss can be calculated:

$$I_1^2 \times R_1 = 4.28^2 \times 1.86 \quad \text{or} \quad 34 \text{ watts}$$

$$\text{Motor input} = \text{rotor input} + \text{stator copper loss}$$

$$245 + 34 = 279 \text{ watts}$$

$$\begin{aligned} \text{Apparent power} &= I_1 \times V \\ &= 4.28 \times 110 \quad \text{or} \quad 471.5 \text{ volt-amperes} \end{aligned}$$

$$\begin{aligned} \text{Power factor} &= \frac{\text{watts}}{\text{apparent power}} \\ &= \frac{279}{471.5} \quad \text{or} \quad 0.589 \end{aligned}$$

$$\begin{aligned} \text{Efficiency} &= \frac{\text{rotor output}}{\text{stator input}} \\ &= \frac{189.5}{279} \quad \text{or} \quad 0.679 \end{aligned}$$

279. Treatment of Core Loss. Mention has already been made of the fact that, in place of the magnetizing reactance X_m in each axis, an impedance can be substituted containing a resistance in which a copper loss occurs. The correct calculation of this resistance results in a copper loss equivalent to half of the core loss, the basic idea being that approximately half of the core loss occurs in each axis.

Another method which does not interfere with the equations as shown (without core loss) but which is only approximate, is based on the idea of subtracting half of the core loss from the rotor output and adding the

other half to the stator input. This method is simple to apply and often acceptable.

A third method sometimes suggested is the simple addition of the entire core loss to the stator input, adding an appropriate in-place component of current. This is more suitable for conditions of heavy load on the motor.

280. Ratios of Constants. In some of the equations which follow, and also in dealing with test data, it will be found convenient to use certain combinations and ratios of winding constants. Thus:

Let

$$X'_0 = X_m + X_1 \qquad [412]$$

and

$$X''_0 = X_m + X_2 \qquad [413]$$

or, if it is assumed that the leakage reactances of the primary and secondary are equal,

$$X_0 = X'_0 = X''_0 \qquad [414]$$

Such a term is a simple convenience for combining terms.

We can also define the ratio of mutual reactance to *total* primary reactance as K_p. That is,

$$K_p = \frac{X_m}{X'_0} \qquad [415]$$

Also let

$$K_s = \frac{X_m}{X''_0} \qquad [416]$$

Let the product of these factor be designated K_r. Then:

$$K_r = K_p \times K_s = \frac{X_m^{\ 2}}{X_0^{\ 2}} \qquad [417]$$

or

$$K_r = \frac{X_0 - X}{X_0} \qquad [418]$$

where

$$X = \text{the total leakage reactance}$$

$$\approx X_1 + X_2 \qquad [419]$$

The terms K_p, K_s, and K_r are really coupling coefficients, leakage factors, or other designated terms. We need not concern ourselves with these details but can look on them as ratios used for numerical simplification of formulas. Their application will be noted in the material which follows.

281. Torque. The torque of the single-phase induction motor can be obtained by subtracting the rotor losses from the rotor input and dividing the resulting watt output by S. This gives the torque in synchronous watts. A more direct method takes into account the fact that torque results from the effect of the current in one axis upon the flux of the other axis. To evaluate, we will start with the y-axis current and the x-axis flux.

$$\phi_x \text{ is proportional to } I_x(X_m + X_2) = I_x(X_0)$$

The numerical value of I_xX_0 is

$$\frac{VX_mR_2SX_0}{\sqrt{U^2 + W^2}} \qquad \text{volts} \qquad [420]$$

The in-phase portion of the current I_y is

$$VX_m \frac{X_0(1 - S^2)}{\sqrt{U^2 + W^2}} \qquad \text{amperes} \qquad [421]$$

Their product is the useful torque in synchronous watts:

$$T_{\text{useful}} = \frac{V^2X_m{}^2X_0{}^2R_2S(1 - S^2)}{U^2 + W^2} \qquad [422]$$

The x-axis current and the y-axis flux react to produce a retarding torque, which is a measure of the cross-axis losses (which would include the iron loss of that axis had they been considered in the equations).

The y-axis flux is proportional to $X_m(I_1 - I_y)$, and, with the action of the leakage flux considered as it has been in this analysis, the term I''_yX_2 should be added. Since I''_y and I_y are exactly opposite, ϕ_y is proportional to $X_m(I_1 - I_y) - I_yX_2$. Since $X_m + X_2 = X_0$, this expression becomes

$$X_mI_1 - I_yX_0 = X_m\left(-V\frac{-R_2{}^2 + (1 - S^2)X_0{}^2 - j2R_2X_0}{U + jW}\right) - X_0\left(-VX_m\frac{(1 - S^2)X_0 - jR_2}{U + jW}\right) \qquad [423]$$

Combining and using the real terms,[4] the final expression is

$$\frac{VX_mR_2{}^2}{\sqrt{U^2 + W^2}}$$

[4] The current I_x with which ϕ_y is associated contains no j term and is the axis of reference for this calculation. Since the projection of ϕ_y on I_x is all that is significant in producing torque, the j terms are omitted.

This reacts with the x-axis current:

$$\mathrm{I}_x = \frac{VSX_mR_2}{\sqrt{U^2 + W^2}} \qquad [424]$$

Their product is the retarding torque in synchronous watts.

$$T_{\text{retarding}} = \frac{V^2X_m{}^2R_2{}^3S}{U^2 + W^2} = \text{cross-axis losses} \qquad [425]$$

The net torque developed is then their difference:

$$T = \frac{V^2X_m{}^2R_2S[(1 - S^2)X_0{}^2 - R_2{}^2]}{U^2 + W^2} \text{ synchronous watts} \qquad [426]$$

282. No-load Conditions. Examination of the torque formula indicates that the torque must be zero when

$$\sqrt{1 - S^2} = \frac{R_2}{X_0} \qquad [427]$$

Applying this to the motor just considered:

$$\frac{R_2}{X_0} = \frac{3.56}{55.8} \quad \text{or} \quad 0.064$$

For $\sqrt{1 - S^2}$ to equal 0.064, S equals 0.998.

This indicates a fact well known experimentally: The no-load speed of the single-phase induction motor is not synchronous speed as is the ideal case for the polyphase machine. Actually, the no-load speed of the single-phase motor is less than that indicated in the example because of the further retarding effect of cross-axis losses, as well as the friction and windage.

Ideal no-load speed:

$$S_0 = \sqrt{1 - \left(\frac{R_2}{X_0}\right)^2} \qquad [428]$$

If this value of S is substituted in the current equation, we obtain an expression for no-load current.

$$I_0 = \frac{2VX_0}{2R_1X_0 + \dfrac{R_2X_m{}^2}{X_0} + jX_m[X_m + 2(X_1 + X_2)]} \qquad [429]$$

$$= \frac{2V}{2R_1 + K_rR_2 + j[K_rX_0 + 2\sqrt{K_r}(X_1 + X_2)]} \qquad [430]$$

In dealing with test data we need an expression for no-load current. Once its value is known, X_0 or X_m can be calculated. The above

expression is too unwieldy, and a simpler equation will be obtained by assuming the no-load speed to be synchronous ($S = 1$). Then:

$$I_0 \approx \frac{2V}{X_0(2 - K_r)} \tag{431}$$

This considers the no-load current as made up of only the out-of-phase or magnetizing component and as capable of being divided into two parts thus:

$$I_0 \approx \frac{V}{X_0} + \frac{V}{X_0}\left(\frac{K_r}{2 - K_r}\right) \tag{432}$$

In the polyphase induction motor, the no-load current, or at least its quadrature lagging component, is V/X_0. If the magnetizing current of *both axes* is considered for the single-phase induction motor, a first approximation would be the assumption that

$$I_0 \approx 2\frac{V}{X_0} \tag{433}$$

Actually, because of the reduction effective between the main and the cross-axis values, the magnetizing current of the cross-axis at no load is more nearly

$$I_{x0} \approx \frac{V}{X_0}\left(\frac{K_r}{2 - K_r}\right) \tag{434}$$

This is still an approximation, but checks on many designs against test data indicate a good degree of accuracy for equations 432 and 434. The additional fact is now brought out that a single-phase induction motor may take almost twice the no-load current of a similar polyphase machine (when the latter is in equivalent single-phase values) with resulting lower pf at all loads.

283. Example of Torque Calculation. Making use of the $\frac{1}{4}$ hp motor data shown in Articles 266 and 278, the torque in synchronous watts, developed at a slip of 5 per cent, will be (equation 426):

$$T = \frac{110^2 \times 53.5^2 \times 3.56 \times 0.95(0.1 \times 55.8^2 - 3.56^2)}{12{,}725^2 + 108^2}$$

$$= 215 \text{ synchronous watts}$$

$$\text{The power developed} = S \times \text{synchronous watts}$$

$$= 0.95 \times 215 \quad \text{or} \quad 204 \text{ watts}$$

From this the friction loss is subtracted, leaving an output of 190.5 watts versus 189.5 by the previous calculations.

284. Maximum Torque.[5] No simple expression exists for either the maximum torque or the speed at which the maximum torque occurs, as is true for the polyphase motor. This torque is not independent of the rotor resistance; increased rotor resistance reduces the maximum value obtainable and also the speed at which it occurs (S_{po}). Because of the

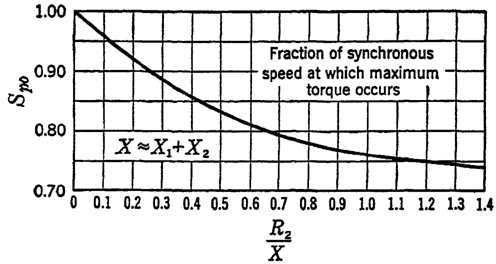


FIG. 227.

complex nature of the derivation no expression will be given here, but by substituting the correct value of S in the usual analyses, the maximum torque can be calculated. Satisfactory, approximate values of S are shown in Fig. 227.

[5] Maximum torque equation is derived in "Single-phase Induction Motor Performance," by A. F. Puchstein and T. C. Lloyd, *Elec. Eng.*, October, 1937.

CHAPTER XXXIV

THE EQUIVALENT CIRCUIT. CROSS-FIELD THEORY

285. Chapter Outline.

Derivation of One Equivalent Circuit; Core Loss Omitted.
Second Derivation; Core Loss Considered.
Example.
No-load Current and Losses.
Contrasted Results.

286. Introduction. A great variety of equivalent circuits are possible for the single-phase induction motor, the multiplicity arising from the different viewpoints and from the various degrees of approximation which are assumed for convenience in setting up the circuits. Since any combination of parallel impedances can be replaced by equivalent series impedances, it is readily possible to represent equivalent networks in a number of different ways.[1]

Our first concern will be with the setting up of a network equivalent to the cross-field equations just presented. This will be done by assuming that the total rotor circuit is made up of two parallel impedances, in series with an additional impedance. If this assumption is in error, the operations will indicate that one or more of the impedances reduces to zero. The rotor circuit assumed is shown in Fig. 228*a*. The current taken by this circuit, with an impressed voltage of E, will be I_y, and we know the relationship from equation 404 to be

$$I_y = -E\left[\frac{R_2 + jX_0(1 - S^2)}{R_2{}^2 - X_2(1 - S^2)X_0 + j(R_2X_0 + R_2X_2)}\right] \qquad [435]$$

The reciprocal of the expression in brackets must equal the impedance of the equivalent network. Referring to Fig. 228*a*, we know that the equivalent impedance is

$$Z_T = \frac{Z_aZ_b + Z_aZ_c + Z_bZ_c}{Z_b + Z_c} \qquad [436]$$

[1] V. Karapetoff, "On the Equivalence of the Two Theories of the Single-phase Induction Motor," *J. Am. Inst. Elec. Engrs.*, August, 1921.

The problem now is to determine Z_a, Z_b, and Z_c, so that

$$Z_b + Z_c = R_2 + jX_0(1 - S^2) \qquad [437]$$

$$Z_aZ_b + Z_aZ_c + Z_bZ_c = R_2^{\,2} - X_2(1 - S^2)X_0 + j(R_2X_0 + R_2X_2) \qquad [438]$$

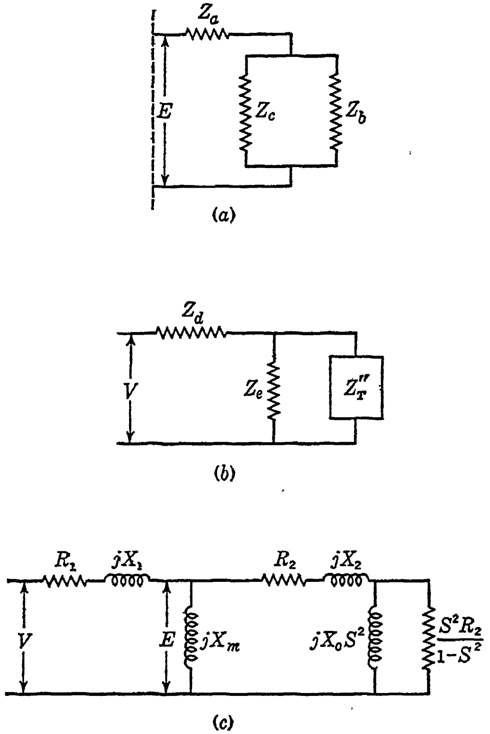


FIG. 228. Derivation of the equivalent circuit.

To do so, multiply both numerator and denominator of equation 435 by $S^2/1 - S^2$. Note also that

$$\frac{R_2}{1 - S^2} = \frac{S^2R_2}{1 - S^2} + R_2 \qquad [439]$$

We then have for $Z_b + Z_c$:

$$\frac{S^2R_2}{(1 - S^2)} + jX_0S^2 \qquad [440]$$

and for $Z_aZ_b + Z_aZ_c + Z_bZ_c$:

$$\frac{S^2R_2^{\,2}}{1 - S^2} - X_2X_0S^2 + j\,\frac{R_2X_0S^2}{1 - S^2} + j\,\frac{R_2X_2S^2}{1 - S^2} \qquad [441]$$

Expanding the third term by equation 439,

$$\frac{S^2R_2{}^2}{1-S^2} - X_2X_0S^2 + jX_0S^2\left(\frac{S^2R_2}{1-S^2} + R_2\right) + jX_2\frac{S^2R_2}{1-S^2}$$

Rearranging terms,

$$R_2\frac{S^2R_2}{1-S^2} + jX_2\frac{S^2R_2}{1-S^2} + R_2(jX_0S^2) + jX_2(jX_0S^2) + (jX_0S^2)\left(\frac{S^2R_2}{1-S^2}\right)$$

For the final expressions we have

$$Z_b + Z_c = \frac{S^2R_2}{1-S^2} + jX_0S^2 \qquad [442]$$

$$Z_aZ_b + Z_aZ_c + Z_bZ_c$$

$$= (R_2 + jX_2)\frac{S^2R_2}{1-S^2} + (R_2 + jX_2)(jX_0S^2) + \frac{S^2R_2}{1-S^2}(jX_0S^2) \qquad [443]$$

Now by similarity it is plain that:

$$Z_a = R_2 + jX_2 \qquad [444]$$

$$Z_b = \frac{S^2R_2}{1-S^2} \qquad [445]$$

$$Z_c = jX_0S^2 \qquad [446]$$

This same process can be used with the equation for I_1 and the entire equivalent network to evaluate Z_d, Z_e, and Z''_T in Fig. 228*b*. However, we know from the vector diagram of Fig. 226*a* and the relationships of equation 389 that the primary impedance must be connected in series with a magnetizing branch which is paralleled with the rotor circuit. Hence, without further work, we can construct the entire circuit as shown in *c*.

287. Equivalent Circuit with Core Loss Considered. To illustrate an additional equivalent circuit, and at the same time consider the core losses, a further calculation will be made from the fundamental equations. These will be kept in terms of E_{TMy} or E. From equation 393:

$$I_xZ_2 = SX_m(I_1 - I_y) - jX_mI_x - SI_yX_2$$

$$E = -jX_m(I_1 - I_y)$$

$$I''_y = -I_y$$

To consider the core losses, jX_m is replaced by $R_n + jX_n$. Hence:

$$I_xZ_2 = jSE - I_x(R_n + jX_n) + SI''_yX_2 \qquad [447]$$

Let

$$R_n + jX_n = Z_n$$

Then:

$$I_x(Z_n + Z_2) = jS(E - jI''_y X_2) \tag{448}$$

or

$$I_x = \frac{jS(E - jI''_y X_2)}{Z_n + Z_2} \tag{449}$$

From equation 398 we know that the main-axis relationships are

$$I_y Z_2 = jX_m(I_1 - I_y) + SI_x(X_m + X_2)$$

or

$$-I''_y Z_2 = -E + SI_x(X_m + X_2)$$

Considering the core losses,

$$E - I''_y Z_2 = -jSI_x(R_n + jX_n + jX_2) \tag{450}$$

Solve for I_y by substituting for I_x from equation 449:

$$E - I''_y Z_2 = -jS(Z_n + jX_2)\,\frac{jS(E - jI''_y X_2)}{(Z_n + Z_2)}$$

From this,

$$I''_y = E\,\frac{(1 - S^2)(Z_n + Z_2) + R_2 S^2}{(Z_2 - jX_2 S^2)(Z_n + Z_2) + jR_2 X_2 S^2} \tag{451}$$

Solving for I_x,

$$I_x = E\,\frac{-jR_2 S}{(Z_2 - jX_2 S^2)(Z_n + Z_2) + jR_2 X_2 S^2} \tag{452}$$

Note that expressions for I''_y and I_x have already been obtained in Article 277 in terms of counter emf. In the previous case the core losses were not considered, but even so, the terms are combined in a very different manner in the final equations.

288. The Equivalent Circuit. The equivalent circuit for the rotor only will be set up, using these equations. Again we will assume that, since I''_y is the current that reflects into the stator, the impedance of the combined rotor circuits is the reciprocal of the factor used with E in equation 451. This circuit will be assumed to be made up of a series impedance Z_a, with two parallel impedances Z_b and Z_c as shown in Fig. 228*a*.

Since

$$Z_T = \frac{Z_a Z_b + Z_a Z_c + Z_b Z_c}{Z_b + Z_c}$$

the problem is to determine Z_a, Z_b, and Z_c, so that

$$Z_b + Z_c = (1 - S^2)(Z_n + Z_2) + R_2S^2 \qquad [453]$$

and

$$Z_aZ_b + Z_aZ_c + Z_bZ_c = (Z_2 - jX_2S^2)(Z_n + Z_2) + jR_2X_2S^2 \qquad [454]$$

Divide both numerator and denominator by $S^2(1 - S^2)$. Then

$$Z_b + Z_c = \frac{Z_n + Z_2}{S^2} + \frac{R_2}{1 - S^2} \qquad [455]$$

and for the numerator,

$$\frac{R_2 + jX_2 - jX_2S^2}{S^2(1 - S^2)}(Z_n + Z_2) + j\frac{R_2X_2}{(1 - S^2)}$$

$$= \left[\frac{R_2}{S^2(1 - S^2)} + j\frac{X_2(1 - S^2)}{S^2(1 - S^2)}\right](Z_n + Z_2) + j\frac{R_2X_2}{(1 - S^2)} \qquad [456]$$

Rearranging:

$$jX_2\frac{R_2}{(1 - S^2)} + jX_2\frac{1}{S^2}(Z_n + Z_2) + \frac{R_2}{(1 - S^2)}\frac{1}{S^2}(Z_n + Z_2)$$
$$= Z_aZ_b + Z_aZ_c + Z_bZ_c$$

From which it follows that

$$Z_a = jX_2 \qquad [457]$$

$$Z_b = \frac{R_2}{1 - S^2} \qquad [458]$$

$$Z_c = \frac{1}{S^2}(Z_n + Z_2) \qquad [459]$$

These values are located on Fig. 229*a* as shown. Following the usual procedure of adding the main-axis magnetizing branch and the primary impedance, the completed equivalent circuit is shown in *b*.

289. Method and Example. To illustrate the method of evaluating the impedance Z_n and determining the performance, consider the $\frac{1}{4}$-hp single-phase induction motor used previously (Article 278).

$$X_0 = 55.8 \qquad X_m = 53.5 \qquad Z_1 = 1.86 + j2.56$$
$$\text{Fe} = 36 \text{ watts} \qquad Z_2 = 3.56 + j2.56$$

Assuming that half of the core loss takes place in each axis and that E is 100 volts

$$I_F \approx \frac{0.5 \text{ Fe}}{100} \quad \text{or} \quad 0.180 \text{ ampere}$$

Equivalent resistance causing the core loss:

$$R_F = \frac{0.5\text{ Fe}}{I_F^2} = \frac{18}{0.18^2} \quad \text{or} \quad 555\text{ ohms}$$

The parallel resistance R_F and reactance X_m are equivalent to the series impedance Z_n.

$$Z_n = \frac{1}{\dfrac{1}{R_F} + \dfrac{1}{jX_m}} \quad \text{or} \quad 5.1 + j53.0\text{ ohms} \tag{460}$$

Refer to Fig. 229*b* and note the letters whereby various parts of the circuit will be identified. Although there are many methods for attack-

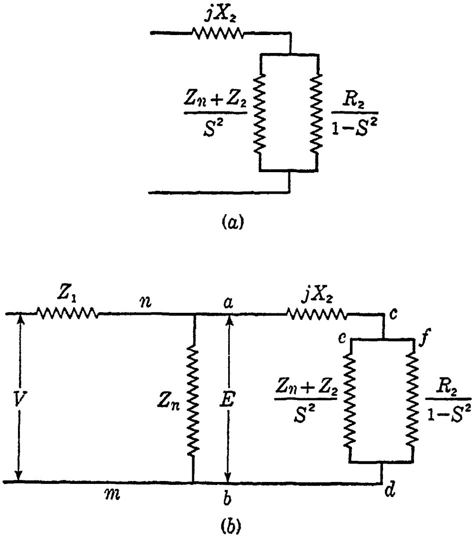


FIG. 229. Derivation of the equivalent circuit with core loss considered.

ing such a circuit problem, we will proceed on the assumption that E is 100 volts, later correcting by proportion. For a slip of 5 per cent, S equals 0.95.

$$\frac{Z_n + Z_2}{S^2} = 9.64 + j61.9$$

$$\frac{R_2}{1 - S^2} = 35.6$$

$$Z_{cd} = 25.8 + j13.4$$

$$Z_{ab} = 25.8 + j15.96$$

$$\text{Rotor current} = \frac{E}{Z_{ab}} = 2.8 - j1.74$$

$$\text{Magnetizing branch current} = \frac{E}{Z_n} = 0.18 - j1.87$$

$$\text{Primary current} = \text{sum} = 2.98 - j3.61$$

$$\text{Stator impedance drop} = I_1 Z_1 = 14.8 + j0.95$$

$$\text{Applied voltage } V = 114.8 + j0.95 \quad \text{or} \quad 114.8$$

Correction factor for actual applied voltage of 110:

$$\frac{110}{114.8} = 0.96$$

$$\text{Correct primary current } I_1 = 2.86 - j3.465 \quad \text{or} \quad 4.5 \text{ amperes}$$

$$\text{Correct rotor current } I_{ac} = 2.69 - j1.67 \quad \text{or} \quad 3.162 \text{ amperes}$$

Since the counter emf and applied voltage are so nearly in phase, the in-phase component of primary current will also be considered as in phase with applied volts.

$$\text{Watts input} = V \times I_1 \text{ (in phase)} = 110 \times 2.86 \quad \text{or} \quad 315 \text{ watts}$$

The losses will now be determined. First determine the voltage E_{cd} to determine the individual currents.

$$I_{ac} \times jX_2 = 4.29 + j6.9$$

$$E_{cd} = 96 - (4.29 + j6.9) = 91.71 - j6.9$$

$$I_{fd} = \frac{E_{cd}}{Z_{fd}} = 2.56 - j0.193 \quad \text{or} \quad 2.562 \text{ amperes}$$

$$I_{ed} = \frac{E_{cd}}{Z_{ed}} = 0.13 - j1.477 \quad \text{or} \quad 1.48 \text{ amperes}$$

$$\text{Main-axis rotor losses} = I_{fd}^2 R_2 \quad \text{or} \quad 23.4 \text{ watts}$$

$$\text{Cross-axis core loss} = I_{ed}^2 R_n \quad \text{or} \quad 11.18 \text{ watts}$$

$$\text{Cross-axis copper loss} = I_{ed}^2 R_2 \quad \text{or} \quad 7.8 \text{ watts}$$

$$\text{Total rotor losses} = 42.38 \text{ watts}$$

To determine the stator losses:

$$I_{\text{mag}} \text{ (main axis)} = \frac{E}{Z_n} = 0.1728 - j1.796 \quad \text{or} \quad 1.805$$

$$I_{\text{mag}}{}^2 \times R_n = 16.6 \text{ watts}$$

$$I_1{}^2R_1 = 37.7 \text{ watts}$$

$$\text{Total stator losses} = 54.3 \text{ watts}$$

$$\text{Input to rotor} = \text{motor input—stator losses}$$

$$\text{Input to rotor} = 260.7 \text{ watts}$$

$$\text{Power developed} = \text{rotor input—rotor losses}$$

$$= 218.3 \text{ watts}$$

Subtracting friction and windage loss of 13.5 watts leaves a net output of 204.8 watts or 0.274 hp.

$$\text{Efficiency} = \frac{204.8}{315} \quad \text{or} \quad 0.647$$

$$\text{Power factor} = \frac{315}{110 \times 4.5} \quad \text{or} \quad 0.635$$

Note that this same motor was used with core losses neglected in the analysis of Article 278. In that case the pf was lower and the efficiency was higher. These would be the natural effects of omitting a loss component.

290. No-load Current and Losses. Although the single-phase induction motor does not run at synchronous speed at no load, if it is assumed to do so, the no-load current can be determined conveniently with close approximation. With $S = 1$, the current through $R_2/(1 - S)$ is obviously zero, and the impedance of the rotor circuit is $Z_n + Z_2 + jX_2$. Applied to this example we have

$$I_{\text{no load}} = 1.252 - j3.49 \quad \text{or} \quad 3.71 \text{ amperes}$$

$$I_{0_{\text{main}}} = 0.178 - j1.85 \quad \text{or} \quad 1.86 \text{ amperes}$$

$$I_{0_{\text{cross}}} = I_{ed} = 1.074 - j1.64 \quad \text{or} \quad 1.96 \text{ amperes}$$

If the iron loss component is neglected and the approximations of Article 282 are applied,

$$I_\phi = \frac{2V}{X_0(2 - K_r)} \quad \text{or} \quad 3.64 \text{ amperes (versus 3.71)}$$

$$I_{y0} = \frac{V}{X_0} \quad \text{or} \quad 1.97 \text{ amperes (versus 1.86)}$$

$$I_{x0} = \frac{V}{X_0}\left(\frac{K_r}{2 - K_r}\right) \quad \text{or} \quad 1.67 \text{ amperes (versus 1.96)}$$

These approximations neglect the core loss in the cross-axis of the rotor, or in fact, any in-phase component of current.

At no load, the copper loss in the rotor is not zero (as is theoretically the case in the polyphase machine) since the rotor cross-axis always carries a magnetizing current. In this case the rotor copper loss at no load is

$$(I_{0_{\text{cross}}} \quad \text{or} \quad I_{x0})^2 \times R_2$$

Using the more accurate value, we obtain

$$I_0^2 R_2 = 1.96^2 \times 3.56 \quad \text{or} \quad 13.7 \text{ watts}$$

Note that this is higher than the cross-axis copper loss under load.

TABLE XVII

CONTRASTED PERFORMANCE *

($\frac{1}{4}$-hp, 4-pole, 60-cycle, 110-volt, Single-phase Induction Motor)

Method	Double-revolving Field		Cross-field	
Item	Without core loss	With core loss	Without core loss	With core loss (equivalent circuit)
Slip	0.05	0.05	0.05	0.05
I_1	4.27	4.46	4.28	4.50
Input	275.0	314.0	279.0	315.0
Output	188.5	181.5	189.5	204.8
Efficiency	0.68	0.578	0.679	0.647
Pf	0.585	0.635	0.589	0.635

* C. T. Button, "Single-phase Motor Theory—A Correlation of the Cross-field and the Revolving Field Concepts." *A.I.E.E. Paper*, pp. 40–151, 1940.

CHAPTER XXXV

DETERMINATION OF SINGLE-PHASE INDUCTION-MOTOR CONSTANTS FROM TESTS

291. Chapter Outline.

Running-light and Rotor-blocked Tests.
Determination of Resistance and Reactance Values.
Losses.

292. Description of Tests. By applying a load to the single-phase induction motor, direct readings can be made of output, input, speed, and current. From these readings, efficiency, losses, and pf can be obtained. Because such motors are usually small in ratings, these direct readings form the common method of obtaining the performance.

However, for the purposes of checking designs and exploring performance calculations, it is necessary to make various tests for determining the machine constants or parameters.

The following tests are usually made.

(*a*) Measurement of the resistance of each of the stator windings. (Split-phase or capacitor start; in the latter case the condenser is omitted.)

(*b*) Blocked rotor. (1) The input volts, amperes and watts are measured for the main winding with the auxiliary winding disconnected. (2) The test is repeated with the auxiliary winding, the main winding being disconnected. When it is undesirable to use full-rated voltage because of excessive short-circuit current and heating effects, about 40 per cent of rated voltage is often used.

(*c*) No load. This test is made at rated voltage, exciting the main winding only. Input watts, volts, and amperes are measured as well as the speed.

If the voltage is reduced until the motor will just run, the power input can be used as an approximate measure of the friction and windage losses. A more accurate method involves the extropolation of volts-versus-watts curve until the zero voltage intercept is obtained. This method has been described in connection with polyphase motors.

Once such data have been obtained, several methods of treatment are available in order to determine the parameters. They will be shown below.

293. Test Results. A $\frac{1}{4}$-hp, 110-volt, single-phase, 60-cycle, 4-pole motor shows the following test results:

(*a*) Blocked rotor. Main winding.

$V_L = 40 \qquad I_L = 5.45 \qquad W_L = 152 \qquad R_1 = 1.86$ ohms

Start winding.

$V_L = 30 \qquad I_L = 1.19 \qquad W_L = 33 \qquad R_1 = 17.0$ ohms

(*b*) No load. Main winding.

$V_n = 110 \qquad I_n = 3.48 \qquad W_n = 90$

Lowest operating voltage.

$V_n = 22 \qquad I_n = 0.905 \qquad W_n = 15$

First Method. For approximate work, this assumes a simple series circuit as equivalent to the motor.

$$Z_e = \frac{V_L}{I_L} = \frac{40}{5.45} \quad \text{or} \quad 7.32 \text{ ohms} \tag{461}$$

$$R_e = \frac{W_L}{I_L^2} = \frac{152}{5.45^2} \quad \text{or} \quad 5.11 \text{ ohms} \tag{462}$$

$$R_2 \approx R_e - R_1 \tag{463}$$

$$\approx 5.11 - 1.86 \quad \text{or} \quad 3.25 \text{ ohms}$$

$$X_e \approx \sqrt{Z_e^2 - R_e^2} \quad \text{or} \quad 5.24 \text{ ohms} \tag{464}$$

If the magnetizing reactance is assumed to be the same in both the cross and main axes,

$$X_0 \approx \frac{2V_n}{I_n} = \frac{2 \times 110}{3.48} \quad \text{or} \quad 63.2 \text{ ohms} \tag{465}$$

Second Method. For more accuracy, the presence of the magnetizing branch in parallel with the rotor circuit, should be taken into account. To do this conveniently we will use the flux factors described in Article 280.[1] Successive approximations will be required.

$$K_r = \frac{X_0 - X_e}{X_0} = \frac{63.2 - 5.24}{63.2} \quad \text{or} \quad 0.915 \tag{466}$$

[1] The use of such terms as $X_0 = X_m + X_1$ can be readily accepted as a convenient combination of constants. Similarly, the definition of K_r as $(X_0 - X_e)/X_0$ can be looked upon as a significant ratio expressing magnetic coupling. But the use of such flux factors as corrective measures for dealing with theoretically exact equivalent circuits

Refer to the expression for no-load magnetizing current (Article 282).

$$I_{\text{mag}_{\text{main}}} = \frac{I_n}{1 + \dfrac{K_r}{2 - K_r}} = I_n \frac{2}{2 - K_r} \tag{467}$$

$$= \frac{3.48}{1.844} \quad \text{or} \quad 1.885 \text{ amperes}$$

That is, of the total of 3.48 no-load amperes, 1.885 is assumed to be that required to magnetize the main axis only. The cross-axis magnetizing component is then

$$I_{\text{mag}_{\text{cross}}} = \frac{K_r}{2 - K_r} I_{\text{mag}_{\text{main}}} \tag{468}$$

$$= 0.844 \times 1.885 \quad \text{or} \quad 1.595 \text{ amperes}$$

Or, by direct subtraction,

$$I_{\text{mag}_{\text{cross}}} = I_n - I_{\text{mag}_{\text{main}}}$$

$$= 3.48 - 1.885 \quad \text{or} \quad 1.595$$

If the magnetizing current in the main axis is 1.885, it follows that a more accurate expression for X_0 would be

$$X_0 = \frac{V_n}{I_{\text{mag}}} = \frac{110}{1.885} \quad \text{or} \quad 58.3 \text{ ohms} \tag{469}$$

The second approximation for K_r is then

$$K_r = \frac{X_0 - X_e}{X_0} = \frac{58.3 - 5.24}{58.3} \quad \text{or} \quad 0.911$$

The rotor resistance in stator terms is then, more accurately,

$$R_2 = \frac{R_e - R_1}{K_r} = \frac{5.11 - 1.86}{0.911} \quad \text{or} \quad 3.56 \text{ ohms} \tag{470}$$

The equivalent reactance may be divided into its primary and secondary components, using the approximate relationship

$$X_1 = X_2 \approx \frac{X_e}{2} \tag{471}$$

is not obvious, but involves a rather obscure technique in so far as the physical interpretations are concerned. Rather than become involved in the extensive derivations required, it is better for the student to look upon the use of these terms here as merely corrective flux factors which do account for the effect of magnetizing current on the test readings.

but to correct for the presence of magnetizing reactance in the test readings, more correctly,

$$X_1 = X_2 = X_e \frac{1 - \sqrt{K_r}}{1 - K_r} \quad [472]$$

$$= 5.24 \frac{1 - \sqrt{0.911}}{1 - 0.911} \quad \text{or} \quad 2.65 \text{ ohms}$$

Third Method. Veinott suggests a test procedure as a part of the problem of separating the losses.[2] The procedure is outlined below, applied to these test figures.

First approximation:

$$R'_2 = \frac{W_L}{I_L^2} - R_1 \quad \text{or} \quad 3.25 \text{ ohms} \quad [473]$$

$$X'_e = \sqrt{\left(\frac{V_L}{I_L}\right)^2 - \left(\frac{W_L}{I_L^2}\right)^2} \quad \text{or} \quad 5.24 \text{ ohms} \quad [474]$$

Corrected:

$$X_e = \frac{V_n}{I_n} - \sqrt{\left(\frac{V_n}{I_n} - X'_e\right)^2 + (R'_2)^2} \quad \text{or} \quad 5.06 \text{ ohms} \quad [475]$$

$$X_0 = \frac{2V_n}{I_n} - X \quad \text{or} \quad 58.14 \text{ ohms} \quad [476]$$

$$R_2 = \frac{R'_2 X_0}{X_0 - X'_e} \quad \text{or} \quad 3.6 \text{ ohms} \quad [477]$$

$$K'_r = \frac{X_0 - X_e}{X_0} \quad \text{or} \quad 0.914$$

More accurately,

$$K_r = \frac{X_0 - X'_e}{X_0} \quad \text{or} \quad 0.905$$

Contrasted Results

Method	(1)	(2)	(3)
X_e	5.24	5.24	5.06
X_0	63.2	58.3	58.14
R_2	3.25	3.56	3.60
K_r		0.911	0.905

[2] C. G. Veinott, "Segregation of Losses in Single-phase Induction Motors," *Elec. Eng.*, December, 1935.

294. Tests with Repulsion-start Motors. If the motor to be tested utilizes the repulsion-start method, the no-load test is made in exactly the same way as the similar test described above. With the rotor blocked, the starting mechanism should also be blocked in the running position so that the values are obtained with the rotor closed as when operating normally. If the starting mechanism cannot be conveniently blocked, the same effect can be obtained by wrapping the commutator with several turns of bare copper wire so as effectively to short-circuit it. The test data so obtained can be used as indicated above, to obtain the motor constants for the running connection.

CHAPTER XXXVI

CAPACITOR MOTORS

295. Chapter Outline.

Capacitor-motor Construction and Characteristics.
Sizes and Applications.
Cross-field Theory of Analysis.
Special Starting Methods.
Speed Control of Single-phase Induction Motors.

296. Motor Construction and Characteristics. We have already discussed briefly the use of capacitors for starting duty on single-phase induction motors. In such cases, many microfarads are required to permit a large current in the starting winding, displaced approximately 90 electrical degrees in time from the current of the main winding. If a condenser is connected permanently in the auxiliary or start winding of such a motor, the current in this winding will be of such value and time phase as to produce an imperfect two-phase motor for running as well as starting duty.

Such a machine is called a capacitor motor, or more generally a permanently split capacitor motor. If the two windings are identical, a comparatively large condenser is needed to bring about balanced two-phase operation. Even then, perfect two-phase conditions occur at only one value of load for any given condenser value. As the load changes, unbalances in current and phase angle take place.

Commercially, capacitor motors are usually built with more turns of comparatively smaller wire in the auxiliary than in the main winding. This saves condenser capacitance. Furthermore, capacitor motors are rarely designed for balanced operation because of the capacitor cost.[1]

A condenser which results in satisfactory running performance for the motor is usually only of small capacity, and hence does not result in large starting torque. Commercial designs are nearly always of fractional

[1] P. H. Trickey, "Design of Capacitor Motors for Balanced Operation," *Trans. A.I.E.E.*, Vol. 51, 1932.

P. H. Trickey, "The Equal-volt-ampere Method of Designing Capacitor Motors," *Elec. Eng.*, November, 1941.

horsepower ratings, requiring 2 to 20 μf and giving starting torques of 35 to 50 per cent of full-load torque. This might be only one-tenth of the starting torque found in capacitor-start, induction-run designs. Furthermore, since the condenser is connected in the circuit permanently (that is, so long as the motor is in operation), the intermittent-duty electrolytic condenser is unsuitable and oil-insulated, foil, paper condensers of comparatively large bulk per microfarad are required. Table XVIII shows reasonable commercial designs for 110-volt motors.

TABLE XVIII

PERMANENT-SPLIT CAPACITOR MOTORS

(Oil Condensers)

Hp	Rpm	Typical Condenser Capacity*	Approximate Dimensions of Oil Condenser (Inches)
$\frac{1}{100}$ to $\frac{1}{40}$	3400 1700 1100 850	2 to 3 μf	
$\frac{1}{20}$	1725 1140 840	3 μf	$3\frac{1}{2} \times 3\frac{1}{2} \times 1\frac{1}{4}$ (330 volts)
$\frac{1}{12}$	1725 1140 840	5 μf	$3\frac{1}{2} \times 3\frac{1}{2} \times 2$ (330 volts)
$\frac{1}{8}$	1725 1140 840	5 μf	$3\frac{1}{2} \times 3\frac{1}{2} \times 2$ (330 volts)
$\frac{1}{6}$	1725 1140 840	8 μf	$3\frac{1}{2} \times 3\frac{1}{2} \times 2$ (220 volts)
$\frac{1}{4}$	1725 1140 840	8 μf	$3\frac{1}{2} \times 3\frac{1}{2} \times 2$ (220 volts)
$\frac{1}{3}$	1725 1140 840	15 μf	$4\frac{1}{2} \times 3\frac{3}{4} \times 2$
$\frac{1}{2}$	1725 1140	15 μf	$4\frac{1}{2} \times 3\frac{3}{4} \times 2$
$\frac{3}{4}$	1725	20 μf	$3\frac{1}{2} \times 3\frac{1}{2} \times 4\frac{1}{2}$ (330 volts)

* Yield starting torques of only 35 to 50 per cent of full-load torque.

Applications. The past ten years have seen a great increase in the use of this type of motor for many applications, although the largest single field is probably for fan drive. This is true for small desk fans as well as for larger ventilating types of approximately $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$ hp. For small ratings, the permanently split capacitor motor is about twice as efficient as the shaded-pole motor which it often replaces, but it offers other advantages as well. Single-phase induction motors, in general, produce a pulsating torque, resulting in motor vibration and noise, which is amplified by the fan blades. Approaching, as it does, a two-phase motor, this type develops a more constant instantaneous torque in time, with little cogging, and hence is nearly always quieter than the ordinary single-phase machines. It also presents speed-control possibilities which will be discussed later.

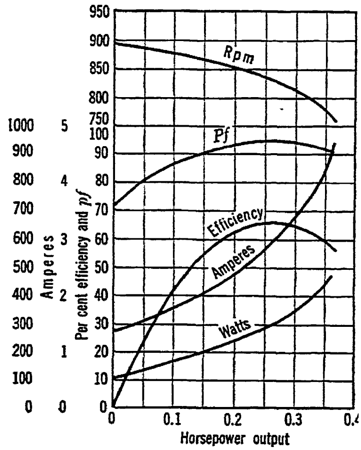


FIG. 230. Slow-speed motor for fan duty, $\frac{1}{4}$ hp. This is a permanent-split capacitor motor, using an oil condenser large enough to give unusually high pf. The starting torque of 10.7 oz-ft is less than half of the full-load torque.

297. Methods of Analysis. Except for balanced operation, no simple method is available for determining the performance from machine constants. No simple equivalent circuit can be drawn for this motor because of the interaction of the two fields. Calculations are complicated by the dissimilarity of the constants for the two windings.

The pioneer effort in English, for predicting motor performance, was based on the double-revolving-field theory,[2] which, incidentally, when solved for a slip of unity, presents an expression for starting torque of capacitor motors.[3]

A recent analysis, based on the cross-field theory, will be presented here.[4] The problem is to determine complete running performance of

[2] Wayne J. Morrill, "The Revolving Field Theory of the Capacitor Motor," *Trans. A.I.E.E.*, Vol. 48, April, 1929.

[3] A second analysis, generalized to consider a capacitor motor in which the main and start windings were not in quadrature, but making any angle with each other in space, was presented by the present writers: "Capacitor Motors with Windings Not in Quadrature," *Elec. Eng.*, November, 1935.

A similar analysis using symmetrical components is given by Lyon and Kingsley, "Analysis of Unsymmetrical Machines," *Elec. Eng.*, May, 1936.

[4] A. F. Puchstein and T. C. Lloyd, "The Cross-field Theory of the Capacitor Motor," *Trans. A.I.E.E.*, Vol. 60, pp. 58–61, 1941.

the motor for any assumed slip, given the capacitance and the constants of the windings.

298. The Cross-field Theory of the Capacitor Motor. The equations for the capacitor motor are somewhat similar to those for the plain single-phase motor, based on the cross-field theory. The two windings on the stator are assumed to be in space quadrature. The winding having the oil condenser in series is designated as the start winding although it is used continuously. The subscript S will be used for terms pertaining to this winding, contrasted to M for the main winding. The rotor bars will be assumed to generate speed voltages by cutting both rotor leakage and mutual or air-gap fluxes, as was done in the plain single-phase induction-motor equations.

All rotor values are given in main-winding-stator terms. Usually there are more turns on the start winding than on the main, so that rotor constants in start-winding terms would be higher (or at least different). The ratio is

$$a = \frac{\text{effective start-winding turns}}{\text{effective main-winding turns}} \qquad [478]$$

Now, in dealing with rotor currents which will be operative in the start axis, they should be multiplied by a before combining with other values. Similarly, any magnetizing reactance reflected to the rotor, operative in the start axis, should be divided by a^2 if it is necessary to combine its effect with main-axis terms. This does not hold for the rotor impedance since its values are in main-winding terms throughout.

With these preliminary statements, we can write the fundamental equations.

(*a*) The voltage applied to the main winding overcomes a local stator impedance drop and the counter emf built up by the mutual flux of the main axis.

The counter emf will be the reference vector. It is, numerically,

$$E_M = I_{\phi M} Z_M \qquad [479]$$

where

$I_{\phi M}$ = the magnetizing current of the main winding

Z_M = the magnetizing impedance of the main winding

If the core loss is neglected, Z_M reduces to jX_{mM}, where

jX_{mM} = the magnetizing reactance of the main winding

(*b*) The load will require a current component in the main or y axis which is designated I''_y. Reflected to the stator, this will be I_y, and the

total stator current of the main winding is the vector sum of this current and the "exciting-branch current." (See Fig. 232.)

$$I_M = I_y + I_{\phi M} \tag{480}$$

or

$$I_{\phi M} = I_M - I_y \tag{481}$$

Combining equations 479 and 481,

$$E_M = (I_M - I_y)Z_M \tag{482}$$

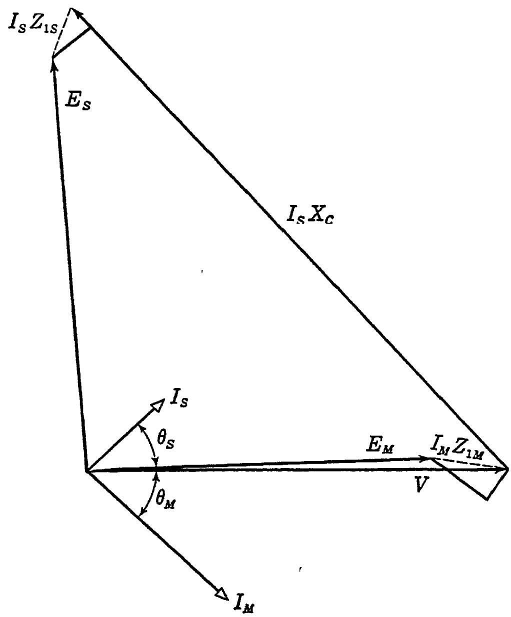


FIG. 231. Vector diagram showing stator currents and voltage components. Note on this analysis that the counter emf of the main axis is used as reference vector. Values are expressed by equations 483 and 489.

The equation for the condition expressed in item *a* is, then,

$$V = Z_M(I_M - I_y) + I_M Z_{1M} \tag{483}$$

where

$$Z_{1M} = \text{the impedance of the stator main winding}$$
$$= R_{1M} + jX_{1M} \tag{484}$$

See the lower vectors of Fig. 231.

(*c*) The voltage applied to the start winding overcomes the local stator impedance drop, the reactance or impedance of the condenser,

and the counter emf built up by the mutual flux of the cross or start axis.

The counter emf will be, numerically,

$$E_S = I_{\phi S} Z_S \qquad [485]$$

Z_S = the magnetizing impedance of the start winding

Neglecting the core loss, Z_S reduces to jX_{mS}.

(*d*) The stator current of the start winding is the resultant of a load component (I_x) reflected by transformer action from the rotor, and a

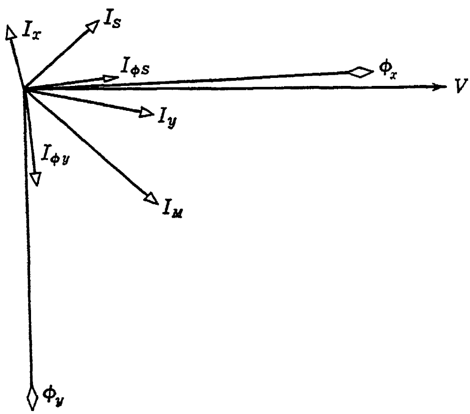


FIG. 232. The stator current components and the flux vectors.

start-axis magnetizing current similar to that of the main winding but usually differing in magnitude. (See Fig. 232.)

$$I_S = I_x + I_{\phi S} \qquad [486]$$

or

$$I_{\phi S} = I_S - I_x \qquad [487]$$

I_x = the rotor current I''_x reflected in the stator

Combining equations 485 and 487,

$$E_S = (I_S - I_x) Z_S \qquad [488]$$

The equation for the condition expressed in item *c* is, then,

$$V = Z_S(I_S - I_x) + I_S(Z_{1S} + Z_C) \qquad [489]$$

where

Z_{1S} = the impedance of the stator start winding.

$$= R_{1S} + jX_{1S} \qquad [490]$$

Z_C = the condenser impedance

If its small resistance is neglected, this becomes

$$-jX_C = \frac{-j10^6}{2\pi f \times \text{microfarads}} \qquad [491]$$

(See the upper portion of the diagram of Fig. 231.)

(*e*) In the rotor, the main or y-axis current results from two emf's:

(1) The transformer voltage opposite in phase to the stator counter emf, but of the same magnitude with the rotor constants in main-winding terms, $Z_M(I_M - I_y)$.

(2) A speed voltage, resulting from the rotor bars' cutting the cross-axis mutual flux and the rotor cross-axis leakage flux. It must be remembered that a speed voltage is S times as large as the transformer voltage caused by the same flux.

Speed voltage, cross-axis mutual flux:

$$SZ_S(I_S - I_x) \qquad [492]$$

To combine with main- or y-axis terms:

$$\frac{SZ_S}{a^2}(I_S - I_x)a = S\frac{Z_S}{a}(I_S - I_x) \qquad [493]$$

Speed voltage, leakage flux, in stator terms:

$$aI_xX_2 \qquad [494]$$

Combining, and considering that a speed emf reaches its maximum at the same time as the flux causing it,

$$\text{Final speed emf} = S\left[j\frac{Z_S}{a}(I_S - I_x) + aI_xX_2\right] \qquad [495]$$

By Kirchhoff's law the vector sum of the voltages of items *e*1, *e*2 and the rotor impedance drop caused by the main-axis current equals zero. (See Fig. 233*a*.)

$$Z_M(I_M - I_y) + S\left[j\frac{Z_S}{a}(I_S - I_x) + aI_xX_2\right] + I''_yZ_2 = 0$$

or

$$Z_M(I_M - I_y) + S\left[j\frac{Z_S}{a}(I_S - I_x) + aI_xX_2\right] - I_yZ_2 = 0 \qquad [496]$$

(*f*) In the start or x axis of the rotor, two emf's similar to those of item *e* are present.

(1) A transformer voltage opposite in phase to the cross-axis counter emf, but different in magnitude by the ratio of transformation.

$$\frac{Z_S}{a^2}(I_S - I_x)a = \frac{Z_S}{a}(I_S - I_x) \qquad [497]$$

(2) A speed voltage, resulting from the rotor bars' cutting the main-winding mutual and rotor-leakage fluxes.

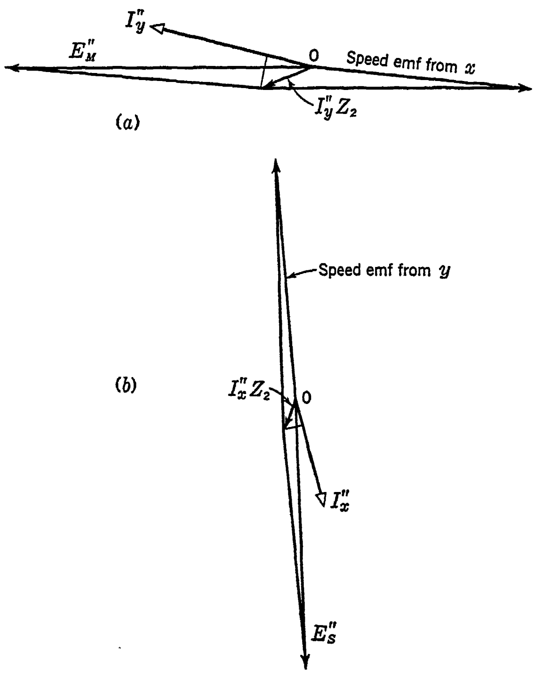


FIG. 233. Rotor current and voltage vectors. In (a) the relationships are shown by equation 496, in (b) by equation 501.

For the mutual flux, the emf from speed action is

$$SZ_M(I_M - I_y) \qquad [498]$$

For the leakage flux, in stator terms,

$$SI_yX_2 \qquad [499]$$

Combining and locating the vector position,

$$S[-jZ_M(I_M - I_y) - I_yX_2] \qquad [500]$$

By Kirchhoff's law the vector sum of these voltages and the rotor impedance drop of the cross-axis rotor current equals zero. See Fig. 233*b*.

$$\frac{Z_S}{a}(I_S - I_x) + S[-jZ_M(I_M - I_y) - I_yX_2] + I''_xZ_2 = 0$$

or,

$$\frac{Z_S}{a}(I_S - I_x) + S[-jZ_M(I_M - I_y) - I_yX_2] - aI_xZ_2 = 0 \quad [501]$$

We now have four fundamental equations, 483, 489, 496, and 501, involving four currents I_M, I_S, I_y, and I_x. They must be solved simultaneously.

299. Final Expressions for Current. Using determinants for the solution of the equations mentioned above, we obtain (without giving the detailed solution)

$$I_M = \frac{C_1B_2 - C_2B_1}{A_1B_2 - A_2B_1} \quad [502]$$

$$I_S = \frac{A_1C_2 - A_2C_1}{A_1B_2 - A_2B_1} \quad [503]$$

where

$$A_1 = S\left[jZ_{1M} - X_2\left(1 + \frac{Z_{1M}}{Z_M}\right)\right] \quad [504]$$

$$B_1 = -\left[aZ_2 + (Z_{1S} + Z_c)\left(\frac{1}{a} + a\frac{Z_2}{Z_S}\right)\right] \quad [505]$$

$$C_1 = V\left[S\left(j - \frac{X_2}{Z_M}\right) - \left(\frac{1}{a} + a\frac{Z_2}{Z_S}\right)\right] \quad [506]$$

$$A_2 = \left[Z_{1M}\left(1 + \frac{Z_2}{Z_M}\right) + Z_2\right] \quad [507]$$

$$B_2 = S\left[-aX_2 + (Z_{1S} + Z_c)\left(\frac{j}{a} - a\frac{X_2}{Z_s}\right)\right] \quad [508]$$

$$C_2 = V\left[\left(1 + \frac{Z_2}{Z_M}\right) + S\left(\frac{j}{a} - a\frac{X_2}{Z_S}\right)\right] \quad [509]$$

From equations 483 and 489,

$$I_y = I_M\left(1 + \frac{Z_{1M}}{Z_M}\right) - \frac{V}{Z_M} \quad [510]$$

$$I_x = I_S\left(\frac{Z_s + Z_{1S} + Z_C}{Z_S}\right) - \frac{V}{Z_S} \quad [511]$$

The counter emf's in the separate windings are

$$E_m = V - I_M Z_{IM} \tag{512}$$

$$E_S = V - I_S(Z_{IS} + Z_C) \tag{513}$$

The rotor input, or the power transferred across the gap, is the sum

$$E_M I_y \cos(E_M, I_y) + E_S I_x \cos(E_s, I_x) \tag{514}$$

The power converted to mechanical form:

$$\text{Power across gap by equation 514} - (I_y^2 + a^2 I_x^2)R_2 \tag{515}$$

$$\text{Net output} = \text{power converted to mechanical form} - F \text{ and } W \tag{516}$$

The torque in synchronous watts is the sum

$$aE_M I_x \sin(E_M, I_x) + \frac{E_S}{a} I_y \sin(E_S, I_y) \tag{517}$$

The stator input is the sum

$$VI_M \cos\theta_M + VI_S \cos\theta_S \tag{518}$$

The stator losses:

$$I_M^2 R_{1M} = \text{copper loss in the main winding} \tag{519}$$

$$I_S^2(R_{1S} + R_c) = \text{copper loss in start-winding circuit} \tag{520}$$

The rotor losses:

$$I_y^2 R_2 = \text{copper loss in the y axis} \tag{521}$$

$$a^2 I_x^2 R_2 = \text{copper loss in the x axis} \tag{522}$$

300. Example. A 4-pole, $\frac{1}{4}$-hp, 60-cycle, 110-volt capacitor motor displays the following values:

Main winding.	$R_{1M} = 2.02$ ohms	$X_{1M} = 2.79$ ohms
Magnetizing reactance	$X_{mM} = 66.8$ ohms	$R_2 = 4.12$ ohms
	$X_2 = 2.12$ ohms	
Start winding.	$R_{1S} = 7.13$ ohms	$X_{1S} = 3.22$ ohms
Magnetizing reactance	$X_{mS} = 92.9$ ohms	$X_C = 172.0$ ohms
	$R_C = 9.00$ ohms (condenser resistance)	

Ratio of transformation. $a = 1.18$

F and $W = 13$ watts. Core loss will be neglected in the "magnetizing branch," but it is 24 watts.

The performance will be calculated at a slip of 0.04, and hence S equals 0.96.

$$A_1 = -4.80 + j2.00 \qquad \text{[From equation 504]}$$
$$-B_1 = 10.14 - j145.9 \qquad \text{[From equation 505]}$$
$$A_2 = 6.367 + j4.876 \qquad \text{[From equation 507]}$$
$$B_2 = 139.1 + j13.55 \qquad \text{[From equation 508]}$$
$$C_1 = -96.2 + j114.7 \qquad \text{[From equation 506]}$$
$$C_2 = 113.5 + j85.6 \qquad \text{[From equation 509]}$$
$$I_M = 2.64 \ \overline{|58^\circ\ 36'} \qquad \text{[From equation 502]}$$
$$I_S = 0.954 \ \underline{|39^\circ\ 0'} \qquad \text{[From equation 503]}$$
$$I_y = 1.55 \ \overline{|28^\circ\ 30'} \qquad \text{[From equation 510]}$$
$$I_x = 0.757 \ \underline{|131^\circ\ 25'} \qquad \text{[From equation 511]}$$

The magnetizing current ($I_{\phi M}$) in the main axis $= I_M - I_y$

$$= 0.015 - j1.51$$

In the cross-axis ($I_{\phi S}$):

$$I_S - I_X = 1.241 + j0.034$$
$$E_M = 102.67 \ \underline{|0^\circ\ 19'} \qquad \text{[From equation 512]}$$
$$E_S = 113.5 \ \underline{|91^\circ\ 16'} \qquad \text{[From equation 513]}$$

The rotor input:

$$E_M I_y \cos(E_M, I_y) = 102.67 \times 1.55 \times \cos(28^\circ\ 30' + 0^\circ\ 19')$$
$$= 140 \text{ watts from the main axis}$$
$$E_S I_x \cos(E_S, I_x) = 113.5 \times 0.757 \cos(131^\circ\ 25' - 91^\circ\ 16')$$
$$= 66 \text{ watts from the cross-axis.}$$

Total input to rotor = 206 watts

The rotor losses:

$$I_y^2 R_2 = 1.55^2 \times 4.12 \quad \text{or} \quad 9.9 \text{ watts}$$
$$a^2 I_x^2 R_2 = 1.18^2 \times 0.757^2 \times 4.12 \quad \text{or} \quad 3.3 \text{ watts}$$

Friction and windage = 13.0 watts

Total rotor losses = 26.2 watts

Net output = 206 − 26.2 or 179.8 watts

The stator input:

$$VI_M \cos\theta_M = 110 \times 2.64 \times \cos 58^\circ\ 36'$$
$$= 151 \text{ watts}$$
$$VI_S \cos\theta_S = 110 \times 0.954 \times \cos 39^\circ$$
$$= 82 \text{ watts}$$

Total input = 233 watts

The stator losses:

$$I_M{}^2R_{1M} = 2.64^2 \times 2.02 \quad \text{or} \quad 14.1 \text{ watts}$$

$$I_S{}^2(R_{1S} + R_C) = 0.954^2(7.13 + 9) \quad \text{or} \quad \underline{14.7 \text{ watts}}$$

$$\text{Total stator losses} = 28.8 \text{ watts}$$

The principal performance items can now be tabulated. At a slip of 4 per cent:

Net output = 179.8 watts or 0.241 hp
Input (neglecting core loss) = 233 watts
Input by adding core loss = 257 watts
Efficiency = $\dfrac{179.8}{257}$ or 0.70
Main-winding current = 2.64 amperes
Auxiliary winding or condenser current = 0.954 ampere

301. Starting Conditions and Example. When the motor is stationary, the term S reduces to zero, and the currents become

$$I_M = \frac{V}{Z_{1M} + \left(\dfrac{Z_2}{1 + \dfrac{Z_2}{Z_M}}\right)} \tag{523}$$

$$I_S = \frac{V}{(Z_{1S} + Z_C) + \left(\dfrac{aZ_2}{\dfrac{1}{a} + a\dfrac{Z_2}{Z_S}}\right)} \tag{524}$$

Suppose we consider the motor just used with the condenser changed so that $X_C = 14.5$ ohms and $R_c = 3$ ohms. Then, from equation 523,

$$I_M = 14.2 \overline{\left| 40° \, 0' \right.}$$

and, from equation 524,

$$I_S = 6.25 \underline{\left| 27° \, 50' \right.}$$

The starting torque can be calculated from equation 517 but I_x and I_y must be determined. A convenient method is to use equations 483 and 489. Thus:

$$I_x = \frac{j78.6 I_S}{4.86 + j81.1} \quad \text{or} \quad 6.06 \underline{\left| 31° \, 20' \right.}$$

This is in terms of S winding; in terms of M:

$$I_x = 1.18 \times 6.06 \quad \text{or} \quad 7.15 \text{ amperes}$$

Similarly:

$$I_y = \frac{j66.8 I_M}{4.12 + j68.92} \quad \text{or} \quad 13.8 \overline{\left| 36° \, 34' \right.} \text{ amperes}$$

The counter emf:

$$E_M = jX_M(I_M - I_y)$$
$$= 63.6 \,|\, 10^\circ\, 47'$$
$$E_S = jX_S(I_S - I_x)$$
$$= 38.8 \underline{|\, 57^\circ\, 0'}$$

The starting torque:

$$aE_M I_x \sin (E_M, I_x) = 1.18 \times 63.6 \times 6.06 \sin 42^\circ\, 7' \quad \text{or} \quad 306$$

$$\frac{E_S}{a} I_y \sin (E_S, I_y) = \frac{38.8}{1.18}\, 13.8 \sin 93^\circ\, 34' \quad \text{or} \quad 453$$

In ounce-feet:

$$\text{Starting torque} = \frac{112.7 \times (306 + 453)}{1800} \quad \text{or} \quad 47.5$$

302. Methods of Obtaining High Starting Torque. Because of the relatively small capacitance of the oil-type capacitors used with permanently split motors of this type, the starting current which can flow

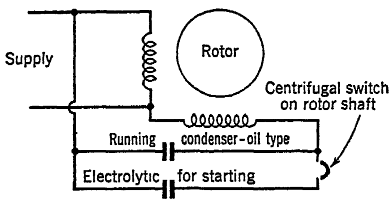


FIG. 234. Double-value capacitor motor with centrifugal switch that permits the electrolytic condenser to be in circuit only long enough to produce good starting torque.

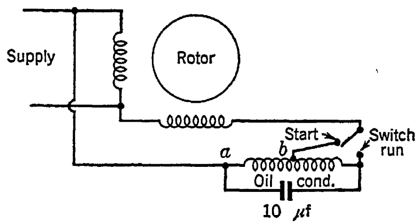


FIG. 235. The effects of high capacity for starting and of low capacity for effective running operation are obtained in this design by varying the voltage across the condenser.

through the auxiliary winding (and hence the starting torque) is low. Yet, because of the favorable running conditions, it is often desirable in practice to make use of the capacitor motor, even though high starting torque may be required.

One obvious plan for obtaining the dual advantages of high starting torque and efficient, quiet, running performance employs an electrolytic condenser of large capacitance for starting duty. This is disconnected by a centrifugal switch (or relay), permitting the motor to run with the smaller oil-type capacitor. A diagram is shown in Fig. 234.

Similar effects can be obtained by using an autotransformer across the condenser terminals as shown in Fig. 235. Note that a 10-μf condenser

is shown here, paralleled with a midtapped autotransformer. When the manually operated starting switch is connected to the transformer tap, the condenser voltage is doubled, giving an effect of 40 μf and resulting in large starting torque. Connecting the switch on the running position puts normal voltage across the condenser for favorable load performance.

303. Speed Control. Attempting to modify the speed of the single-phase induction motor is only slightly more successful than similar attempts on the polyphase machine. The flux rotates at a synchronous speed, fixed by the number of poles and the frequency, and, except for providing for excessive slip, little can be done to modify the rotor speed. In the polyphase motor the use of high rotor resistance to bring about, say, 20 per cent slip, results in 20 per cent of the rotor input power being dissipated as rotor copper loss. In the single-phase induction motor, although rotor copper loss is a function of slip, it is not such a direct, simple relationship.[5]

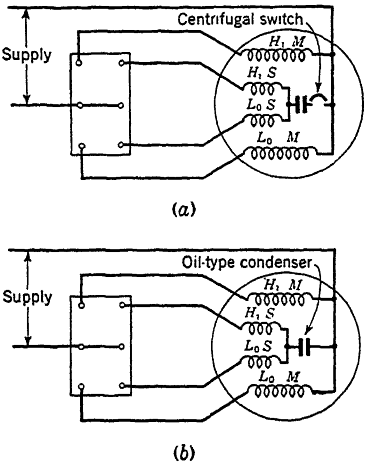


FIG. 236. Two-speed capacitor motors with complete main and auxiliary windings wound for different numbers of poles. A double-pole, double-throw switch is used to transfer from one set of windings to the other. In (*a*) the capacitor is used for starting only.

Furthermore, because of the comparatively small power handled by single-phase induction motors, excessive slip to obtain a slight reduction in speed is not so serious from the standpoint of wasted power. For that reason in various cases, such as for fan and blower applications where speeds of 1200 or 1500 rpm might be required, it is not unusual to use small 4-pole motors with high slip. This can be accomplished (*a*) by high rotor resistance, in which case no variation in speed is introduced, but the motor displays a drooping speed curve; (*b*) by weakening the motor to such an extent that its rated load is near the breakdown point. This same effect can be produced by lowering the applied voltage, and for that reason variable resistance or tapped reactance coils in series with the motor can be used for speed variation. The method is inefficient, gives large variations in speed with load change, but it is much used in spite of these drawbacks.

[5] See curves showing this ratio in Veinott's "Segregation of Losses in Single Phase Induction Motors," *Elec. Eng.*, December, 1935.

Multispeed Motors. As a means of obtaining two or three speeds on single-phase induction motors, the stators may be provided with two or three windings, each for a different number of poles. Suitable

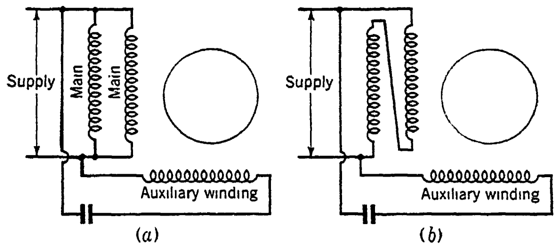


FIG. 237. One patented connection for obtaining two speeds. In normal operation two halves of the main winding are connected in parallel across the supply. In (*b*) the two halves are connected in series, resulting in reduced torque from the main winding and reduced speed under load.

switching gives speed control, but no nicety of adjustment is possible. This is sometimes used on split-phase or capacitor-start motors, and of course the auxiliary or starting windings are in multiple as well as the main. One difficulty with this arrangement is in connection with the centrifugal switch, which must be set to open up below the lowest speed rating. As a result, on the highest speed it will open at a low point on the speed-torque curve which yields low "pull-up" torque.

Figure 236*a* shows a schematic diagram for a two-speed, capacitor-start motor, using the same capacitor with both speeds. In *b* is shown the same type of two-speed motor of the permanently split-capacitor type, requiring no centrifugal switch.

Because of the possibility of varying (within limits) the relative strengths of the main and condenser windings and of obtaining various speed torque curves, capacitor motors are widely used on variable speed motors of fractional horsepower ratings. One scheme used with fans involves the normal parallel connection of two halves of the main winding, which, when reconnected in series, weakens the motor and results

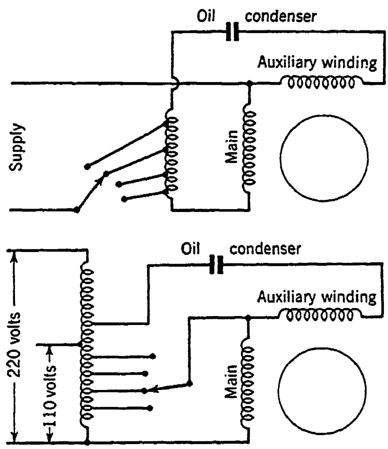


FIG. 238. Two other connections for obtaining speed control by means of auxiliary autotransformers. Both circuit arrangements are patented.

in a speed of about 60 to 70 per cent of the stronger arrangement. That is, the speed-torque requirements of most fans cause the motor to operate at these speeds as the connections are changed. The arrangement is shown in Fig. 237. Other speed-control connections for capacitor motors are shown in Fig. 238, utilizing autotransformers for voltage control.

SYNCHRONOUS MOTORS

CHAPTER XXXVII

THEORY OF OPERATION AND CHARACTERISTICS

304. Chapter Outline.

Theory of Operation.
- Effect of Field Strength.
- Effect of Load.
- Hunting.
- Damping.
- Starting.

The Synchronous Condenser.

305. Operation. If two alternators are connected in parallel, supplying a common load, and the motive power is removed from one, it will continue to operate as a motor, drawing power from the line to supply its own losses. An alternator so operated would be called a synchronous motor. The synchronous motor in its fundamental theory and construction is identical with the alternator. Hence, just as with the d-c dynamo, we have a reversibility of processes: supplying the machine with mechanical energy and taking out electrical energy gives us a generator; supplying electrical energy and obtaining mechanical energy results in motor action.

Except for the fact that synchronous motors are nearly always built with salient poles and are usually provided with windings to give more damping effect than is found in alternators, the two are entirely identical.

The synchronous motor can also be compared to the induction motor, as its operation depends upon the rotating flux, or rather rotating mmf, built up by a distributed stator winding supplied from a polyphase source. In induction motors, the rotating flux built up in the air gap cuts the turns of the rotor winding and induces in them the voltage and current which produce the motor torque. In the synchronous motor, the flux in the physical rotating field structure "locks" with the rotating mmf. The field structure, then, turns at synchronous speed if it has been synchronized previously. A mechanical load applied to this motor

results in a retarding torque which causes the rotating field structure to drop slightly behind its no-load position with reference to the rotating mmf. The actual number of revolutions per minute does not decrease; there is merely an instantaneous reduction in speed, resulting in an increase in the angle between the physical pole and the rotating mmf.

The addition of too much load may result in the retarding load torque exceeding the developed motor torque. In this case the motor pulls out of synchronism and comes to a stop. The motor would then draw an excessive current from the supply lines, and should be protected by fuses or circuit breakers against heat and mechanical stresses on the winding caused by the abnormal current. Of course, if a squirrel-cage winding or other means is used for bringing the motor up to synchronism, and if this means is made automatic in its operation, the motor will not actually stop if it is pulled out of synchronism, but at least it does not run as a synchronous motor.

306. Power Factor. One outstanding characteristic of the synchronous motor, besides its constant-speed characteristic (which may be either an advantage or a disadvantage, depending upon the application), lies in its ability to operate at either leading or lagging pf's. The pf can be varied by change in the field excitation. This feature makes the motor valuable for regulating the delivery voltage of a transmission line and also the system pf. One explanation of the variation in pf follows:

The machine field flux at no load (ideally, with zero armature current) is built up by the d-c field. When the motor is running synchronously and carrying an armature current, poles of mmf are produced on the armature surface. These poles travel in space with the revolving field structure and take a position relative thereto which is fixed mainly by the machine load and pf. When the pf is zero lagging, field and armature mmf's add arithmetically;[1] when the pf is zero leading, field and armature mmf's subtract arithmetically. For other pf's, the addition or subtraction is vectorial when sine distributions prevail. The pf and armature current always take such values that a flux sufficient to generate the necessary value of counter emf is set up. Hence it can be seen that, if the d-c excitation is too low, the armature current will lag the voltage just enough to bring the air-gap flux *up* to the required value. If the d-c field excitation is too high, the armature current will become leading just enough to bring the air-gap flux *down* to the required value.

[1] Since motor action is being described, the armature current with respect to the voltage is considered in a reversed sense from its position in a generator. Compare with applied and counter emf's and the current in separately excited d-c generators and motors.

307. The Vector Diagram. A number of different analyses are possible for the explanation of synchronous-motor operation. The one given below, though somewhat inaccurate, offers some advantages for an understanding of the effect of both load and field variation.

Suppose that a motor is operating at no load and with a d-c field excitation such that the voltage generated in its armature is exactly equal and opposite to the applied voltage. Under these conditions we will assume that no current flows. The voltage vectors are then as shown in Fig. 239*a*.

The d-c field is next over-excited so that, if no current were flowing in the armature to give a demagnetizing effect, the field would produce a generated voltage of E_g. This voltage is greater than the applied voltage by the resultant voltage E_R. As a result, the motor draws current from the line until the demagnetizing effect of this current reduces—by armature reaction—the generated voltage. This voltage is further modified by the resulting IR_e and IX_l drops until the sum of these drops and the counter emf is equal to the applied voltage. Or, if the armature reaction effect is considered as a voltage drop,[2] we could say that the current from the line increases until the synchronous impedance drop it causes equals E_R. Figure 239*b* shows such a diagram. The current I_c results from E_R and lags it by the impedance angle β.

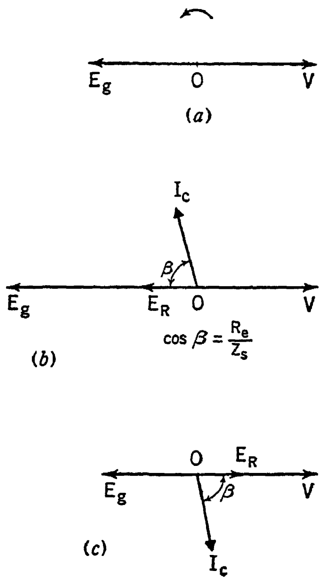


FIG. 239.

$X_s =$ the synchronous reactance of the armature

$R_e =$ the effective resistance of the armature

The position of I_c is then fixed, *lagging the voltage which caused it, but leading the applied voltage.*

Figure 239*c* shows the same motor with reduced excitation. The generated voltage is reduced so that a voltage E_R results as the difference between the applied voltage and the counter emf. Again such a current flows through the armature so that it "uses up" E_R as a synchronous

[2] Considering the effect of armature reaction as a voltage drop, equal numerically to I_aX_s, involves the fictitious synchronous reactance X_s. This has been explained under Alternators.

impedance drop. This current lags behind the voltage which causes it by an angle equal to the angle between the effective armature resistance and the synchronous impedance of the impedance triangle. The current is thus lagging not only E_R, but the applied voltage as well, in this case.

We can see from the above that the current drawn from the line by this motor can be made to vary in magnitude and in position by change

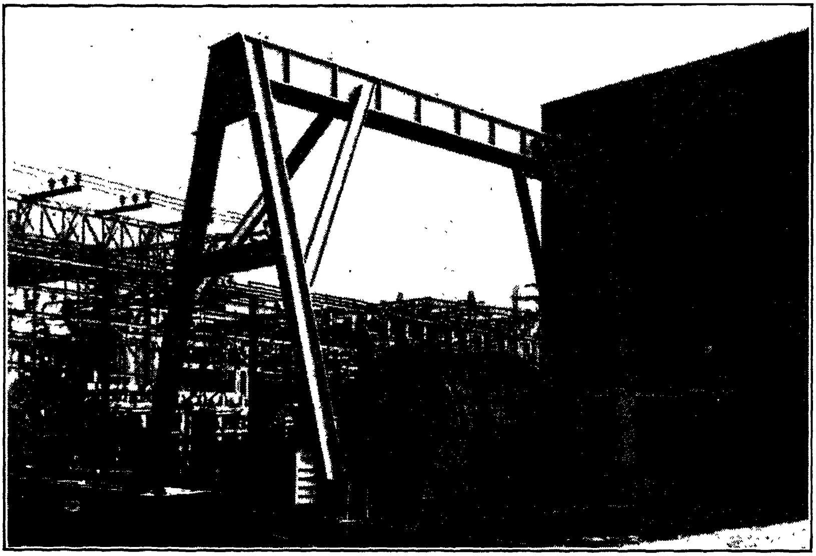


FIG. 240. Outdoor hydrogen-cooled, 11,500-volt synchronous condenser. (*Courtesy of the General Electric Co.*)

in the excitation. The only in-phase component of this current is that which represents the copper loss of the armature; in an actual motor this in-phase current would also contain a component such as is necessary to represent the core, friction, and windage losses.

308. The Effect of Load. When a synchronous motor is loaded, its angular velocity reduces for an instant so that the rotating field structure drops slightly behind its initial position with respect to the rotating mmf. This is illustrated in Fig. 241. The flux lines can be considered as a number of rubber bands stretched across the air gap radially from fixed positions on the field poles to a number of moving points on the stator. A load acts as a retarding torque and, when applied, stretches the bands. During the instant of stretching, there is a reduction in speed, but the actual number of revolutions per minute remains the same as the synchronous speed. If an excessive load is connected to the motor, the

bands (to continue the analogy) reach their elastic limit and break; the motor pulls out of synchronism. At that point we say that the "pull-out torque" has been reached. In motors of ordinary design, the shift in

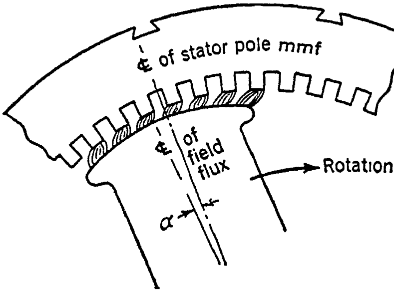


FIG. 241.

field position from no load to full load may reach values up to 60 electrical degrees and more when calculated from the synchronous reactance basis. The change in field position is measured by the "torque angle." A machine so designed as to give a small torque angle at rated load is "hard coupled"; if the field structure shifts back a considerable distance with load the machine is said to be "soft coupled."

To apply this to the vector diagram it will be necessary to shift the position of the internally generated voltage backwards through a number

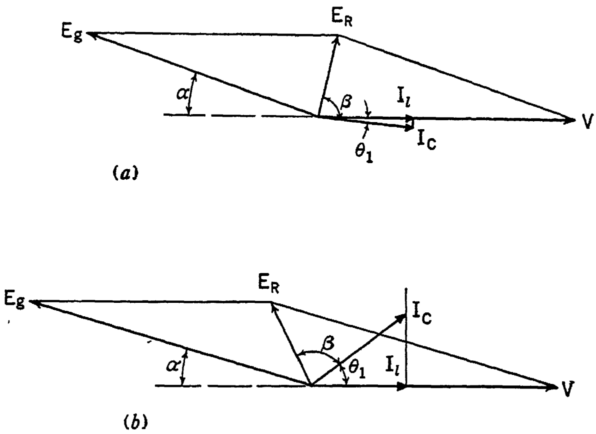


FIG. 242.

of electrical degrees, depending upon the load. This is illustrated in Fig. 242. In *a* the field excitation is such as to make the applied and internal generated voltages equal, but the load causes E_g to lag back

α degrees. The resultant voltage E_R causes the motor current I_c, lagging behind it by the synchronous impedance angle. This current lags the applied voltage by the pf angle θ_1 and has an in-phase component I_l which represents the losses and mechanical output. In Fig. 242*b* the excitation is increased and the mechanical load is assumed to be the same. Hence, except for slight changes in the losses, the in-phase component I_l will remain the same as in the previous case. The difference in voltages causes E_R and the motor current I_c, which now leads the applied voltage.

Thus we see the combined effect of excitation and load upon the current and pf. If other loads, and corresponding angles (α), are drawn on these diagrams, it will be seen that the pf varies with the load for fixed field excitation; in other words, the same excitation does not produce the same pf at all loads.

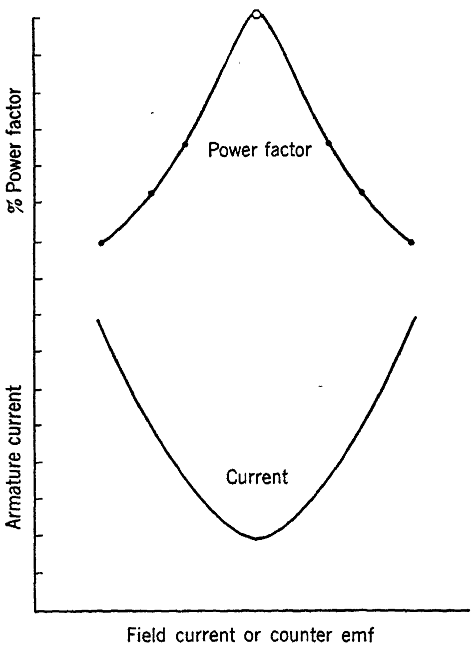


FIG. 243. V-curve and power-factor curve for a synchronous motor.

309. Characteristic Curves. The curve showing the relationship between field current and armature current for a constant mechanical output is known as the V curve on account of its characteristic shape. A V curve is shown in Fig. 243. For a constant output, constant counter emf, and varying armature current, the pf would have to change in

inverse proportion to the armature current. This is so because the power relation

$$\text{Power (constant)} = \sqrt{3}\, VI \cos\theta \text{ (for three-phase)}$$

must be satisfied at all times. Thus, if the pf doubles because of field-current adjustment, the armature current must be cut in half for the power to remain constant.[3] Actually, the counter emf will change more

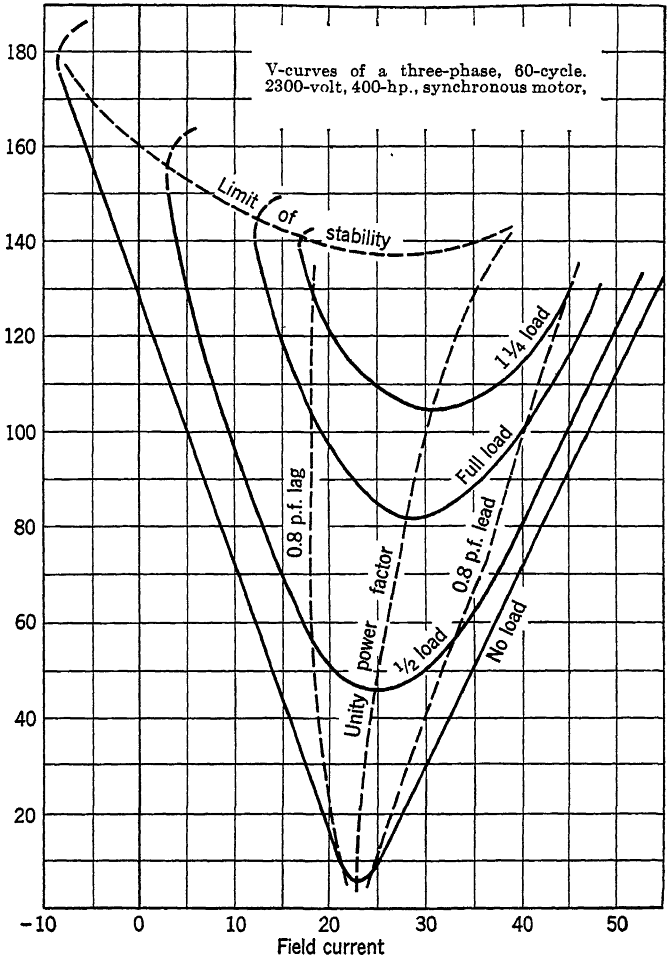


Fig. 244.

[3] Because of non-sinusoidal wave shape it is possible for the pf at no load to be less than unity (as indicated by supply meters) even though the fundamentals of the current and voltage are in phase. This is discussed in Article 354.

or less with the field current, and this will modify the numerical values, but the essential facts remain. If this pf for constant output is plotted against the field current, it will be an inverted V curve as shown in Fig. 243. Likewise, if the armature current is plotted against the field current, the V curve is obtained. Such curves are not quite regular, even for zero saturation, and the branches for leading pf move more and more to the right as the saturation increases.

Figure 244 illustrates actual V curves taken at various loads on a three-phase synchronous motor. Note the shift of the bottom of the V, representing unity pf, at increased loads. The lines drawn through points of equal pf at different loads are known as compounding curves.

It will be noted that the no-load current rises sharply, with change in field current, on each side of its unity pf point. A larger range of excitation makes relatively less difference in the armature current at full load.

As the excitation is reduced, the limit of stability (for the given load) is reached; at this point the motor pulls out of step. Later analyses will show why such limits vary with the load. Under light load, say, up to 40 per cent of rated, the excitation can be reduced to zero in most motors without causing them to pull out of step; in fact, the field current can be reversed and increased in the negative direction up to a certain point.

310. Hunting. When a synchronous motor is operating under steady load conditions and an additional load is suddenly applied, the developed torque is less than that required by the load and the motor starts to slow down. A slight reduction in speed increases the phase angle of the generated voltage and permits more power current to flow through the armature. Some of the kinetic energy of the rotating parts is given out to the load during speed reduction. When the motor is slowing down, it cannot cease deceleration at exactly the correct torque angle corresponding to the increased load. It passes beyond this point, develops more than required torque, and increases in speed. This is followed by a reduction in speed and a repetition of the entire cycle. Such a periodic change in speed is called *hunting*. The mass of the rotating part and the "spring effect" of the flux lines are the necessary elements which give the rotating field structure a natural oscillating period. If the load varies periodically with this same frequency or some multiple of it, the tendency

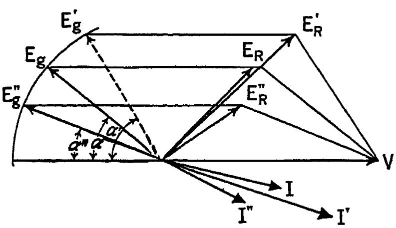


FIG. 245.

is for the amplitude of these oscillations to increase cumulatively until the motor is thrown out of step, causing corresponding current and power pulsations. The mechanical stresses are likely to be severe, and of course the armature current greatly increases when the motor leaves synchronism.

Figure 245 shows the vector diagram of a synchronous motor supplying a load such that its torque angle is α; the current is I. If the field structure oscillates so that α changes over a range of α'' to α', the current will change from I'' to I'. This is revealed by periodic swings of the ammeter and wattmeter in the supply circuit.

311. Damping. Figure 246 shows one form of damping winding in the pole shoes of a synchronous motor. Such a winding is similar to the

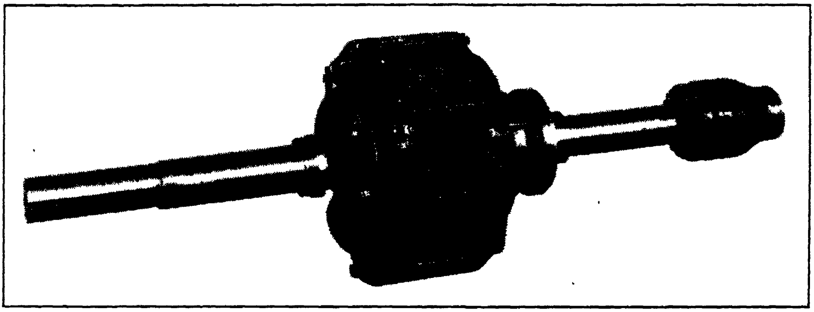


FIG. 246. Revolving member of a 6-pole, 75-hp synchronous motor. (*Courtesy of the General Electric Co.*)

squirrel cage of an induction machine. When hunting occurs it is accompanied by a shift of flux across the faces of the pole shoes, owing to the variation of the effect of armature reaction upon the field flux. Any closed circuit about such a field pole would have circulating currents induced in it by the shifting flux. These currents act, as explained by Lenz's law, to oppose any change in the relative positions of the field and the armature flux. In other words, such an arrangement is an effective "damper." The most common form is a winding similar to the squirrel cage of an induction motor. Phase-wound damper windings with adjustable resistance have been used to obtain high starting torque with moderate starting currents and high pull-in torques. Double squirrel cages and similar arrangements have been used also.

A solid pole gives a damping action but has the disadvantage of excessive pole-face losses unless closed slots are used on the armature. The most satisfactory arrangement from the standpoint of simplicity seems to be laminated poles with the squirrel-cage "amortisseur" or damping

winding. The effectiveness of the damper depends upon its resistance and to a lesser extent upon the length of the air gap. A low-resistance damping winding produces the stronger damping effect, but if the synchronous motor is to be started by induction-motor action, using this squirrel cage, the winding resistance should be fairly high to produce good starting torque. Because of these opposing tendencies, a compromise usually must be reached in damping winding design, or else a polyphase winding with adjustable external resistances must be used.

312. Starting Synchronous Motors. A synchronous motor, as such, has no starting torque. However, when its armature winding is connected to the proper supply, the eddy-current and hysteresis losses set

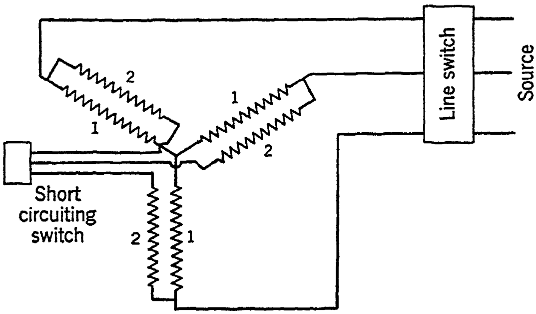


FIG. 247. Arrangement of the double-winding synchronous motor. For starting, the short-circuiting switch is open and only winding 1 is utilized. The short-circuiting switch closes the neutral of the second winding for parallel operation at normal load.

up in the pole faces produce a torque which might be sufficient to start a small motor. A large motor with normal friction could not start by such means, and would take an excessive current when connected to the line. Artificial means must be provided for starting.

External Source. If the d-c field of a synchronous motor is supplied by a direct-connected exciter, this exciter can be used as a starting motor. When synchronous motors are used to drive motor-generator sets, again a source of direct current can be used to start the set from the d-c end. This method is now rarely used.

Induction-motor Start. If a squirrel-cage winding is constructed in the pole faces of the synchronous motor, it can be used to develop a starting torque (as well as provide damping) similar to that of the ordinary induction motor. As the motor reaches about 95 per cent of synchronous speed (it is operating as an induction motor with 5 per cent slip) its field is excited and the motor pulls into step.

For such a setup, the problem of limiting the starting current without too low a value of starting torque is met in several ways.

(*a*) Starting compensators are used which are similar in design to those employed on induction motors. The motor is connected to the reduced voltage supply of the compensator until synchronous or near-synchronous speed is reached, and is then connected to the full voltage.

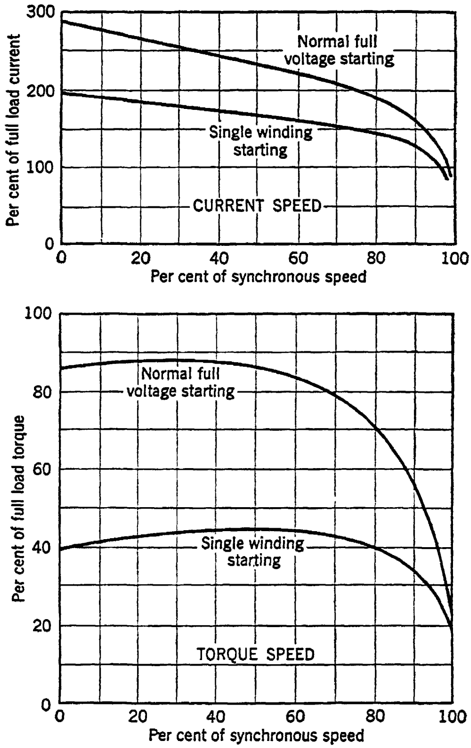


FIG. 248. Comparison of the current-speed curves and the torque-speed curves when starting a 300-hp, 125-rpm, 25-cycle, 6600-volt synchronous motor by the normal full-voltage method and by the method employing but one circuit of a multiple winding.

(*b*) Compensators or autotransformers can be eliminated in some instances by the use of series reactors in the supply lines. These reactors give a large voltage drop due to the heavy current and low pf at starting.

(*c*) Various types of rotor windings are used either independently or in conjunction with the above methods. These may be double squirrel cage,[4] or use may be made of special bar cross-sections (T bar, L bar, or simply deep, narrow bars) to give high skin effect to the squirrel cage and thus limit the starting current. Phase windings with terminals

[4] H. V. Putman, "Starting Performance of Synchronous Machines," *Trans. A.I.E.E.*, Vol. 46, p. 39, 1927.

connected to slip rings for connection to external resistances have been used to give desirable starting characteristics similar to those of wound-rotor induction motors.[5]

(*d*) Multiple winding.[6] This involves a special arrangement of armature coils so that two or more complete windings are paralleled for normal operation. The paralleled windings have normal values of

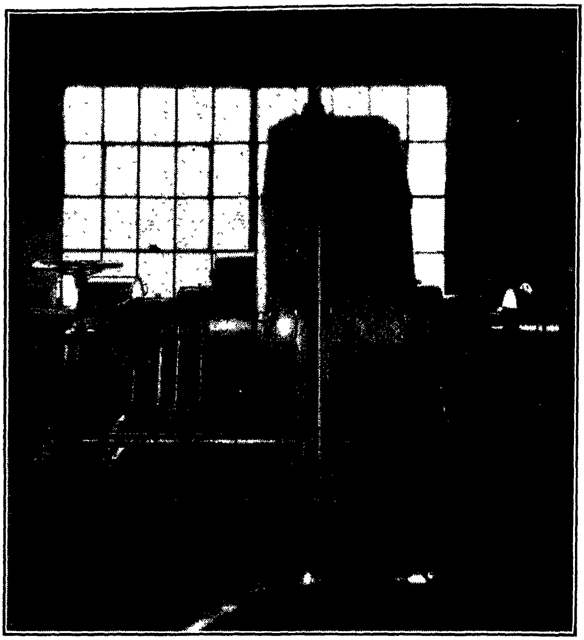


FIG. 249. The supersynchronous motor. (*Courtesy of the General Electric Co.*)

reactance. If one winding is left open during the starting process the reactance and resistance will be approximately double the normal values and will result in a reduced current without the utilization of an autostarter. Switching arrangement and comparative curves are shown in Figs. 247 and 248, respectively.

313. The Supersynchronous Motor. A special method of obtaining good starting torque is embodied in the "supersynchronous" motor, so

[5] M. A. Hyde, "Simplex Synchronous Motor," *Elec. J.*, p. 77, February, 1931.

[6] D. W. McLenegan and A. G. Ferriss, "Multiple Winding Starting Method for Synchronous Motors," *Gen. Elec. Rev.*, Vol. 33, p. 574, October, 1930.

Shutt and Dawson, "Angle Switching of Synchronous Motors," *Elec. Eng.*, November, 1935; July, 1936.

J. K. Kostko, "Self-excited Synchronous Motors," *J. Am. Inst. Elec. Engrs.*, pp. 605–612, June, 1925; p. 897, August, 1925.

constructed that all its pull-out torque is made available for starting the load.

The stator, which is stationary in the common type of motor, is in this case constructed so that it is able to rotate outside of the field structure, although a band brake is provided for locking it in place. The starting sequence illustrates its advantages:

The brake is released and the starting switch is closed. Since the rotor is connected to the load, it remains stationary; but the stator rotates at slightly less than synchronous speed in a direction opposite to normal rotor rotation, owing to its starting torque from the squirrel-cage damping winding. Exciting the field then pulls the motor into synchronism. As the band brake on the stator is tightened (see Fig. 249), the stator gradually reduces speed, and the rotor with its load begins to turn. By the time the stator is brought to rest, the rotor is running synchronously, and the operation of the machine from then on is similar to that of any synchronous motor.

The starting current of such a machine need not be excessively large, and all the pull-out torque is available for starting the actual load.

314. Important Characteristics. The preceding paragraphs have given a general discussion of some of the characteristics of the synchronous motor. What are the important points in which the designer or the operator is interested? Briefly, they are as follows:

(*a*) The starting method, the starting torque, and the starting current.

(*b*) The pf: the variation in pf with excitation, and the limiting pf's, lead or lag, at which the motor will operate for a given output.

(*c*) The losses and the efficiency.

(*d*) Hunting, damping, and stability.

(*e*) The degree of "coupling": whether "hard," "medium," or "soft."

In the chapters which follow, more detailed analyses of the characteristics will be given, with graphical and mathematical methods of obtaining quantitative data.

315. The Need for Leading Current. All a-c machinery requires "wattless" current for magnetization. This alternating current it may draw from its supply, or an internal arrangement to supply it may be provided. In transformers the magnetizing current is relatively small, but in induction motors it may amount to a considerable part of the total current on account of the air gap required by the motor construction. Since the pf of an operating system is the composite pf of all units supplied, a large number of transformers, induction furnaces, or more

especially induction motors, would result in the operation of generating stations with lagging current at fairly low pf. This is especially the case with slow-speed induction motors or any induction motor operated at light loads.[7]

From the standpoint of the generating and distributing systems, low pf with current lagging has several detrimental effects:

(a) It results either in excessive line loss or necessitates an increase in conductor size and weight.

(b) It requires larger sizes of transformers and generators.

(c) It results in bad voltage regulation of generators, transformers, and transmission lines.

The synchronous motor is useful in improving and controlling the pf of a system because of the ease with which its own pf can be varied. Because of this property, a synchronous machine may be used also to regulate, or to maintain constant, the delivered voltage of a transmission line.

316. Synchronous Condensers. When a synchronous motor is used for pf correction with no mechanical output, it is known as a synchronous condenser. The term *condenser,* applied to such a device, arises from the fact that it draws leading current as does a static condenser. There is a considerable increase in leading kilovolt-amperes available when the horsepower load of a synchronous motor is less than its rated load. The following equation expresses this relationship:

$$\text{Reactive kilovolt-amperes in percentage of rated} = \sqrt{100^2 - (\text{percentage of kilowatt output})^2}$$

The synchronous condenser is especially designed so that practically all its rated kilovolt-amperes are available for pf correction. Owing to the absence of shaft load, the mechanical design is modified from the ordinary motor standards. Because of the pf adjustment possible, the synchronous condenser is particularly applicable to transmission line control. In the United States alone over 5,000,000 kv-a are so used.[8]

[7] Charles R. Underhill, "Power Factor Wastes," McGraw-Hill Book Co.

[8] See A. Still's "Electric Power Transmission," Third Edition, Chapter X, McGraw-Hill Book Co.

CHAPTER XXXVIII

VECTOR DIAGRAMS

317. Chapter Outline.

Methods of Analysis by Vector Diagrams.
  General or Potier Diagram.
    Transition from Generator Action.
  Synchronous Impedance Diagrams.
  American Standards Association Method.
  Two-reaction Theory.
    Examples.

318. Introduction. In this chapter will be given a brief outline of the various types of vector diagrams that have been used for studying the behavior of synchronous motors. Those given in the preceding chapter at various loads and pf's are useful as aids in understanding the action of the motor, but are not in the most convenient form for calculation. In order to develop working diagrams the methods of analysis which have been already used on alternators (and with which the student is assumed to be familiar) will be applied to synchronous motors. These are:

The general or Potier diagram.
The synchronous-impedance diagram.
The two-reaction diagram.

319. The General Diagram. Transition from Generator Action. This analysis assumes that the field windings and the field iron are distributed to give a sinusoidal wave shape. It considers separately the effects of armature leakage reactance, armature reaction, and armature resistance. The effects of the load and pf, and the influence of the no-load saturation curve, are all included; correction may be made for the change in full-load field leakage.

Because of the similarity between the alternator- and synchronous-motor actions, transition diagrams will be shown to illustrate the change.

Figure 250 depicts the vector diagram of a non-salient-pole machine based on the general or Potier analysis. As used with the alternator:

V = the applied voltage per phase
I = the current in amperes per terminal

M_a = the mmf of armature reaction, in ampere-turns per pole
M_f = the mmf of the field alone, in ampere turns per pole
M_r = the ampere turns or mmf producing the air-gap flux. It is the resultant of M_f and M_a
IR_e = the effective resistance drop of the armature in volts per phase
IX_l = the leakage reactance drop of the armature in volts per phase

This alternator is assumed to be connected to an "infinite bus." That is, the capacity of the system to which the alternator is connected is such

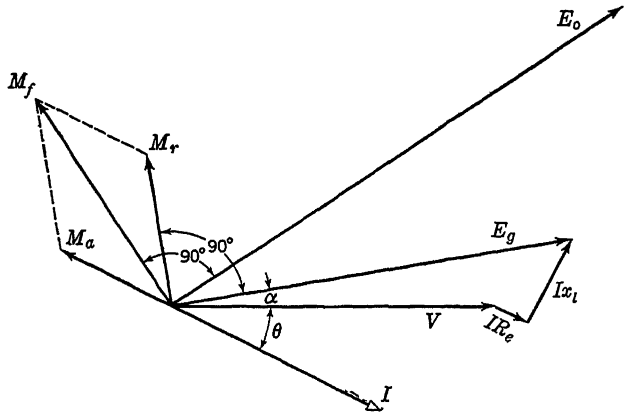


FIG. 250. Vector diagram oi one phase of an alternator.

that no change taking place in this machine will alter the magnitude or position of the bus voltage.

As shown on this figure, the alternator is supplying an output of

$$P_{\text{output}} = mVI \cos \theta \text{ watts}$$

and an input of

$$P_{\text{input}} = mE_gI \cos (\theta + \alpha) + (\text{rotational losses})$$

If the power supplied to the shaft of this alternator is reduced, it momentarily slows down, causing the position of its generated voltage to lag behind its former position. This change is shown in Fig. 251. The output is now less, matching the decreased input. Periodic swings of the vector E_g may have accompanied the transition to the reduced output, but the final steady state is, of course, one in which mechanical input equals output plus losses. Or, with θ equal to 90°, the output is obviously zero.

Note the reduction in the torque angle α. As will be shown in more detail, the change in torque or output is always accompanied by a change in torque angle.

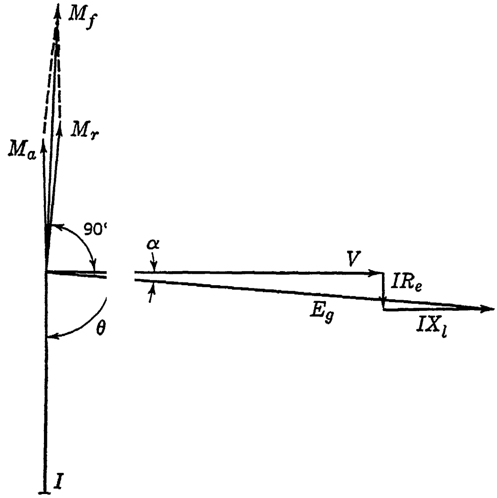


FIG. 251. As compared with the preceding figure, the alternator has had its mechanical driving power reduced so that E_g has fallen back in its phase position, and the current is at zero power factor. The machine is "floating" at this point, ready to become a generator or motor, depending upon the mechanical power at the shaft.

As a next step, suppose the driving power is removed from this alternator and a mechanical load added to the shaft. Electrical power is now supplied to the machine from the bus. Accompanying this change is a further momentary decrease in speed and a new position for the

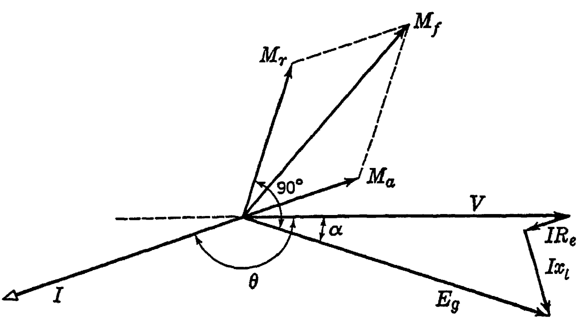


FIG. 252. A mechanical load has been added to the shaft so that the motor draws electrical power from the bus. It is operating as a synchronous motor with leading current.

generated voltage vector E_g behind its previous positions. Figure 252 shows this condition, which is the general vector diagram for the synchronous motor. Note that, in dealing with generator action, the current has a positive projection on the generated and terminal voltage. In motor action, this is not the case, as the angle θ is greater than 90°. The power input is $mVI \cos \theta$, which is then negative. To avoid the use of negative pf's and to deal with slightly simpler diagrams, the theoretically correct position of the vectors (as would follow from the transition from generator action) is neglected, and the diagram is commonly shown as in Fig. 253. In both of these diagrams (Figs. 252 and 253), the torque angles are negative with respect to their positions for generator action, although in Fig. 252 the motor is operating with a leading current; in Fig. 253, with a lagging current.

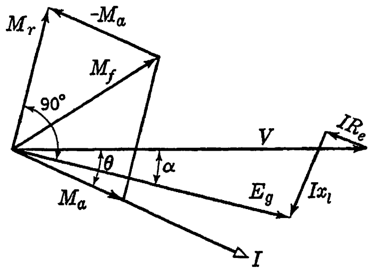


FIG. 253. The synchronous motor of FIG. 252 has now had its excitation reduced so that it is operating at lagging current. But more important than that, the current vector has been swung so that it projects on the voltage vectors rather than on the *negative* voltage vectors, as would be true, strictly speaking, for motor action. This change in convention is purely one of convenience.

Application of This Diagram. In order to make use of such a diagram the following information must be known:

(*a*) The usual name-plate rating.

(*b*) The no-load saturation curve in ampere turns per pole or field amperes.

(*c*) The effective resistance and the leakage reactance per phase.

(*d*) The value of armature reaction turns or ampere turns per pole.

Methods of obtaining such data have been shown under Alternators. The same rules apply to the motor.

Although fairly accurate [1] results can be obtained, the application of this analysis is inconvenient. For example, if the excitation, the saturation curve, and the mechanical load are known, it is difficult to find the

[1] Contrary to what one would expect from comparative results on alternators, the *general diagram analysis* gives less accurate results for salient-pole motors than does the synchronous-impedance method. It is more applicable for salient-pole generators than for motors. See:

The writings of E. Arnold.

A series of articles by V. Karapetoff, "Essays on Synchronous Machinery." *Gen. Elec. Rev.*, 1910 and 1911.

current and the pf. It is likewise difficult to calculate the pull-out torque for any given excitation, or to design the machine so as to obtain a prescribed overload capacity. It is, however, easy to find the field excitation when the other quantities are given, the procedure being almost the same as for the generator. The following procedure is suggested:

(*a*) V, X_l, R_e, M_a, and the saturation curve are known.

(*b*) Assume values of θ from, say, 75° lagging to 75° leading, in intervals of 10° or 15°.

(*c*) For each angle assume the input current per line varies from 0 to 250 per cent of rated, in steps of 25 per cent.

(*d*) For each angle and current, work through to find the required ampere turns of the field, and then field current.

(*e*) Plot the results in a series of V curves or in any other desired form.

320. The Synchronous-impedance Diagram. In this analysis the assumption is made that the armature leakage reactance and the armature reaction can be replaced by an equivalent reactance, which produces

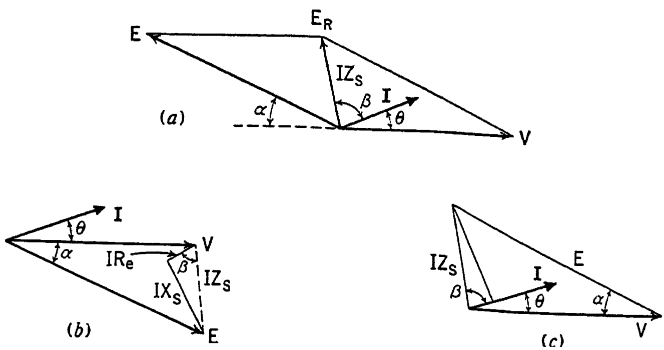


FIG. 254. Various equivalent forms of the synchronous impedance diagram for the synchronous motor with current leading.

similar effects. In strictness, it should be applied only to the non-salient-pole or cylindrical-rotor machine, but in spite of its limited accuracy on salient-pole motors it is much used. Vector diagrams based on the use of synchronous impedance are shown in Fig. 254. In each case the internal power developed, regardless of the method of analysis, is equal to the product of counter emf, the current, and the cosine of the angle between them. Note the torque or power angle on all three diagrams.

Inherent errors in the synchronous-impedance method affect the torque angle and the calculation of excitation for a given pf. This latter

error varies between leading and lagging current, as can be seen from the following considerations.

In the synchronous motor, with fixed applied voltage, the generated voltage and pole-flux leakage are greater with leading than with lagging

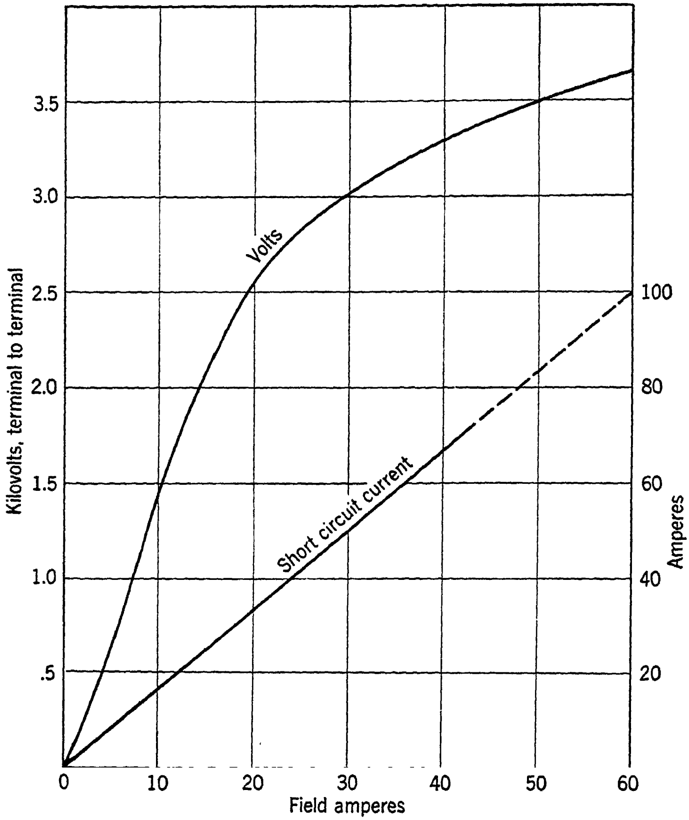


FIG. 255. Test curves. Synchronous motor rated at 187 kv-a, 225 rpm, 60 cycles.

currents. This implies greater saturation in the former case, and more saturation would result in a smaller value of X_S. Hence, inherently, X_S would be larger for lagging-current calculations and smaller for leading currents. Actually, a constant value is assumed when calculations are made. Calculated V curves obtained from such calculations are likely to be shifted from test values. In salient-pole machines, change in pf changes the distribution and value of gap, tooth, and pole-leakage fluxes, and this together with changes in saturation exacts further changes in X_S, and thus in the V curves and in performance.

321. Example. A salient-pole synchronous motor is rated as follows:

187 kv-a	three-phase	2300 volts
47 amperes per terminal		60 cycles
225 rpm	32 poles	
R_e = 1.5 ohms per phase		

The saturation and the short-circuit curves are shown in Fig. 255. The synchronous impedance was calculated at the highest point on the short-circuit curve, and equals 21.1 ohms per phase.

Determine the field current necessary to give (*a*) unity pf, and (*b*) 0.8 leading pf for loads requiring 47 amperes in each case. (*c*) Determine the power developed.

Solution:

$$X \approx Z = 21.1 \text{ ohms}$$

$$IX = 994 \text{ volts}$$

$$IR = 70.5 \text{ volts}$$

$$V_{\text{per phase}} = \frac{2300}{\sqrt{3}} \quad \text{or} \quad 1328 \text{ volts}$$

The diagrams are laid out to scale as shown in Fig. 256. The value required in each case is E. This can be found analytically by using the cosine law, or graphically, as was done here.

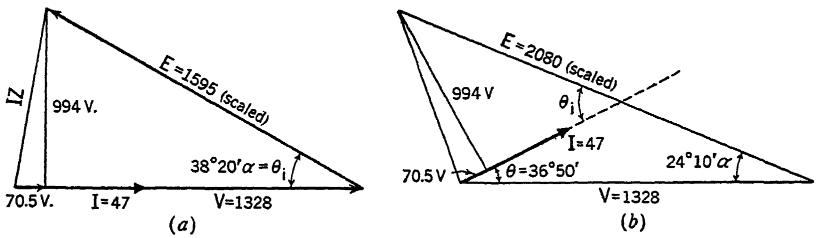


FIG. 256.

(*a*) E (scaled) at unity pf = 1595 volts

$$\sqrt{3}E = 2760 \text{ volts}$$

This corresponds to an excitation of 23.5 amperes.

(*b*) E (scaled) at 0.80 pf, leading = 2080 volts

$$\sqrt{3}E = 3610 \text{ volts}$$

This corresponds to an excitation of 55.5 amperes

(*c*) Internal power developed in (*a*):

$$\theta_i \text{ (scaled)} = 38° \, 20'$$

$$\cos \theta_i = 0.784$$

$$P_{\text{per phase}} = 1595 \times 47 \times 0.784 \quad \text{or} \quad 58{,}800 \text{ watts}$$

$$P_{\text{total}} = 176.4 \text{ kw} \quad \text{or} \quad 237 \text{ hp}$$

Internal power developed in (*b*):

$$\theta_i \text{ (scaled)} = 59° \, 18'$$

$$\cos \theta_i = 0.510$$

$$P_{\text{per phase}} = 2080 \times 47 \times 0.510 \quad \text{or} \quad 50{,}000 \text{ watts}$$

$$P_{\text{total}} = 150.0 \text{ kw} \quad \text{or} \quad 201 \text{ hp}$$

The power input to the motor, minus the copper losses in the armature, represents the power developed, or converted power: i.e., electrical power converted into mechanical power. The friction, windage, and core losses are looked upon as giving retarding torques. If these, as watts, are subtracted from the power converted, the result is the useful output. In the analyses which follow, the term "power developed" is used to signify this converted power. Thus in the above example:

At unity pf:

(*a*) Input $= 3 \times 47 \times 1328 \times 1 = 186.4$ kw

(*b*) $I^2R = 3 \times 47^2 \times 1.5 = 10$ kw

Power converted $= (a) - (b) = 186.4 - 10 = 176.4$ kw

This agrees with the value determined from consideration of the internal voltage and pf angle.

322. American Standards Association Method. Example. In addition to the use of the general and synchronous-impedance methods for obtaining vector diagrams for motor analysis, the American Standards Association method, described under Alternators, is also applicable to motor analysis. Since the general theory has already been given, the work will be applied at once to an example.

Synchronous Motor.

100 hp	8 poles	60 cycles
three-phase	Y-connected	440 volts

Unity power factor.
Resistance of armature per phase (effective at 75° C) = 0.055
Leakage reactance per phase = 0.25 ohm.
Full-load current = 105 amperes.

Although this motor is intended for operation at unity pf, the process will be examined for both unity and 0.8 pf leading. The curves of Fig. 257 pertain to this machine. Note that they are plotted in volts per phase, the normal rated voltage of 440 between terminals resulting in 254 volts to neutral.

Determine the excitation required at full load.

$$IR_e = 105 \times 0.055 \quad \text{or} \quad 5.78 \text{ volts}$$

$$IX_l = 105 \times 0.25 \quad \text{or} \quad 26.2 \text{ volts}$$

Lay off the diagram (Fig. 258*a*) and calculate E_g (250.0). At the 250-volt value on the saturation and air-gap lines read the horizontal distance between the two, indicated as I_{FS}. This is read as 0.4 amperes, and is the correction for saturation. Read the value I_{FG} as 5.6 amperes at normal voltage. Read I_{FSI} as 5.5 amperes.

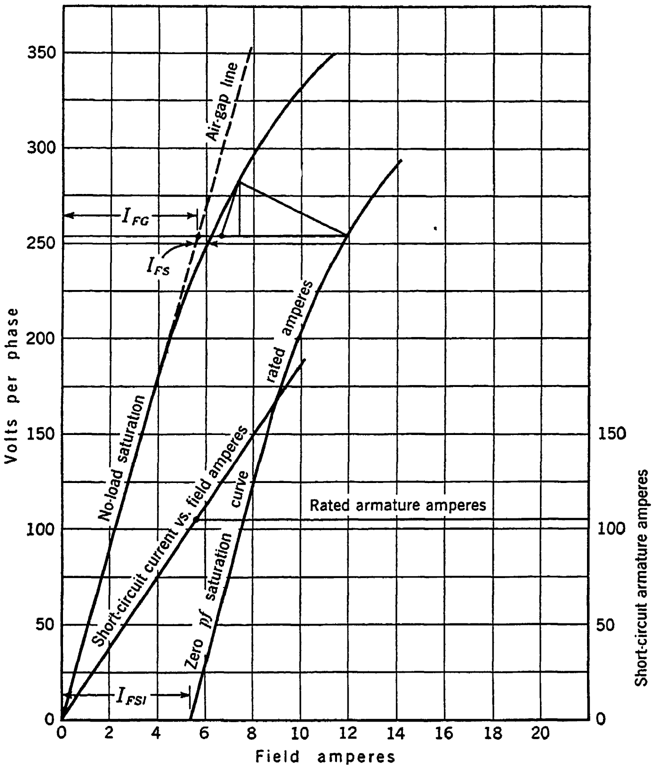


FIG. 257.

The diagram of Fig. 258*b* can now be drawn, noting that I_{FSI}, which makes an angle with the vertical equal to the pf angle, is, of course, vertical for unity pf. The value to be obtained is I_{FL}, which can be calculated or scaled as 8.25 amperes, corresponding on the open-circuit saturation curve, to 303 volts. The full-load field current at unity pf is then 8.25 amperes, and, by selecting other loads or pf's, a series of points could be obtained for the construction of V curves.

To show the effect of pf, and the method of dealing with various pf's by this method, we will assume that the synchronous motor operates with less mechanical load, but with a leading pf of 0.8. The full-load current will again be 105 amperes.

As before, IR_e is 5.78 volts, and IX_l is 26.2 volts. Lay off the diagram of Fig. 259*a*, calculating or scaling the value of E_g as 268 volts.

At the 268-volt ordinate on Fig. 259 note that the horizontal distance between the air-gap line and the open-circuit saturation curve is $I_{FS} = 0.6$ amperes. With $I_{FG} = 5.6$, and $I_{FSI} = 5.5$, lay off the diagram of Fig. 259*b*, noting that I_{FSI} makes

an angle with the vertical equal to the pf angle. For underexcited motors (lagging current), the vector I_{FSI} projects to the left of the vertical.

Calculate or scale I_{FL}, equaling 10.52 amperes field current. This is the excitation required to deliver approximately 80 hp at a pf of 0.8 leading.

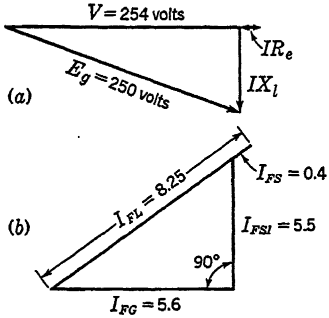


FIG. 258.

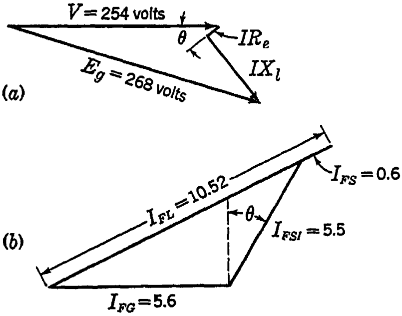


FIG. 259.

323. Two-reaction Method. All the diagrams presented up to this point suffer in accuracy when applied to salient-pole machines because of the non-uniformity of the gap reluctance and the resulting distortions in the space distribution of flux in the air gap. Since most synchronous motors are built with salient poles, the methods are not entirely accurate although experience indicates that the synchronous-reactance method is not so inaccurate for motors as for generators.

The Doherty-Nickle extension of the Blondel two-reaction theory can be applied with great accuracy to the salient-pole synchronous motor not only for vector-diagram representation of the operation but also for the mathematical expression of it. It will be given here in its simplest and least exact form. As explained under Alternators, the armature current is divided into two components, in phase and in quadrature, respectively, with the position of the air-gap flux or excitation voltage. The effect of armature reaction will be combined with the leakage reactance into two components, represented by equivalent reactances, X_d and X_q.

Compare diagrams, illustrating the change between generator and motor action, are shown in Fig. 260. Note that the armature current is shown for the motors in its correct position with zero projection on the voltage vectors. It will be noted that, with a change in pf from lead to lag, the component I_d changes in direction, which is matched by a corresponding change in the I_dX_d drop. Physically, of course, this agrees with the fact that at leading currents the direct component of armature reaction demagnetizes the impressed field; at lagging currents armature reaction assists the impressed field. Examination of these facts is an aid in determining the correct construction for the vector diagrams.

This method can be used to determine the excitation for any load and pf (points on the V curves) or for study of the torque angle and per-

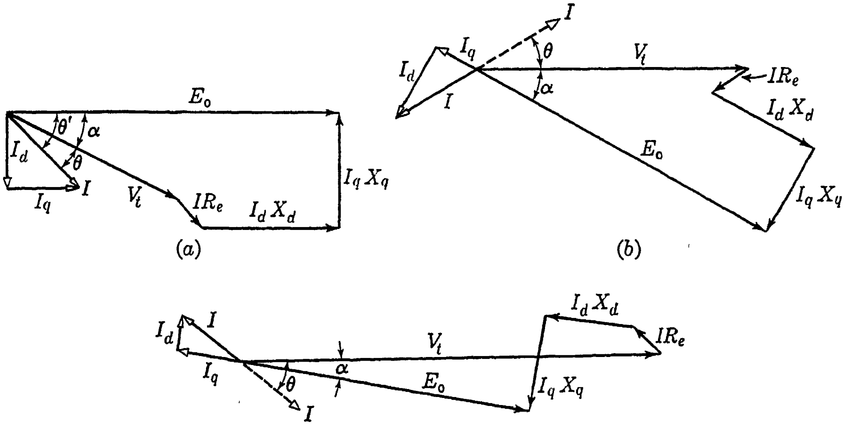


FIG. 260. Vector diagrams for the two-reaction method. (a) The usual diagram for generator action. (b) Leading current. For motor action the correct current vector position is such that it has no projection on the voltage vector. (c) Lagging current changes the direction of I_d. On all of these diagrams the resistance drop is exaggerated.

formance characteristics. Like the Blondel analysis, the diagram cannot be laid out directly because several positions must be known before it can be begun. Therefore, further examination of relationships will be of value.

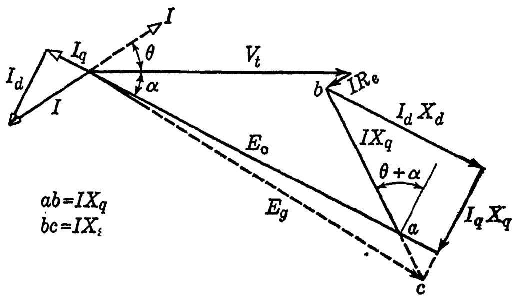


FIG. 261.

324. Analysis of the Diagrams. The vector diagram for leading current is repeated in Fig. 261, in more detailed form. Again the resistance drop has been exaggerated, although in cases of large machines little error is introduced by neglecting it entirely.

To determine the angle α:

$$ab = \frac{I_q X_q}{\cos(\theta + \alpha)} \qquad [525]$$

$$I_q = I \cos(\theta + \alpha) \qquad [526]$$

$$\therefore\ ab = \frac{I X_q \cos(\theta + \alpha)}{\cos(\theta + \alpha)} = I X_q \qquad [527]$$

Starting with the vector V_t for terminal volts per phase and an assumed pf, the position and magnitude of IR_e and IX_q are known so that the point a can be located. This fixes the direction of E_0.

Analytically, using V_t as the reference vector:

$$\begin{aligned} Oa &= V_t - I(R_e + jX_q)(\cos\theta + j\sin\theta) \qquad [528] \\ &= V_t - I(R_e\cos\theta - X_q\sin\theta) - jI(R_e\sin\theta + X_q\cos\theta) \end{aligned}$$

$$\therefore\ \tan\alpha = \frac{-I(R_e\sin\theta + X_q\cos\theta)}{V_t - I(R_e\cos\theta - X_q\sin\theta)} \qquad [529]$$

This fixes the value of α.

An expression for E_0 is of value. Using it as the reference vector,

$$\begin{aligned} E_0 = {} & V_t(\cos\alpha + j\sin\alpha) \\ & -IR_e[\cos(\theta + \alpha) + j\sin(\theta + \alpha)] \\ & -jIX_q[\cos(\theta + \alpha)] + IX_d[\sin(\theta + \alpha)] \qquad [530] \end{aligned}$$

For leading current, both θ and α are positive for motor action with respect to the vector E_0. Since E_0 is the reference vector, the j terms of the above equation will cancel numerically.

It is of interest to note in dealing with this diagram that the length ab is proportional to the IX_q drop. Suppose that we were dealing with the motor by the synchronous-reactance method, the IX_S drop would be in the same phase position as the IX_q vector (in quadrature with the current) but would be larger. That is so because synchronous reactance and the direct-axis component X_d of the two-reaction theory are identical. The quadrature reactance X_q is always smaller than X_d because of the greater reluctance in the air gap of the quadrature axis. Accordingly, by the synchronous-reactance method, the generated voltage would be E'_0, and the end of this vector would lie on the projection of I_qX_q. Usually there may be very little difference in the magnitude of E_0 and E'_0. As a result, the excitation required for a given set of conditions, or calculated by the synchronous-impedance and the two-reaction theories, may not differ greatly. But note that the angle between the

excitation and terminal voltages may be greatly increased by the use of synchronous impedance. This is the torque angle, and, for certain problems requiring the value of this angle, the more exact two-reaction analysis is essential.

Unity Power Factor. In this case the torque angle expression reduces to the simple formula:

$$\tan \alpha = \frac{-IX_q}{V_t - IR_e} \qquad [531]$$

and, using E_0 as the reference vector for which $\sin \alpha$ is not negative,

$$E_0 = (V_t - IR_e) \cos \alpha + IX_a \sin \alpha \qquad [532]$$

This diagram is shown in Fig. 262.

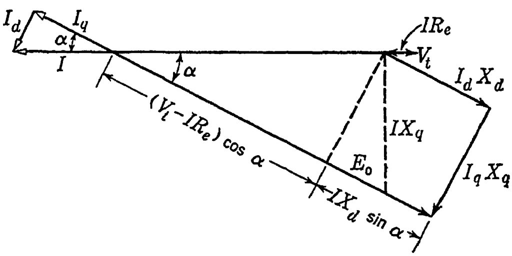


FIG. 262. Vector diagram for unity pf; motor action.

325. Example. Two-reaction Method. The 100-hp 8-pole, synchronous motor (with salient poles), used in Article 322, will be examined by the two-reaction method, obtaining expressions for torque angle and field excitation. The 100 per cent pf condition will be assumed, giving 105 amperes per terminal.

As before, IR_e is 5.78 volts, but we must know in addition that X_d equals 2.25 ohms and X_q equals 1.28 ohms. They are presumed to be obtained from test, and hence are unsaturated values. For a first example, we will use them directly as given, repeating the work with a more elaborate correction for saturation.

From equation 531:

$$\tan \alpha = \frac{-105 \times 1.28}{254 - 105 \times 0.055} \quad \text{or} \quad -0.544$$

$$\alpha = -28^\circ\, 33' \sin \alpha = -0.478$$

$$\cos \alpha = 0.878$$

From equation 532:

$$E_0 = (254 - 105 \times 0.055)0.878 + 105 \times 2.25 \times 0.478$$

$$= 217.0 + 113.0 \quad \text{or} \quad 330.0 \text{ volts}$$

This corresponds to an excitation of 9.9 amperes, field current, on the no-load saturation curve. Hence we have a value contrasting with 8.25 amperes by the A.S.A. method, which is not strictly applicable to this salient-pole machine.

More Accurate Method. To correct for saturation, determine the internally generated voltage.

$$E_g = V_t - I(R_e + jX_l) \qquad [533]$$

In this case this is 250 volts, and for this excitation the same field current would give 263 volts on the air-gap line. (See Fig. 257.) This ratio (for these calculations) will be called the saturation factor. The problem is now to correct the *armature reaction portion* of X_q and X_d by the saturation factor. We know that Potier reactance $\approx X_l \approx 0.25$ ohms. Hence the armature reaction portion is

$$X'_d = 2.25 - 0.25 = 2.00$$

$$X'_q = 1.28 - 0.25 = 1.03$$

$$X_d(\text{saturated}) = 2.00 \times \frac{250}{263} + 0.25 \quad \text{or} \quad 2.15$$

$$X_q(\text{saturated}) = 1.03 \times \frac{250}{263} + 0.25 \quad \text{or} \quad 1.234$$

The above method assumes (1) leakage reactance is not affected by saturation; (2) saturation is the same in both direct and qradrature axes. Neither is strictly true.

We now have (presumably) two more correct values for reactances which will be applied as before.

$$\tan \alpha = \frac{-105 \times 1.234}{254 - 5.78} \quad \text{or} \quad -0.522$$

$$\alpha = -27^\circ\, 34' \sin \alpha = -0.4628 \cos \alpha = 0.8865$$

Then

$$E_0 = (254 - 105 \times 0.055)0.8865 + 105 \times 2.15 \times 0.4628$$

$$= 220 + 104.6 \quad \text{or} \quad 324.6 \text{ volts}$$

This value, being already corrected for the proper degree of saturation, is now referred to the *air-gap line* (Fig. 257) in order to determine the field current. The value read is 7.6 amperes (versus 9.9 by uncorrected reactances, and 8.25 by the A.S.A. method).

CHAPTER XXXIX

MATHEMATICAL ANALYSIS OF MOTOR PERFORMANCE

326. Chapter Outline.

Mathematical Analysis of Synchronous Motor Performance.[1]
- Torque Angle.
- Limits of Operation.
  - Maximum Power under Various Conditions.
  - Maximum Excitation.
- Stiffness of Coupling.
- Two-reaction Theory.
  - Example.

327. Introduction. The internal power developed by a synchronous motor is always equal to the product of the counter or generated emf, the armature current, and the cosine of the angle between them. This is true regardless of the method of analysis (synchronous impedance, mmf, etc.) used.

The following analysis is made in order to present the classical synchronous-impedance theory in analytical form. It is not as truly representative of the actual phenomena as the more elaborate methods, but it will serve for most of the analyses given herein, except for the salient-pole machine where greater accuracy is desired. As these formulas are supposed to hold for the whole range of operation, the assumption of constant x_s, which is made, is obviously in error. So also is the assumption that the effective resistance of the armature remains constant. The possible forms of the work are widely different, but the results (though not the form in which they appear) are the same for all, and are subject to all the limitations of the synchronous-impedance method.

[1] References:

R. R. Lawrence, "Principles of A. C. Machinery," McGraw-Hill Book Co.

C. P. Steinmetz, "A. C. Phenomena" and "Theoretical Elements of Electrical Engineering," McGraw-Hill Book Co.

E. Arnold and J. L. LaCour, "Die synchronen Wechselstrommachinen," Julius Springer, Berlin.

M. Mauduit, "Machines électriques," Dunod, Paris.

Consider the vector diagram of Fig. 263, in which the effect of armature reaction is replaced by synchronous reactance.

Let V be the reference vector. Then

$$E = e - je' \qquad [534]$$

$$I = i - ji' \qquad [535]$$

Let r_e = the effective resistance of the armature.

x_s = the synchronous reactance.

Then

$$Z_s = \sqrt{r_e^2 + x_s^2}$$

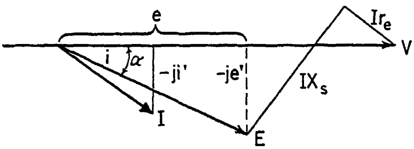


FIG. 263.

The effective voltage capable of forcing current through the motor is $V - E$, subtracted vectorially. Then

$$I = \frac{V - E}{Z_s} \qquad [536]$$

$$= \frac{V - (e - je')}{r_e + jx_s} \qquad [537]$$

This can be rationalized by multiplying by the conjugate function $(r_e - jx_s)/(r_e - jx_s)$:

$$\frac{V - (e - je')}{r_e + jx_s} \cdot \frac{r_e - jx_s}{r_e - jx_s}$$

$$= \frac{(Vr_e - r_e e + x_s e') - j(-r_e e' + x_s V - ex_s)}{r_e^2 + x_s^2} \qquad [538]$$

Since the power is equal to $ei + e'i'$ and equation 538 is an expression for current, equivalent to $i - ji'$, then

$$P_{\text{developed}} = \frac{e(Vr_e - r_e e + x_s e') + e'(-r_e e' + x_s V - ex_s)}{r_e^2 + x_s^2} \qquad [539]$$

From the diagram it is obvious that

$$e = E \cos \alpha \quad \text{and} \quad e' = E \sin \alpha$$

where α is the torque angle.

Substituting in equation 539,

$$P_{\text{developed}} = \frac{VE(r_e \cos \alpha + x_s \sin \alpha) - r_e E^2}{r_e^2 + x_s^2} \qquad [540]$$

Another interesting relationship can be obtained by the use of the internal angle between r_e and Z_s. This is shown in Fig. 254 as β. Then

$$r_e = Z_s \cos \beta \quad \text{and} \quad x_s = Z_s \sin \beta \qquad [541]$$

Substituting in equation 540,

$$P_{\text{developed}} = \frac{VE \cos (\beta - \alpha) - E^2 \cos \beta}{r_e^2 + x_s^2} \qquad [542]$$

Let us next assume that the effective resistance of the armature is small as compared to the synchronous reactance. This will be increasingly in error with reduction in size of the motor. Then equation 540 becomes

$$P = \frac{VE \sin \alpha}{x_s} \qquad [543]$$

This is an important relationship as it shows that the developed power varies inversely as the synchronous reactance of the machine, and directly as the sine of the torque or power angle α. This is in keeping with previous statements to the effect that no change in power can take place in either the alternator or synchronous motor with fixed voltage and excitation, other than by a change in torque angle. Strictly speaking, in generators the torque angle is positive and in motors it is negative, although that distinction is not entirely maintained here in dealing with only one type of action because various common forms of the vector diagram are being illustrated.

If resistance of the armature winding can be neglected, the torque angle is zero at zero power, approaching a maximum when the sine of α is a maximum. Obviously this would be unity at 90°. If the resistance is considered, the torque angle may be greater than zero before mechanical power is delivered by the motor, and the position of maximum power is not 90° but an angle whose cosine is r_e/Z_s.

A further exposition of this phenomenon is shown in Fig. 264. If it is assumed that a synchronous motor, delivering full-load output operates at unity pf with a torque angle of 20 electrical degrees; the vector diagram is as shown in Fig. 264*a*. A schematic representation of this condition is illustrated in *b*, salient poles being shown for clarity of pole position. Any increase in mechanical load results in further increase in torque angle and power delivered until a 90° angle is reached. Since the

sine of 20° is 0.34 and the maximum value is 1, it follows that the ratio of pull-out torque to full-load torque in this particular motor is

$$\frac{1.0}{0.34} \approx 3.0$$

If instead, the armature reaction (and hence x_s) were such that at full load α was 30°, the sine of α would be 0.5 and the pull-out torque or overload capacity would be only twice rated value. This is a theoretical

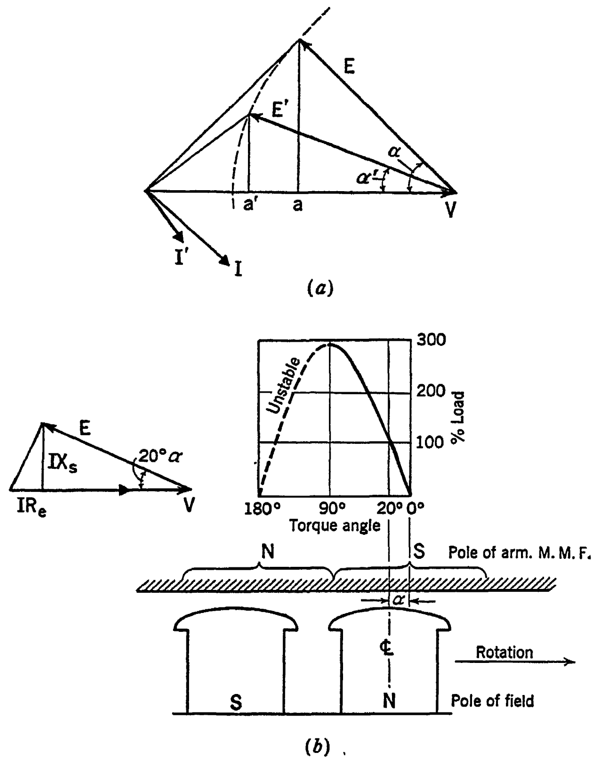


Fig. 264.

ratio, for N.E.M.A. Standards require that a motor shall be capable of delivering the rated pull-out torque at infrequent intervals for periods of at least one-minute duration. Actually, at the pull-out point the motor is very unstable, likely to pass the peak and pull-out of synchronism at the slightest change in voltage or fluctuation of load. Hence the nominal value of pull-out torque must be set at a figure under the theoretical value to provide a reasonable margin for such contingencies.

It will be noted in the equation for torque or power developed that

these values vary directly with the excitation voltage E. This indicates that, for a given power, a decrease in torque angle would result from an increase in excitation. Hence the apparent ratio of maximum to full-load torque would be larger if a motor were operated at 0.80 leading pf, rather than unity pf. While this is true, it must be remembered that a motor built for continuous operation at leading (or lagging) pf must carry more current for a given load than under unity pf operation. Furthermore, for a given load, more exciter capacity and field-coil radiating surface may be required. Hence the apparent increase in "reserve" torque or power available by operating at leading currents must be paid for in more generous motor proportion if the machine is not to overheat.

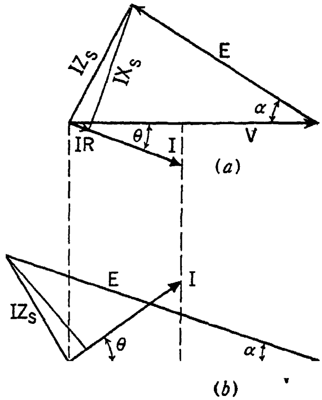


FIG. 265.

The comparative change in torque angle for under- and over-excitation is shown in Fig. 265. In both cases the power input may be the same since the in-phase portions of the current are identical on the two diagrams.

328. Various Conditions of Maxima.

I. *Fixed V, E, r_e, and x_s.* Under these conditions the maximum power developed will occur when $(dP_d)/(d\alpha) = 0$. Thus:

$$\frac{dP_d}{d\alpha} = \frac{VE}{Z_s^2}(-r_e \sin\alpha + x_s \cos\alpha) = 0 \qquad [544]$$

and

$$\frac{\sin\alpha}{\cos\alpha} = \frac{x_s}{r_e}$$

Hence the maximum power is developed by a motor for any given value of applied voltage and excitation when the load is such as to make the torque angle $\tan^{-1} x_s/r_e$. The value of the maximum power can be found by substituting in equation 540. Since $\sin\alpha = x_s/Z_s$, the maximum power under the above conditions is

$$P_{dm} = \frac{VE\left(\frac{r_e^2}{Z_s} + \frac{x_s^2}{Z_s}\right) - r_e E^2}{Z_s^2} \qquad [545]$$

$$= \frac{VE}{Z_s} - \frac{r_e E^2}{Z_s^2} \qquad [546]$$

This is the point at which the motor breaks out of step.

II. *Fixed V, r_e, and x_s.* It must be remembered that the maximum power developed as shown by equation 546 is for any fixed value of excitation or generated voltage, E. Suppose now that the excitation were varied to a value necessary to give the maximum power possible. This excitation can be evaluated by differentiating equation 546, with respect to E, and equating to zero. Thus:

$$\frac{dP_{dm}}{dE} = \frac{V}{Z_s} - \frac{2r_eE}{Z_s^2} = 0 \quad [547]$$

and

$$E = \frac{VZ_s}{2r_e} \quad [548]$$

This is the value of the motor-generated voltage to give the maximum power, but it is not the maximum possible value of generated voltage at which the motor will operate. The maximum motor power possible can be found by substituting E from equation 548 in equation 546. Then

$$P'_{dm} = \frac{V^2}{4r_e} \text{ watts per phase} \quad [549]$$

III. *Maximum Excitation.* The maximum excitation at which the machine will operate as a motor is that which makes the power developed equal to zero; any increase in excitation beyond that value would give generator, rather than motor, action.

$$P_d = \frac{VE}{Z_s} - \frac{r_eE^2}{Z_s^2} = 0 \quad [550]$$

$$E = \frac{VZ_s}{r_e} \quad \text{or} \quad \frac{V}{\cos\beta} \quad [551]$$

This is the maximum excitation. The minimum from this analysis is zero, although physically the machine will remain in step and carry some load, even with a negative field excitation, but will finally break out of step as I_f increases in the negative direction. Refer to Fig. 266*a*. When E is increased to the point where I is 90° from V, the input is zero. When I is 90° from E, the power converted is zero. Any further increase in E would result in generator action.

Comparison of equations 548 and 551 shows that the generated voltage for the maximum possible power converted is one-half of the theoretical maximum of generated voltage for motor action.

A problem of some practical importance is that of determining the limits of excitation for any load or value of developed power. Refer to equation 546.

$$P_{dm} = \frac{VE}{Z_s} - \frac{r_e E^2}{Z_s^2}$$

Solving for E,

$$E = \frac{Z_s}{2r_e}\left(V \pm \sqrt{V^2 - 4r_e P_{dm}}\right) \qquad [552]$$

The two values of E which satisfy this equation for a fixed value of power developed are the excitation limits.

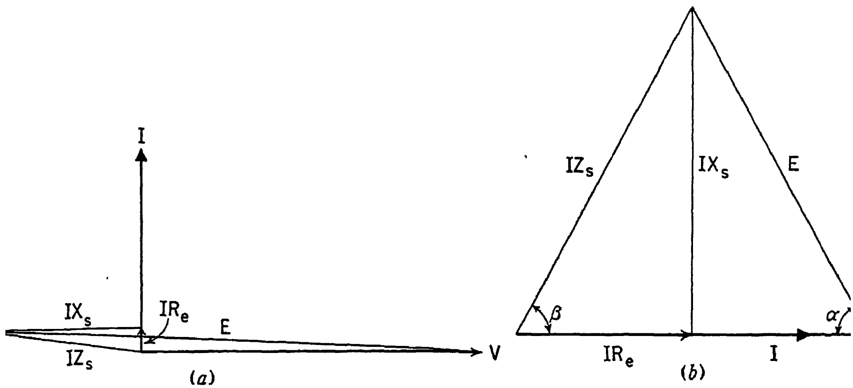


FIG. 266. (a) Vector diagram for maximum excitation. (b) Vector diagram for maximum power.

Attention should be given to such equations as 548 and 551, indicating excitation voltages which would be very large, owing to the fact that the effective resistance of the armature winding is always comparatively small. Usually it is physically impossible to achieve these maximum conditions because of saturation and practical limits on excitation. Hence it must be emphasized that they are of theoretical interest only because the assumptions on which they are based are not in sufficient agreement with the physical facts.

329. Current and Power Factor under Conditions of Maximum. It has been shown (equation 544) that the maximum power for given values of E, V, r_e, and x_s will occur when the torque angle α is increased to the point where $(x_s)/(r_e)$ equals the tangent of α. Examination of Fig. 266b shows that this condition can occur only when α is equal to the internal impedance angle β. The diagram then takes the form indicated.

Under these conditions:

$$E = \frac{VZ_s}{2r_e} \qquad [553]$$

$$= IZ_s \qquad [551]$$

or

$$I = \frac{V}{2r_e} \qquad [555]$$

The pf is then unity, and the efficiency is 50 per cent. For machines built commercially, the current indicated by equation 555 is an excessive value, far beyond the normal range of the motor.

330. Value of x_s for Maximum Power. For fixed values of generated voltage, applied voltage, and effective resistance, the value of synchronous reactance which will give maximum power can be found by differentiating the power equation, with respect to x_s, and equating the derivative to zero:

$$P_{dm} = \frac{VE}{\sqrt{r_e^2 + x_s^2}} - \frac{r_eE^2}{r_e^2 + x_s^2}$$

$$= \frac{VE\sqrt{r_e^2 + x_s^2} - r_eE^2}{r_e^2 + x_s^2}$$

$$\frac{dP_{dm}}{dx_s} = \frac{(r_e^2 + x_s^2)VE\dfrac{2x_s}{2\sqrt{r_e^2 + x_s^2}} - (VE\sqrt{r_e^2 + x_s^2} - r_eE^2)2x_s}{(r_e^2 + x_s^2)^2} = 0$$

By canceling out $(2x_s)/(r_e^2 + x_s^2)^2$,

$$0 = \frac{-VE\sqrt{r_e^2 + x_s^2}}{2} + r_eE^2$$

and

$$x_s^2 = r_e^2\left(4\frac{E^2}{V^2} - 1\right) \qquad [556]$$

This gives the value of x_s, or synchronous reactance, necessary to develop the maximum power. Let us assume that a motor is to be operated at 100 per cent excitation. That means that E equals V. Then from equation 556,

$$x_s = \sqrt{3}r_e \qquad [557]$$

This implies that for maximum power for 100 per cent excitation the reactance should be $1.73r_e$. To take advantage of the outstanding characteristic of the synchronous motor, i.e., its ability to operate with in-

phase or with leading current, it is necessary that E is at least equal to or greater than V. Under the latter condition, x_s must be greater than $1.73r_e$. Other factors entering into design are more important in determining x_s and fixing it at comparatively larger values than would be indicated from this consideration. Hence this also is an analysis of theoretical interest only, having no practical importance in design or operation.

331. Stiffness of Coupling. This factor has been pointed out previously (Article 308.) It may be defined as the tendency of the motor torque angle to follow irregularities in the speed of its supply generators. Too stiff a coupling causes the motor to be subjected to shocks caused by any pulsations in the load or the sources of supply. Too soft a coupling implies a large torque angle change with load variation, and results in instability. A change in load results in a comparatively great change in pf if the excitation is kept constant.

Stiffness of coupling is influenced by all those factors which affect synchronous reactance. Hence a machine is loosely coupled if it has a large leakage reactance or if it has a large armature reaction effect (which might arise from a small air gap).

It will be seen from the foregoing statements and analyses that armature reaction in a synchronous motor is fully as important in determining its characteristics as it is in the alternator. A motor with a small synchronous reactance (and good regulation as an alternator) would be necessary for large overload capacity. This is true only for loads applied gradually. Pioneer synchronous motors were built with small synchronous impedance, i.e., "hard coupled." They were commercial failures because any suddenly applied load pulled them out of step. Because of their low reactance, the greatly increased current, during the internal readjustments necessary to load increase, gave an excessive line drop. This difficulty is overcome with the more flexible "soft-coupled" motor, which is less sensitive to changes in voltage and load and relatively less expensive. These advantages outweigh the desirability of large overload capacity for the average motor.

Equation 544 gives an expression for the change in power developed with a change in the torque angle, i.e., $dP_d/d\alpha$. It was found that

$$\frac{dP_d}{d\alpha} = \frac{VE(x_s \cos \alpha - r_e \sin \alpha)}{Z_s^2}$$

or, by using equation 542,

$$\frac{dP_d}{d\alpha} = \frac{VE \sin (\beta - \alpha)}{Z_s} \qquad [558]$$

In a sense, $(dP_d)/(d\alpha)$ is a "rigidity factor" inasmuch as it indicates a change in the power developed for a change in torque angle, and such a change expresses the stiffness or rigidity of coupling. Note that it increases directly with the counter emf. Hence an over-excited motor is more rigidly coupled and has greater overload capacity, as pointed out previously.

This factor is the same, numerically, as the synchronizing power, abbreviated P_r and used in the N.E.M.A. Standards.

The synchronizing power is the power at synchronous speed corresponding to the synchronizing torque tending to restore the rotor to the no-load position.

P_r is the rate of change of the steady state synchronizing power with respect to the displacement angle at normal voltage and rated load, power factor and frequency. It is expressed in kw. at synchronous speed corresponding to the torque exerted on the rotor per radian displacement angle.

Because this term will be used elsewhere throughout this text, the ideas will be further illustrated with an example.

Example.

200-hp motor	440 volts	three-phase
8 poles	60 cycles	unity pf

I = 209 amperes

Z_s = 1.625 ohms per phase

r_e = 0.045 ohm per phase

Volts per phase = 254

E_0 at full load = 325

$$\beta = \text{arc tan } \frac{r_e}{x_s} \quad \text{or} \quad 87°\ 29'$$

α = 41° 17′ at full-load, unity pf

From equation 558,

$$\frac{dP}{d\alpha} = \frac{254 \times 325 \times \sin(87°\ 29' - 41°\ 17')}{1.025}$$

$$= 58{,}000 \text{ watts per electrical radian per phase}$$

$$P_r = \frac{58{,}000 \times 3 \text{ phases}}{1000} \quad \text{or} \quad 174 \text{ kw per electrical radian}$$

332. Two-reaction Theory. All the foregoing analyses have been based on synchronous-reactance concepts. Once more it will be pointed out that this implies cylindrical-rotor or non-salient-pole construction for

theoretical accuracy. The two-reaction theory applicable to the salient-pole machine will now be considered. All the investigations just made are not available by the two-reaction method or else they result in expressions so complicated that they militate against their own use. Some of the more fundamental relationships will be derived.

Power Input. We shall first deal with the expression for power input, referring to Fig. 267. This is obviously $V_tI \cos \theta$ watts per phase, but to obtain this in terms of motor constants a further derivation must be made. For simplification, the effects of armature resistance will be neglected.

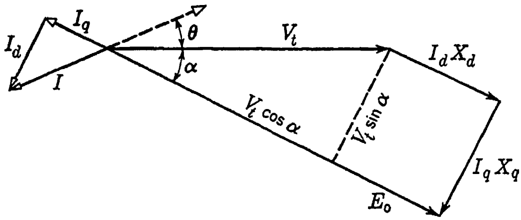


FIG. 267. Synchronous motor vector diagram with leading current.

In general, power is equal to the product of the in-phase portions of current and voltage plus the product of the quadrature components. This is true regardless of the axis of reference. Using E_0 as the reference vector, we obtain $V_t \cos \alpha$ and $V_t \sin \alpha$ as the in-phase and quadrature components, respectively. Then power input is

$$P_{\text{input}} = I_qV_t \cos \alpha + I_dV_t \sin \alpha \qquad [559]$$

also,

$$V_t \cos \alpha = E_0 - I_dX_d \qquad [560]$$

and

$$V_t \sin d = I_qX_q \qquad [561]$$

$$\therefore \; I_d = \frac{E_0 - V_t \cos \alpha}{X_d} \qquad [562]$$

and

$$I_q = \frac{V_t \sin \alpha}{X_q} \qquad [563]$$

Substituting these expressions for current in equation 559 and rearranging,

$$P_{\text{input}} = \frac{V_t^2 \sin \alpha}{X_q} \cdot \cos \alpha + \frac{V_tE_0 - V_t^2 \cos \alpha}{X_d} \sin \alpha$$

$$= \frac{V_tE_0}{X_d} \sin \alpha + \frac{V_t^2(X_d - X_q)}{2X_dX_q} \sin 2\alpha \qquad [564]$$

In dealing with synchronous-reactance theory, we obtained expressions for *power developed*, which is less than the input by the armature copper losses. The power developed by this analysis would be

$$P_d = \frac{V_t E_0}{X_d} \sin \alpha + \frac{V_t^2(X_d - X_q)}{2X_d X_q} \sin 2\alpha - (I_q^2 + I_d^2) r_e \quad [565]$$

It is possible to substitute expanded expressions for the current components in the above equation, in terms of voltages and reactances. When this is done, the equation becomes too unwieldy. Accordingly, we will deal with the expression for power *input* rather than for power *developed*. In a well-designed machine, the difference is only a few per cent.

333. Power and Torque Angle. Thinking of equation 564 as a measure of developed power, we find a very interesting relationship for power as a

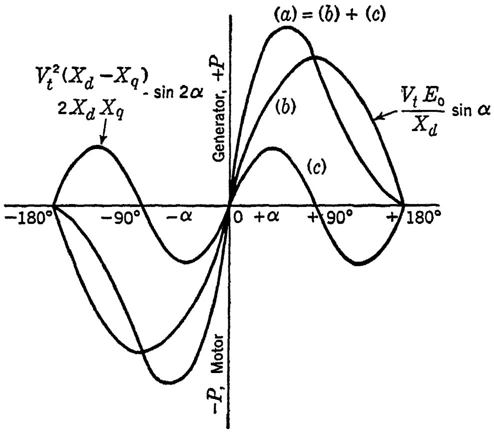


FIG. 268. In a salient-pole machine considered by the two-reaction theory, the torque-angle change brings about a change in developed power or torque as a function of a sine wave plus a second harmonic. This curve is shown as *a*. Motor action is represented by a negative torque angle and negative power. By the synchronous-reactance theory, correctly applied only to the non-salient-pole machines, the power is a simple sine function of the torque angle as shown by *b*.

function of torque angle. Unlike the non-salient-pole machine, the developed torque or power does not vary simply as the sine of the torque angle. Instead, one component varies as the sine of this angle; the other as the sine of twice the angle. Or, stated another way, the torque developed per degree of torque angle in the salient-pole machine, over the usual range of operation as a motor, is greater than in the non-salient machine. Furthermore, the maximum torque occurs at a reduced angle as shown in Fig. 268.

Let us suppose that this equation were applied to the non-salient-pole machine. In the first member of the left-hand side of the equation, X_d equals X_s, and this term becomes identical with that already developed in the first part of this chapter. In non-salient-pole motors, X_d equals X_q and so the second term reduces to zero. We can conclude then that additional torque is exerted by the synchronous motor, owing to the saliency of the poles, equivalent in watts per phase to

$$\frac{V_t^2(X_d - X_q)}{2X_dX_q} \sin 2\alpha \qquad [566]$$

This might be called the "reluctance power" of the salient-pole motor since it arises from the difference in reluctance between the direct- and quadrature-axis paths. Note that it is independent of the internally generated voltage E_0, owing to d-c excitation. Accordingly, this is the power which the synchronous motor will develop if it is unexcited.

334. Additional Expression for Power. A second expression for power input can be obtained by reconsideration of the voltage components $V_t \cos \alpha$ and $V_t \sin \alpha$. Note in Fig. 267 that

$$V_t \cos \alpha = E_0 - I_dX_d$$

$$V_t \sin \alpha = I_qX_q$$

If these expressions are used in equation 559, we obtain

$$P_{\text{input}} = I_q(E_0 - I_dX_d) + I_dI_qX_q \qquad [567]$$

$$= E_0I_q - I_qI_d(X_d - X_q) \qquad [568]$$

This expression for power is also useful in considering the physical action in the motor. Recall that the two-reaction theory assumes that the mmf of the armature can be divided into sets of direct-axis and quadrature-axis poles. Considering only the "main" poles which are in line with the actual physical poles, the emf generated would be $E_0 \pm I_dX_d$. This is the resultant of field and armature mmf's and would take the negative sign for leading currents in the motor. Since this voltage would be in phase with the current component I_q, the power developed by the main poles is $I_q(E_0 - I_dX_d)$. Similarly, the voltage of the quadrature axis, produced by the second set of fictitious poles, is I_qX_q, having the current component I_d in phase with it. Hence the power of the quadrature axis is $I_dI_qX_q$, which corresponds to the "reluctance power" previously mentioned.

In this analysis to achieve sufficient simplicity, the effect of Ir_e drop has been disregarded, and also the fact that the true developed power is less than the power input.

335. Stability and Maximum Torque Angle. By differentiating the equation

$$P = \frac{V_t E_0}{X_d} \sin \alpha + \frac{V_t^2 (X_d - X_q)}{2 X_d X_q} \sin 2\alpha$$

with respect to torque angle, one obtains an expression for the rate of change of power as a function of the torque angle. This has been called the "stability" factor, "rigidity" factor, or simply "stiffness of coupling." Then:

$$\frac{dP}{d\alpha} = \frac{V_t E_0}{X_d} \cos \alpha + \frac{V_t^2 (X_d - X_q)}{X_d X_q} \cos 2\alpha \qquad [569]$$

This is approximate since it is based on power input rather than power developed. However, it indicates that whereas the synchronous-reactance method gave in its approximate form an expression

$$\frac{V_t E_0}{X_s} \cos \alpha$$

equation 569 contains a second term, implying greater stiffness of coupling for the salient-pole motor. This greater stiffness would naturally be expected from examining the curves of Fig. 268.

Equating the right-hand member of equation 569 to zero, we have an equation from which the maximum torque angle can be calculated. That is,

$$\frac{V_t E_0}{X_d} \cos \alpha + \frac{V_t^2 (X_d - X_q)}{X_d X_q} \cos 2\alpha = 0 \qquad [570]$$

Let

$$B = \frac{V_t (X_d - X_q)}{X_q} \quad \text{and solve for } \cos \alpha\text{:}$$

$$\cos \alpha \approx \frac{-E_0}{4B} \pm \sqrt{\frac{1}{2} + \left(\frac{E_0}{4B}\right)^2} \qquad [571]$$

This is the cosine of the maximum torque angle when r_e is neglected.

336. Example. All the items just discussed will be illustrated by a numerical example using the following motor

200 hp   8 poles   60 cycles   440 volts   three-phase

Unity pf $r_e = 0.0285$ ohm per phase

$X_d = 1.27$ ohms per phase $X_q = 0.774$ ohm per phase

Full-load amperes = 210 amperes per terminal

As shown in Article 324, the vector diagram reduces so that $\theta' = \alpha$, and

$$\tan \alpha = \frac{-IX_q}{V_t - Ir_e} = \frac{-210 \times 0.774}{254 - 210 \times 0.0285} \quad \text{or} \quad -0.655$$

Then

$$\alpha = -33^\circ\, 13' \text{ as the full-load torque angle}$$

$$\sin \alpha = -0.548$$

$$\cos \alpha = 0.837$$

The excitation voltage necessary to produce unity pf is, using E_0 as reference vector ($\sin \alpha$ is positive),

$$\begin{aligned} E_0 &= (V_t - IR_e) \cos \alpha + IX_d \sin \alpha \\ &= (254 - 210 \times 0.0285)\, 0.837 + 210 \times 1.27 \times 0.548 \\ &= 354 \text{ volts} \end{aligned}$$

From equation 564:

$$\begin{aligned} P_{\text{input}} &= \frac{254 \times 354}{1.27} (-0.548) + \frac{254^2(1.27 - 0.774)}{2 \times 1.27 \times 0.774} (-0.9166) \\ &= -38{,}800 - 14{,}800 \quad \text{or} \quad -53{,}300 \text{ watts per phase} \end{aligned}$$

The input is 159.9 kw for three phases.
The power developed is input minus armature copper losses, or

$$159{,}900 - 3 \times 210^2 \times 0.0285 \quad \text{or} \quad 155{,}420 \text{ watts}$$

The current components:

$$I_q = I \cos \alpha = 210 \times 0.837 \quad \text{or} \quad 175.5$$

$$I_d = I \sin \alpha = 210 \times 0.548 \quad \text{or} \quad 115.0$$

Check power input by equation 568:

$$\begin{aligned} P_{\text{input}} &= 354 \times 175.5 - 175.5 \times 115\, (1.27 - 0.774) \\ &= 53{,}500 \text{ watts per phase} \end{aligned}$$

Determine the maximum torque angle (pull-out point) and the stability factor.

$$B = \frac{V_t(x_d - x_q)}{x_q} = \frac{254(1.27 - 0.774)}{0.774} \quad \text{or} \quad 162$$

$$\begin{aligned} \cos \alpha &\approx -\frac{354}{4 \times 162} \pm \sqrt{\frac{1}{2} + \left(\frac{354}{4 \times 162}\right)^2} \\ &= -0.544 \pm 0.891 \quad \text{or} \quad 0.347 \end{aligned}$$

The positive sign gives useful values. Hence the torque angle at pull out is $69^\circ\, 40'$. Note how widely this differs from the cylindrical rotor assumption of approximately 90°.

The stability factor:

$$P_r = \frac{dP}{d\alpha} \approx \frac{254 \times 354}{1.27} \times 0.837 + \frac{254^2(1.27 - 0.774)}{1.27 \times 0.774} \times 0.3998$$

$$= 72{,}500 \text{ watts per phase per electrical radian}$$

$$= 217.5 \text{ kw for 3 phases}$$

In the synchronous motor, synchronous watts and watt output are represented by the same figure.

$$\text{Torque in pound-feet} = \frac{7.04 \text{ synchronous watts}}{\text{synchronous rpm}} = \frac{7.04 \times 217{,}500}{900} \quad \text{or} \quad 1720$$

Hence, in terms of pound-feet of torque per electrical radian, the stability factor is 1720; or, per mechanical radian

$$1720 \times \frac{P}{2} = 6880$$

This is, of course, approximate, being based on watt input rather than output. The maximum input, at the point of pull out is

$$P_{\max} = \frac{254 \times 354}{1.27} \sin(-69° \, 40') + \frac{254^2(1.27 - 0.774)}{2 \times 1.27 \times 0.774} \sin(-139° \, 20')$$

$$= -77{,}400 \text{ watts per phase or } 232.2 \text{ kw for 3 phases}$$

It will be recalled that these values are based on *input* not *output*. Neglecting this error, and, since the values obtained are also proportional to torque,

$$\frac{\text{Maximum torque}}{\text{Full-load torque}} \approx \frac{232.2 \text{ kw}}{162.75 \text{ kw}} \quad \text{or} \quad 1.43$$

(This motor does not meet A.S.A. or N.E.M.A. Standards, which require that a 200-hp motor of this speed have a maximum torque of at least 1.75 times the full-load value.)

CHAPTER XL

CHARACTERISTICS BY ANALYTICAL AND GRAPHICAL METHODS

337. Chapter Outline.

Synchronous Motor Characteristics.
Step-by-step Analysis. Synchronous Impedance.
The Circle Diagram. Synchronous Impedance.
Example of Each.
Check by Analytical Formulas.

338. Graphical Analysis of the Synchronous Motor by the Synchronous-Impedance or Reactance Method. By drawing a number of diagrams for the synchronous motor at various torque angles, the V curves and the performance characteristics can be determined. The use of these diagrams, based on synchronous impedance, offers the simplest known method of studying the behavior of the synchronous motor. The results obtained are accurate for cylindrical rotor machines, except for the disturbances caused by saturation.

The calculation process will be illustrated by the following synchronous motor. Some of the data given below are from test runs.

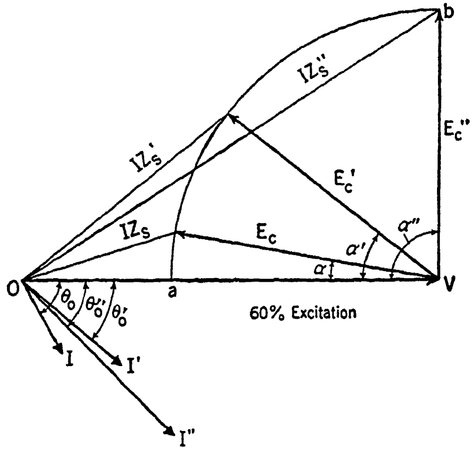
5 hp	220 volts	single phase
60 cycles	1800 rpm	X_s armature $= 4.00$ ohms
R_e armature $= 0.907$ ohm		

Assume an excitation of 60 per cent. The counter emf is then 132 volts; the field current is 0.98 ampere; the excitation loss, including field rheostat, is 107.5 watts; friction, windage, and core losses total 666 watts.

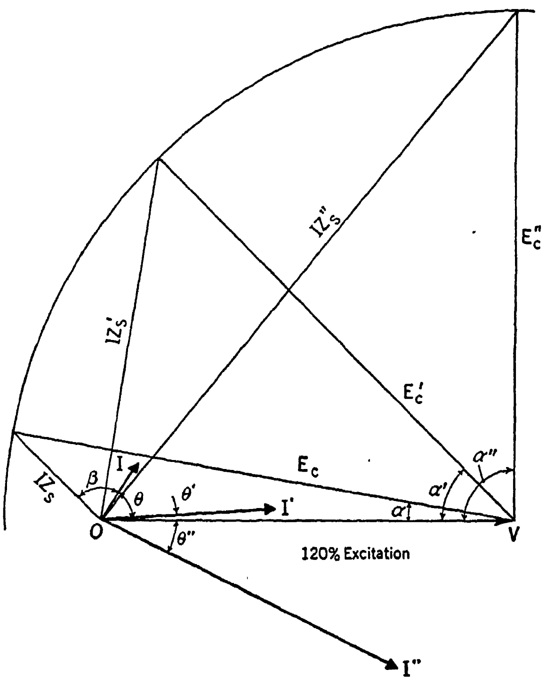
At 100 per cent excitation the counter emf is 220 volts; the field current is 1.925 amperes; the excitation losses are 212 watts; and the friction, windage, and core losses are 666 watts.

At 120 per cent excitation the counter emf is 264 volts; the field current is 2.6 amperes; and the excitation losses are 286 watts. Friction, windage, and core losses are 666 watts.

The performance will be calculated for these three excitations: 60, 100, and 120 per cent, although other values could be assumed as well.


b
IZ''_s
E''_c
IZ'_s
E'_c
IZ_s
E_c
α''
α'
α
O
V
a
θ_0
θ''_0
θ'_0
60% Excitation
I
I'
I''

(a)


IZ''_s
E''_c
IZ'_s
E'_c
E_c
IZ_s
β
I
θ
θ'
I'
α''
α'
α
O
V
θ''
120% Excitation
I''

(b)

FIG. 269.

To lay out the diagram: Use the applied voltage as the reference vector (Fig. 269*a*) equal to OV. From V as the center, strike the arc *ab*, using as radius the counter emf, 132 volts, to the assumed voltage scale. Assume torque angles, say 0, 5, 10, 15, 30, 45, 60, 75, and 90 degrees. Scale off IZ_s for each torque angle and determine I. Obviously,

$$I = \frac{IZ_s}{Z_s}$$

The position of I is determined from the impedance angle β, where $\tan \beta = x_s/r_e$. The pf angle θ_0 is read from the diagram.

The values are tabulated in Table XIX.

The value of the current and pf having been determined, the input can be calculated as

$$VI \cos \theta_0$$

Then by subtracting the armature copper loss for each input, and the constant losses for the particular excitation, the output can be deter-

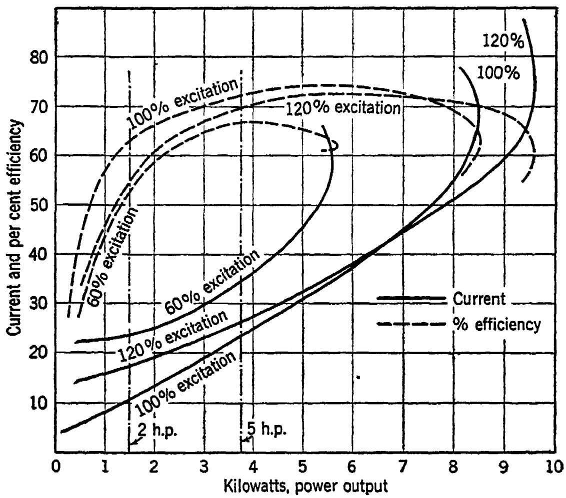


FIG. 270. Single-phase synchronous motor. Calculated performance curves.

mined. The developed power is equal to the input minus the armature copper loss, and is always higher than the true output.

Curves of kilowatt (or horsepower) output versus armature current and efficiency for the various excitations are shown in Fig. 270. The vertical lines for 2- and 5-hp output, respectively, are shown in this figure. The intercept of these lines with the armature-current curves

TABLE XIX

60 Per Cent Excitation

α	IZ_s	I	$\cos\theta_0$	Input $VI\cos\theta_0$	I^2R_e	$P_{\text{developed}}$	Friction and Core Loss	Field Losses	Total Losses	Output, Watts	Efficiency, Per Cent
0	88.0	22.00	0.235	1,138	435	703	666	107.5	1208.5		
5	89.0	22.25	0.340	1,665	440	1,225	666	107.5	1213.5	451.5	27.1
10	92.5	23.13	0.450	2,290	485	1,805	666	107.5	1258.5	1031.5	45.2
15	98.4	24.60	0.560	3,040	550	2,490	666	107.5	1323.5	1716.5	56.5
30	124.0	31.00	0.715	4,900	880	4,020	666	107.5	1653.5	3246.5	66.2
45	156.0	39.00	0.760	6,510	1400	5,110	666	107.5	2173.5	4336.5	66.6
60	191.4	47.85	0.762	8,020	2060	5,960	666	107.5	2833.5	5186.5	64.5
75	224.6	56.20	0.740	9,200	2800	6,400	666	107.5	3573.5	5626.5	61.2
90	256.0	64.00	0.694	9,850	3650	6,200	666	107.5	4423.5	5426.5	61.4
					100 Per Cent Excitation						
0							666	212.0			
5	18.2	4.55	0.988	988	20	968	666	212.0	898.0	90.0	9.1
10	36.8	9.20	0.990	2,000	74	1,926	666	212.0	952.0	1048.0	52.4
15	57.0	14.25	0.992	3,130	185	2,945	666	212.0	1063.0	2067.0	66.1
30	113.6	28.40	0.999	6,210	730	5,480	666	212.0	1608.0	4602.0	74.3
45	167.6	41.90	0.990	9,150	1580	7,570	666	212.0	2458.0	6692.0	73.0
60	219.6	54.90	0.959	11,550	2680	8,870	666	212.0	3558.0	7992.0	69.1
75	268.0	67.00	0.910	13,400	4020	9,380	666	212.0	4898.0	8502.0	63.5
90	310.0	77.50	0.850	14,500	5475	9,025	666	212.0	6353.0	8147.0	56.1
					120 Per Cent Excitation						
0	44.0	11.00					666	286.0			
5	48.0	12.00	0.235	620	130	490	666	286.0	1085.0		
10	60.0	15.00	0.580	1,920	203	1,717	666	286.0	1155.0	765.0	39.4
15	76.0	19.00	0.770	3,220	310	2,910	666	286.0	1262.0	1958.0	60.8
30	131.0	32.80	0.960	6,940	960	5,980	666	286.0	1912.0	5028.0	72.5
45	188.0	47.00	0.990	10,350	2000	8,350	666	286.0	2952.0	7398.0	71.5
60	244.0	61.00	0.990	13,400	3300	10,100	666	286.0	4252.0	9148.0	68.2
75	296.0	74.00	0.950	15,500	5000	10,500	666	286.0	5952.0	9548.0	61.8
90	343.0	85.80	0.900	17,000	6640	10,360	666	286.0	7592.0	9408.0	55.3

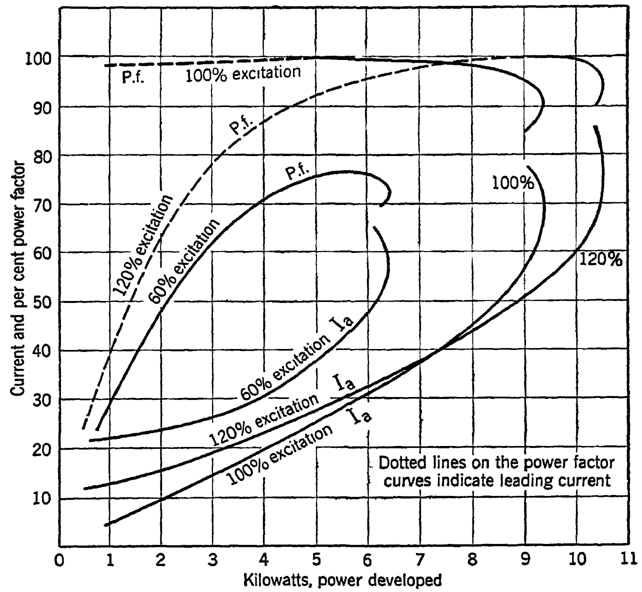


FIG. 271. Single-phase synchronous motor. Calculated performance curves.

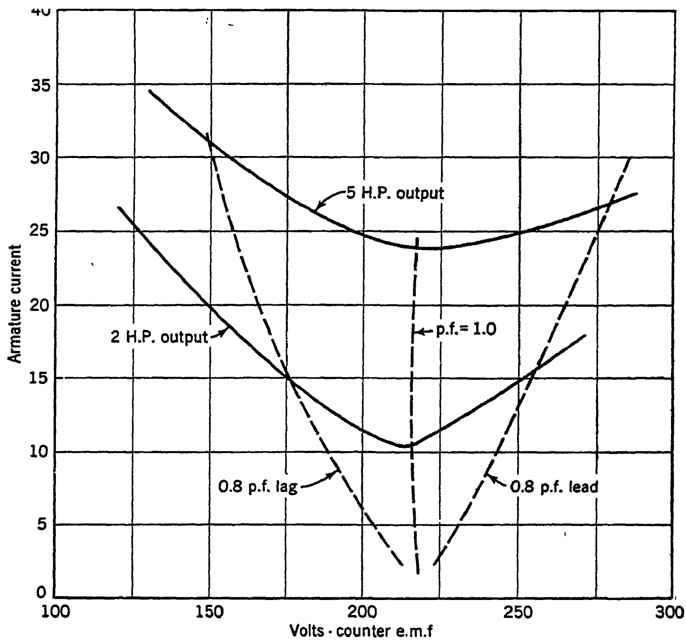


FIG. 272. V curves for a synchronous motor, derived from the synchronous-impedance diagram.

gives data for determining the V curves, two of which are shown in Fig. 272. The vector diagram of Fig. 273 shows a method of determining the compounding curves.

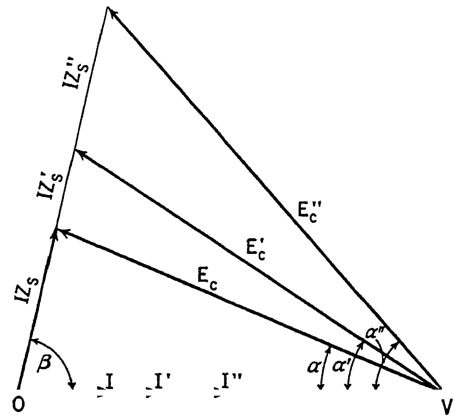


FIG. 273. Synchronous-impedance diagram showing the method of determining excitations required at various loads to give unity pf.

339. The Circle Diagram. It is obvious that the step-by-step process illustrated in the past example is not handy, to be avoided if possible. A more practical method of producing the same results utilizes some form of the circle diagram such as that developed by Dr. A. S. McAllister in 1907.[1] As with the mathematical analysis of motor action, the circle diagram is open to the criticism that it assumes a constant resistance and synchronous reactance.

The vector diagram of Fig. 274*a* with which the student is already familiar can be drawn, for convenience, as shown in Fig. 274*b*. As the torque angle α varies, the point M will describe a circle about O. Since the angle β is constant and as I always equals IZ_s/Z_s, the point I will describe another circle, not shown here. The construction of this second circle will be facilitated if I is moved to the upper left-hand corner of the diagram as shown in Fig. 274*c*. Consider M as a point which can be placed at any point on, or to the right of, the vertical line OG. If M is on OG, then I will be at GH. If M is at O, then I will be at GC, etc. As α increases with load, the vector OM swings over its arc and the

[1] A. S. McAllister, "Circular Current Loci of the Synchronous Motor," *Elec. World*, August, 1907.

André Blondel applied the circle diagram to synchronous machines in 1895. See translation of his work, "Synchronous Motors and Converters," by C. A. Adams, McGraw-Hill Book Co.

current I will trace the semicircle HB. The diameter of the semicircle HB is proportional to the excitation E.

If $E < V$, then $HC < GC$.
If $E = V$, then $HC = GC$.
If $E > V$, then $HC > GC$.

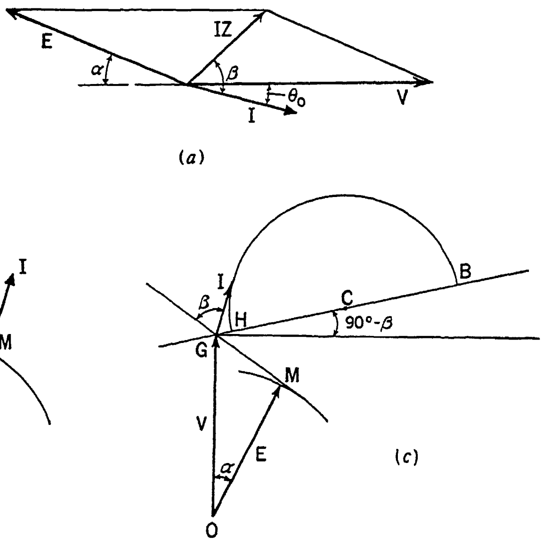


FIG. 274.

When M lies on OG,

$$IZ_s = V - E$$

and

$$GH = \frac{V - E}{Z} \qquad [572]$$

When M is at O (i.e., zero excitation),

$$IZ_s = V$$

and

$$GC = \frac{V}{Z_s} \qquad [573]$$

Equation 573 fixes the center of the circle by data which are readily obtainable. Furthermore, subtracting equation 572 from equation 573 shows that the radius CH is proportional to the counter emf E. For convenience, a family of circles can be drawn about the center point C, representing various excitations or ratios of E/V. The next step is to drop the voltage triangles (GOM in Fig. 274c) as being no longer neces-

sary and to draw the applied voltage vector V up from the point G. These changes are shown in Fig. 275. Under the conditions shown in Fig. 275 it is obvious that for a current I:

$$GT = \text{the in-phase component of the current } GI$$
$$TI = \text{the lagging component of the current } GI$$
$$\frac{GT}{GI} = \text{the pf}$$
$$GT \times GV = \text{the input watts per phase}$$

If the input is maintained constant, the current will follow a horizontal line such as ab, as the excitation is varied.

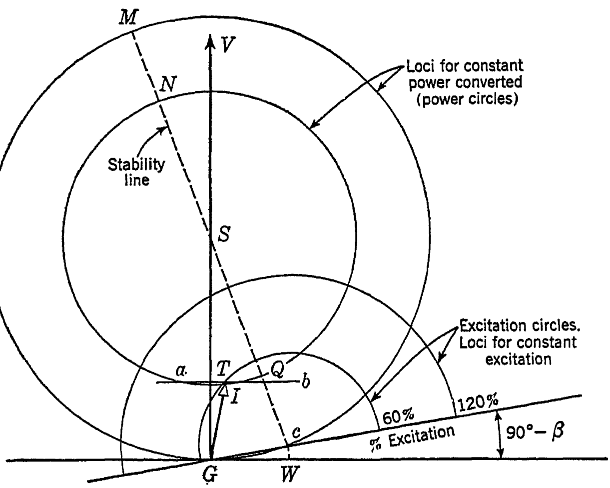


FIG. 275. Circle diagram of the synchronous motor.

For most practical purposes the current locus is desired for various degrees of excitation at any one output. It is convenient to determine these current values for *constant power developed or converted*, rather than for constant input or true constant output. Of the power put into the machine, all is converted into mechanical energy except that dissipated as I^2R_e in the armature circuit. The core, friction, and windage losses act as retarding torques, reducing the mechanical output. Hence:

$$\text{Input} - I^2R_e = \text{power developed}$$
$$\text{Power developed} - (C + F + W) = \text{useful output}$$

The core, friction, and windage losses are assumed constant, and so any characteristic determined for constant power developed will differ from

the true output only by a relatively small value. The assumption of constancy for core loss is only slightly in error, and the degree of error is easily within the total error inherent in the diagram.

340. Power Circle. To aid in determining the current characteristic at constant developed power, a power circle can be drawn which will form a locus of the current vector. To derive this locus:

At zero excitation the current taken by the motor is GC. The vertical line CW then represents the power component of GC. To a different scale, it represents the motor input $V \cdot CW$, and this is exactly equal to the I^2R_e loss, so that the power developed is zero. By similar triangles,

$$\frac{CW}{GC} = \frac{GC}{2GS}$$

from which

$$GS = \frac{(GC)^2}{2CW}$$

With

$$GC = \frac{V}{Z_s}$$

and

$$CW = I \cos \beta = \frac{VR_e}{Z_sZ_s}$$

there is obtained for the radius of the zero power circle,

$$GS = 0.5 \left(\frac{V}{Z_s}\right)^2 \cdot \frac{Z_s}{VR_e} \quad \text{or} \quad 0.5 \frac{V}{R_e}$$

For other loads, the location of the power-circle center remains the same. To obtain the radius, we apply the cosine law

$$(SI)^2 = (GS)^2 + (GI)^2 - 2(GS)(GI) \cos \theta \qquad [575]$$

Transposing,

$$(GS)^2 - (SI)^2 = 2(GS)(GI) \cos \theta - (GI)^2 \qquad [576]$$

Replace $2(GS)$ by V/R_e and note that $GI \cos \theta$ is the power component of the current; then,

$$(GS)^2 - (SI)^2 = \frac{\text{power input}}{R_e} - (GI)^2 \qquad [577]$$

If the second member is written as

$$\frac{\text{Power input} - R_e(GI)^2}{R_e}$$

it is seen that its numerator is the power converted and equation 577 becomes

$$(GS)^2 - (SI)^2 = \frac{\text{power converted}}{R_e} \qquad [578]$$

For a given constant power converted, equation 578 gives the radius SI of a circle with the center at S. Two such circles are shown in Fig. 275; the one passing through G and C represents zero developed power.

To obtain the radius for any desired developed power: Since GS equals $V/2R_e$ equation 578 can be written

$$\left(\frac{V}{2R_e}\right)^2 - (SI)^2 = \frac{\text{power converted}}{R_e} \qquad [579]$$

from which the radius

$$SI = \frac{1}{R_e}\sqrt{\frac{V^2}{4} - R_e \times \text{watts converted}} \qquad \text{amperes} \qquad [580]$$

SI will be called the "power-circle radius."

The end of the current vector will follow this circle, cutting across the various excitation circles.

It will be noted from equation 580 that the circle corresponding to zero power will give a radius of $V/2R_e$. As it can be shown that SG equals SC, this circle will pass through the points G and C. Circles of smaller radii represent greater power. The maximum possible developed power shrinks the power circle to the point S. Under this condition the power converted is

$$P_{\text{converted}} = \frac{V^2}{4R_e}$$

This is a fictitious value inasmuch as the current could not be increased enough actually to convert this amount of power, owing to the limiting effect of saturation, heating, and other factors. This expression agrees with that derived mathematically in Article 327. By this theory, the machine can carry a given load only for excitations whose circle lies within the appropriate power circle.

341. Limiting Conditions from the Diagram. It has already been pointed out that the theoretical maximum power which can be converted by any machine reduces the power circle to a point, and represents $V^2/4R_e$ watts per phase.

The *maximum theoretical excitation* at which the machine can operate as a motor at no load is the distance CM in Fig. 275, and represents the

point of tangency of the zero power and the excitation circles. This is an excessive excitation which cannot be reached in practice.

The *maximum theoretical excitation* for a fixed power, say, that represented by the circle passing through N in Fig. 275, is the excitation CN. The current GN is then leading.

The minimum excitation for the same power is CQ, with lagging current, GQ.

The minimum power factor for any power converted occurs when the current vector is tangent to the power circle of that load.

The limit of stability is represented by the line CM. All currents to the left of that line represent a stable operating condition; all currents to the right or above represent instability. Consider a motor operating with fixed excitation and line current I. For any momentary increase in load, an increase in torque angle occurs, permitting more current to be taken by the motor. If I increases beyond the point Q, any further increase in load swings the current vector along its fixed excitation circle, and across power circles of greater diameter. The increase in power demand then results in less power developed, and the limit of stability is reached.

342. Construction Procedure and Example. This type of diagram will be applied to a synchronous motor to illustrate the procedure, using the same machine for which curves were obtained in the first part of this chapter.

Construction:

$$\tan \beta = \frac{x_s}{r_e} \quad \text{or} \quad 4.41$$

$$90° - \beta = 13.1°$$

(*a*) Lay off the indefinite line GCB (Fig. 276), rotated 13.1° from the horizontal base line. Draw the vertical line GS from the intersection.

(*b*) To locate S, solve for GS thus, selecting an ampere scale:

$$GS = \frac{V}{2R_e} = \frac{220}{2 \times 0.907} \quad \text{or} \quad 121 \text{ amperes}$$

(*c*) To locate the point C:

$$GC = \frac{V}{Z_s} = \frac{220}{4.1} \quad \text{or} \quad 53.6$$

Lay off GC to the current scale. It will be noticed that on commercial machines, owing to the relative values of R_e and Z_s, the excitation circles are comparatively small.

(*d*) To draw the excitation circles:

Divide GC into 100 parts, and using C as the center lay off the semicircles for such excitations as, say, 60, 100, and 120 per cent.

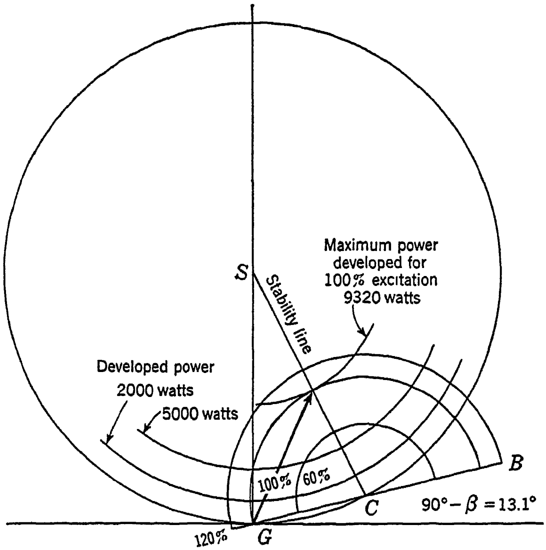


FIG. 276.

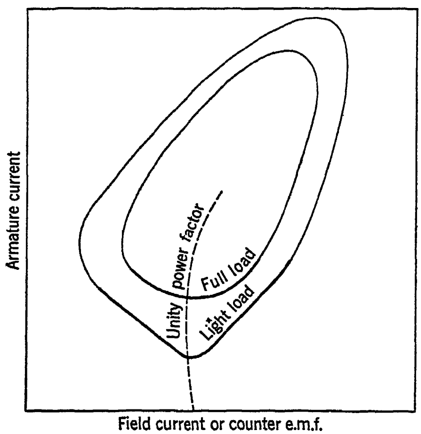


FIG. 277. Typical O curves for a synchronous motor.

(*e*) Assume values of developed power. In this case 2000 and 5000 watts, respectively, will be used. Since this is a single-phase machine these watts are total developed powers. Otherwise the diagram is always drawn per phase.

For 2000 watts:

$$\text{Radius} = \frac{1}{0.907}\sqrt{\frac{220^2}{4} - 0.907 \times 2000}$$

$$= 111 \text{ amperes}$$

For 5000 watts:

$$\text{Radius} = \frac{1}{0.907}\sqrt{\frac{220^2}{4} - 0.907 \times 5000}$$

$$= 95.1 \text{ amperes}$$

The power circles are laid out with these radii as shown in Fig. 276.

(*f*) The current for any power developed and at the various excitations can be scaled off and tabulated.

Only one value will be checked on this diagram.

At 100 per cent excitation the tangent power circle has a radius of 66 amperes. This corresponds to a developed power of 9320 watts, and the pf indicated at this load is 0.91, lagging. Reference to Fig. 271 shows that the maximum power which can be developed at this excitation is 9400 watts at a pf of 0.92, lagging.

If the current vectors for any load are read around the entire circumference of the power circle, "O" curves are obtained as shown in Fig. 277.[2]

343. Check by Analytical Formulas. The results of the vector diagrams and the more comprehensive circle diagram can be checked by several of the formulas given in Chapter XXXIX.

Power developed:

Refer to Table XIX. Select the value of calculated power developed at an excitation of 120 per cent, input of 19 amperes. This is 2910 watts. By formula (equation 540) the predicted value for this single-phase synchronous motor is

$$P_d = \frac{VE(r_e \cos\alpha + x_s \sin\alpha) - r_e E^2}{Z_s^2}$$

wherein

$$\begin{aligned} \alpha &= 15^\circ \\ \cos\alpha &= 0.9659 \\ \sin\alpha &= 0.2588 \\ x_s \approx Z_s &= 4.0 \text{ ohms} \\ r_e &= 0.907 \text{ ohm} \\ E &= 264 \text{ volts} \\ V &= 220 \text{ volts} \end{aligned}$$

Then

$$P_d = \frac{220 \times 264(0.907 \times 0.9659 + 4 \times 0.2588) - 0.907 \times 264^2}{4^2}$$

$$= 3000 \text{ watts}$$

[2] John F. H. Douglas, Eric D. Engeset, and Robert H. Jones, "Complete Synchronous Motor Excitation Characteristics," *Trans. A.I.E.E.*, Vol. 44, p. 164, 1925.

Maximum power:

At this excitation of 120 per cent the maximum power as indicated by the curve on Fig. 271 is 10,520 watts. The calculated value is

$$P_{dm} = \frac{VE}{Z_s} - \frac{r_e E^2}{Z_s^2}$$

$$= \frac{220 \times 264}{4} - \frac{0.907 \times 264^2}{16}$$

$$= 10{,}570 \text{ watts}$$

Maximum excitation for motor action:

From equation (551):

$$E = \frac{220 \times 4}{0.907} \quad \text{or} \quad 970 \text{ volts}$$

Obviously, so high a value could not be reached on a machine of this rated voltage.

This entire method of procedure has been illustrated with a small salient-pole machine, using a method strictly applicable only to the cylindrical rotor type with sinusoidally distributed field winding.

CHAPTER XLI

STARTING SYNCHRONOUS MOTORS

344. Chapter Outline.

345. Starting Synchronous Motors. It has been pointed out in Chapter XXXVII that one of the common methods of starting synchronous motors employs the induction-motor action of the special windings in the pole faces of the rotor. As an induction motor always operates with a slip, such a method cannot bring the motor into synchronism. The action by which the motor is brought into synchronism when the field is excited involves some interesting transient phenomena which will be discussed briefly here.[1] The starting characteristics divide up naturally into two parts: (*a*) those pertaining to the initial period of starting as an induction motor, and (*b*) those displayed on exciting the field, causing the motor to come up to synchronous speed.

Both the leakage reactance and the rotor-cage resistance have relatively large effects on the starting torque. A large rotor resistance may produce a high starting torque, but results in a slip which does not bring the motor sufficiently close to synchronous speed for the final "pull-in" to synchronism. A low resistance may result in a small final slip with reduced starting torque. See Fig. 278. This shows the importance of starting windings which are phase wound with external resistance. After the higher resistance is utilized to give good initial starting torque, it can be reduced to give a minimum slip.

[1] Harold E. Edgerton, Kenneth J. Germeshausen, Gordon S. Brown, and Ralph W. Hamilton, "Synchronous Motor Pulling-into-step Phenomena," *A.I.E.E. Paper* 33–26.

Consider that the load torque is shown in part by the dotted line of Fig. 278. The motor torque is greater than this value at lower speeds and results in acceleration. Motor *A* reaches its maximum speed at *a* as an induction motor; motor *B* reaches its maximum speed at *c*. The torque developed by the motor from its starting winding as it approaches synchronism is considerably larger in the case of *B*. This results in greater useful pull-in torque when the field excitation is applied. We will

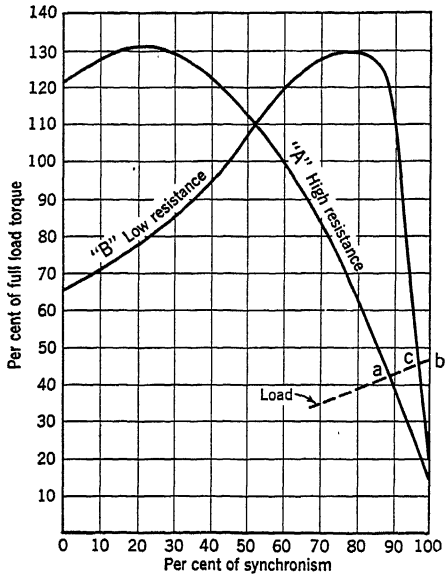


FIG. 278.

see later the items which must be considered in determining whether or not the motor will be able to pull in from such a slip when the field is excited.

The change in effective resistance with frequency change (skin effect) gives to a greater or lesser degree the desirable starting characteristics described above. That is, at low speeds and high slip frequency the effective resistance of a rotor cage may be high; at near synchronism the effective resistance reduces, being nearly the d-c value owing to the lower frequency of slip. If the resistance of the cage could be zero, it would be possible to run in exact synchronism as an induction motor; the decreased resistance from skin effect then results in little slip and a close approach to synchronous speed.

During the initial starting period the required current is limited by the same factors which affect the short-circuit currents of induction motors.

Practical considerations limit the allowable starting current, and minimum values of starting torque are set up by both N.E.M.A. and A.S.A. Standards.

346. Effect of Starting with Field Excited or with Field Short-circuited. When a synchronous motor is in operation, the torque angle increases with load, the torque increasing as the sine of the torque angle. As the motor is being brought up to speed and its field supply is thrown on, the

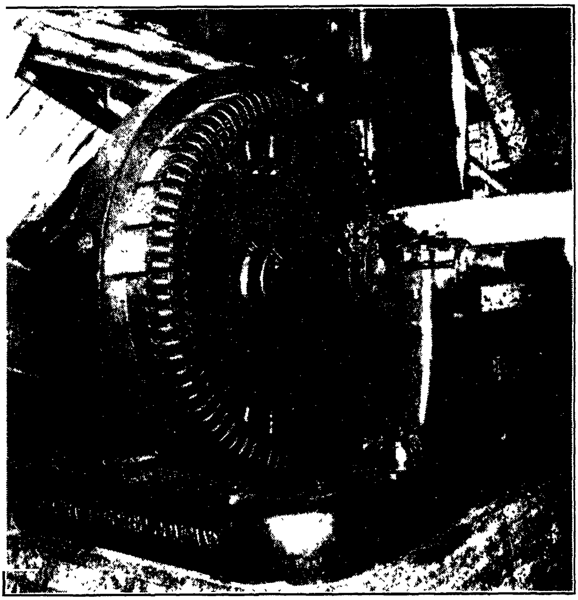


FIG. 279. Synchronous motor. 300 kv-a, 300 rpm, 2200 volts, 0.8 pf.

rotor, as it slips, is alternately going through positions of maximum forward torque and maximum backward (generator action) torque. That is, as the poles slip behind the synchronously revolving flux, the torque is positive through one-half cycle and negative through the other half. This pulsating torque sets up oscillations in the rotor speed and results in large power variations in the supply. Finally, if load inertia and slip are not too great, these speed variations will be arrested by the synchronous motor swinging into step on one of its forward oscillations and remaining in synchronism. This action is aided by the salient-pole feature, providing a path of minimum reluctance alternating with paths of maximum reluctance. Good pull-in characteristics are obtained if the motor will come into synchronism after passing through only one generating cycle.

Exciting the field too early in the process of coming up to speed may so exaggerate this generator action as to prevent the motor from attaining synchronism. As stated above, during each half cycle the motor acts as a generator, driven by a separate source. The field flux cuts the stator winding at varying subsynchronous speeds and gives rise to a variable frequency current in the supply lines, flowing back through the supply generator and other closed circuits. The line current then contains the usual sinusoidal supply and a superimposed lower frequency component, the magnitude of which depends upon the field excitation. To maintain this component by generator action requires torque from the accelerating synchronous motor. In other words the pumping-back generator action gives a retarding torque and may cause the motor to slow down (or at least to accelerate less rapidly) rather than pull into synchronism.

If the d-c field winding of a synchronous motor is short-circuited during the starting cycle, the winding forms a single-phase secondary in a polyphase motor. It can be pointed out that this is, in a sense, an example of internal concatenation, giving a positive torque up to about half speed and a retarding torque thereafter. In spite of this action, tests indicate that the starting torque of a motor with a short-circuited field winding is actually reduced up to and beyond half synchronous speed.[2] The effect is less noticeable near synchronism. This reduction is brought about because of the choking effect of the closed field circuit, which reduces the useful flux and increases that flux forced through the leakage paths of the stator. This decreases the effective torque of the squirrel-cage winding.

The importance of having the excitation applied at the proper point to maintain accelerating motor action has lead to the development of various electronic control systems to assure such action.

347. Pull-in Phenomena. H. E. Edgerton and P. Fourmarier [3] list the following items as influencing the pulling-into-step phenomena which occur between the time when the rotor has reached its maximum speed as induction motor and when it becomes actually synchronized. These apply to salient-pole machines.

(*a*) The amount of load on the motor shaft when starting.

(*b*) The synchronizing torque when the field is excited.

(*c*) The inertia of the rotor and load masses and the natural period of oscillation of these masses.

[2] E. B. Shand, "Starting Characteristics of Synchronous Motors," *Elec. J.*, Vol. 18, p. 309, 1921.

[3] The Pulling-into-step of a Salient-pole Synchronous Motor," *Trans. A.I.E.E.*, Vol. 50, p. 769, June, 1931.

C. J. Fecheimer, "Self-starting Synchronous Motors," *Trans. A.I.E.E.*, Vol. 31, p. 529, 1912.

(*d*) The torque-speed characteristics of the damping winding. The minimum of starting winding resistance brings the rotor closest to synchronous speed and improves the chances of pulling into step. Such a winding exerts a strong damping effect, tending to reduce oscillations of the rotor. This damping may prove detrimental in acting against the oscillation necessary for pulling into step.

(*e*) The angle between stator terminal voltage and the voltage induced in the stator by the field. If the exciter is connected at the instant in which this angle is such as to cause generator action (as previously described) the rotor slips still further from synchronism.

(*f*) The "reluctance torque." The saliency of the rotor gives it paths of maximum and minimum reluctance and results in an additional double-frequency torque pulsation as pointed out in Article 333.

All these effects can be combined in consistent terms in order to give an expression equated to the load torque. Such a differential equation has resisted efforts to obtain a solution, although approximations have been given by several writers, including one quoted in Chapman's "The Induction Motor." The writers cited above, by the use of the "differential analyzer," have obtained a solution, indicating necessary ratios of constants which must hold for certain synchronism even under the least favorable field-switching point.

Thus on the salient-pole machine it is found that, even with the worst switching angle, the motor will pull into step if the following conditions are satisfied:

$$P_L < P_m \times 7.6k \qquad [581]$$

or

$$s < \frac{590}{\text{synchronous rpm}} \sqrt{\frac{P_m}{fWR^2}} \qquad [582]$$

or

$$P_m > 2.86fWR^2(\text{rpm})^2 s^2\, 10^{-6} \qquad [583]$$

wherein:

P_L = load torque in synchronous kilowatts

s = the average slip as a decimal. Both this and P_L can be read from a speed-torque curve of the machine as an induction motor on the straight line portion of the curve, near synchronism

P_m = the synchronous motor power or torque in synchronous kilowatts

$$= \frac{VE_m}{x_d} 10^{-3} \qquad [584]$$

m = number of phases

f = frequency of applied emf

$$k = \frac{P_d}{\sqrt{P_j P_m}} \qquad [585]$$

P_d = kilowatts per electrical degree per second

$$= \frac{P_L}{360fs} \qquad [586]$$

P_j = inertia effect of the rotor in kilowatts per electrical degree per second per second

$$= 18.6 \frac{fWR^2}{p^2} 10^{-6} \qquad [587]$$

WR^2 = moment of inertia in pound-feet [2]
p = number of poles [4]

348. Voltage Induced in the Field. In starting a synchronous motor, the stator winding acts as a primary and the field circuit as the secondary of a transformer. A large number of turns on the field obviously results in a high voltage being induced which dies down as the rotor approaches synchronism. This voltage may be dangerous to life or to the insulation of the field winding. To avoid this hazard the field winding is most usually opened in several places during the starting period, or else closed through a "discharge" resistor.

349. Summary of Starting Procedure.

(*a*) The field switch should be in the "starting" position, when the stator winding is switched onto the supply. Usually a lower voltage is used for starting on all but small motors unless other methods of limiting the current are employed. The field circuit is opened or short-circuited or closed through a resistance.

(*b*) The stator winding is switched over to full voltage if reduced voltage was used in (*a*).

(*c*) The field is excited as the motor reaches nearly synchronous speed.

(*d*) The field is adjusted to provide the desired pf for final operation.

350. Effect of Voltage on the Starting and Running Characteristics. A reduced voltage on the synchronous motor, utilizing the damping winding for starting, will affect the current and torque in the same manner that it does on induction motors. (See Chapter XXVIII.)

[4] M. M. Liwschitz, "Starting Performance of Salient-pole Synchronous Motors," Supplement to *Elec. Eng.*, Trans. Sec., pp. 913–919, December, 1940. See also its list of ten references.

Under normal operation, however, the synchronous motor is much less sensitive to voltage change than is the induction motor. A synchronous motor with the same pull-out torque as an induction motor will continue operation on a voltage at which the induction motor would break down.

The maximum output of a synchronous motor with fixed excitation will vary as the first power of the voltage. (See equations 543 and 569.) If, with every change in voltage, the excitation were changed proportionally so that the operating pf remained constant, the synchronous motor would have the same torque characteristics as the induction motor; the torque would then vary with the square of the voltage.

CHAPTER XLII

SYNCHRONOUS MOTOR TESTS

351. Chapter Outline.

Synchronous Motor Tests. (See Alternator Tests.)
- No-load Saturation Curve.
- Synchronous Impedance.
- Efficiency and Losses.
  - Rated Motor Method.
  - Retardation Method.
- Heat Runs.
- V Curves.
  - Correcting V Curves for Harmonics.[1]
- Standards.

352. Obtaining Data. To obtain the performance characteristics of the synchronous motor from laboratory data a number of test runs must be made:

1. To predict the V curves and the various limits of operation from no-load tests by mathematical analysis, vector diagrams, or the circle diagram, it is necessary to obtain the following data:

(*a*) *Resistance of the Armature per Phase.* This is measured by direct current and corrected to an effective value of from 1.25 to 1.75 times the d-c value. See Article 28.

(*b*) *Synchronous Impedance.* The usual open- and short-circuit characteristics should be obtained as outlined in Article 41.

(*c*) If the excitation for any given load (as for plotting V curves) is to be calculated by the A.S.A. method, it is necessary to obtain the additional data outlined in Article 51 for constructing the curves of Fig. 44. This involves a source of low pf load to obtain the zero pf saturation curve and calculations for the air-gap line.

(*d*) To make the analyses by the *two-reaction theory*, readings of direct- and quadrature-axis reactances must be obtained as outlined in Article 76.

[1] J. R. Collins, "Correcting Synchronous Motor V-curves," *Elec. World*, p. 894, April 17, 1920.

2. (*a*) *Losses and Efficiency.* The losses of a synchronous motor are identical with those of an alternator. The general procedure for determining them by the rated motor or the retardation runs has been given briefly under Alternators.

(*b*) *Heat runs* on synchronous motors and alternators are both made by the variety of methods outlined in Chapter IX.

More detailed methods of making all these tests can be found in the Standards of the A.I.E.E. and of the A.S.A. as previously cited.

353. Experimentally Derived V Curves. If the synchronous motor is of large size, the direct measurement of the V curves requires a large kilovolt-ampere of supply capacity and mechanical load for the motor.

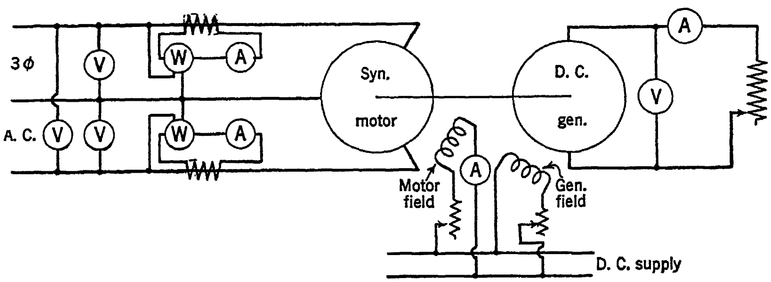


FIG. 280. Connections for determining the V curves of a synchronous motor, loaded on a d-c generator.

These requirements render such tests commercially impractical and necessitate the use of such prediction methods as have been given in the preceding chapters.

For illustrating the characteristics of smaller synchronous motors, V curves can be obtained by direct loading. Connections are shown in Fig. 280 for loading the motor with a d-c generator, the output of which is absorbed by rheostats.

The output of the generator is maintained constant throughout the run, and the field current of the synchronous motor is reduced until the armature current is about 1.5 times normal. The field current is then increased in steps, causing the armature current to go through its minimum and up to the maximum leading value of, say, 1.5 times normal. All meters are read at each step.

This procedure is repeated for other values of constant output.

The no-load V curve is obtained by uncoupling the generator from the motor and operating the motor at no load. The excitation is again varied to swing the armature current from maximum lagging to maximum leading values, within the permissible limits.

The true motor output can be obtained in the load runs by adding the d-c generator losses to the measured generator output. The pf of the motor can be readily obtained through the readings of the voltmeter, ammeters, and wattmeters in the armature circuit or from pf, or reactive-factor, meter readings.

354. Correcting the V Curves When Harmonics Prevent the Indication of Unity Power Factor. When a V curve is obtained experimentally, the observer is frequently surprised to find that a calculation at the lowest point of the V curve does not indicate unity pf. One usually questions the accuracy of the meters or assumes that the lowest current value was passed through but not recorded. The facts are that, with both of these factors taken care of, the departure may be due to harmonics arising from:

(*a*) Deviation of the impressed emf from a simple sine wave.
(*b*) Deviation of the counter emf from a simple sine wave.
(*c*) Both (*a*) and (*b*) together.

(Errors may also be caused by pulsations in the supply voltage and hunting.) Let these harmonics in the current wave be denoted by I_h. Such harmonics will vary with the load and excitation, but the assumption is made that they are constant; the error so caused ordinarily is negligible.

The method of evaluating I_h is shown below. This method is also useful in other measurements where the current and voltage are not sinusoidal.

It is known from a study of harmonics that

$$I^2 = I^2_{\text{fundamental}} + I_h^2 \qquad [588]$$

Denote $I_{\text{fundamental}}$ as I_1.

Also,

$$I_1^2 = I^2_{\text{power}} + I^2_{\text{quadrature}} \qquad [589]$$

Then

$$I^2 = I_p^2 + I_q^2 + I_h^2 \qquad [590]$$

In obtaining the V curves, the impressed voltage is measured with a voltmeter, the current I with an ammeter, and the watts W with a wattmeter. Then

$$I_p = \frac{W}{E}$$

Hence equation 590 becomes

$$I^2 = \left(\frac{W}{E}\right)^2 + I_q^2 + I_h^2 \qquad [591]$$

For the lowest point on the V curve, I_q is zero since the pf would be unity were it not for the fact that the total current I contains a component I_h. Hence for the lowest current value, equation 591 can be written

$$I^2 = \left(\frac{W}{E}\right)^2 + 0 + I_h^2 \qquad [592]$$

This enables I_h to be evaluated. The assumption is then made that I_h remains constant. As the current increases, this component becomes relatively less important.

The method of procedure will be illustrated by an *example.*

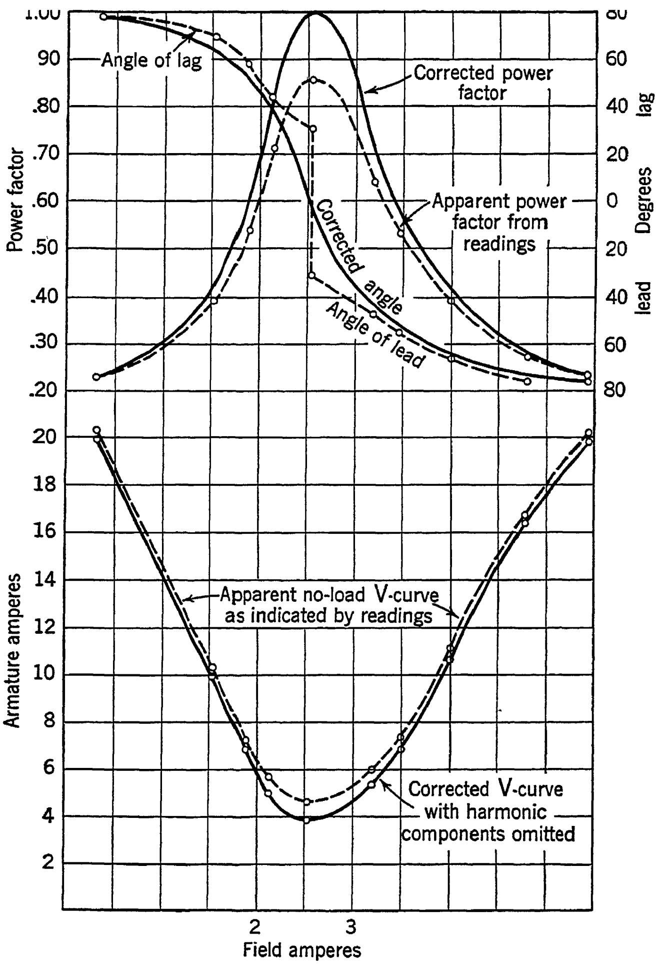


FIG. 281.

Example. Table XX gives partial laboratory data on a single-phase, 220-volt synchronous motor with no load. The complete V curve is given in Fig. 281. The fifth reading of 4.55 is the lowest current obtainable by field adjustment. At this value the pf would be, by calculation,

$$\cos \theta = \frac{858}{4.55 \times 220} \quad \text{or} \quad 0.857$$

TABLE XX

(Lagging current only)

Reading	I_{field}	I_{armature}	Watts	Apparent Power Factor (from Readings)
1	0.31	20.10	1050.0	23.8
2	1.55	10.25	900.0	39.9
3	1.90	7.36	874.5	54.0
4	2.15	5.49	862.5	71.5
5	2.55	4.55	858.0	85.7

Then θ equals 31° 1′.

The pf does not go to unity; note the discontinuity in the curve. The correction will now be applied. From equation 592,

$$I^2 = \left(\frac{W}{E}\right)^2 + 0 + I_h^2$$

$$\overline{4.55}^2 = \left(\frac{858}{220}\right)^2 + I_h^2$$

$$I_h = 2.35 \text{ amperes}$$

The ammeter reads high by this amount, owing to the non-sinusoidal wave form. The true current for the fifth reading is then

$$I_1^2 = I^2 - I_h^2$$

$$= \overline{4.55}^2 - \overline{2.35}^2$$

$$I_1 = 3.9 \text{ amperes}$$

Obviously, from its method of derivation,

$$\cos \theta = \frac{W}{EI}$$

$$= \frac{858}{220 \times 3.9} \quad \text{or} \quad 1.0 \text{ as against } 0.857$$

In general,

$$I_1^2 = I^2 - I_h^2$$

Then, for reading 4, Table XX,

$$I_1^2 = \overline{5.49}^2 - \overline{2.35}^2$$

$$I_1 = 4.97 \text{ amperes}$$

This is the total current exclusive of the error due to harmonics.

The quadrature component of fundamental frequency is

$$I_q^2 = I_1^2 - \left(\frac{W}{E}\right)^2$$

$$= \overline{4.97}^2 - \left(\frac{862.5}{220}\right)^2$$

$$I_q = 3.05 \text{ amperes}$$

The in-phase component of fundamental frequency is

$$\frac{W}{E} = 3.92 \text{ amperes}$$

The pf for the fundamental frequency can be found without solving for the quadrature component, thus:

$$\cos\theta = \frac{3.92}{4.97}$$

$$= 0.79, \text{ as against } 0.715 \text{ in Table XX}$$

This is repeated for the other readings, and a corrected V curve is drawn. The contrasting values of angle of lag or lead, pf, and current against field current, corrected and uncorrected, are shown in Fig. 281.

The method given is not necessary when a special metering arrangement is used, such that volts, amperes, and watts for each important harmonic are measured separately. The attendant complication and labor usually make such elaborate procedure impracticable.

355. Synchronous Motor Standards. The National Electrical Manufacturers Association has standardized many items pertaining to synchronous motors. A few of these items are:

Voltage Ratings. These are 220, 440, 550, and 2200 for ratings of 200 hp or under. Above 200 hp, other standard voltages are 2300, 4000, 6600, 11,000, and 13,200.

Horsepower and Speed. General-purpose motors cover hp ratings of 20 to 200 in speeds corresponding to 4- to 14-pole construction for 60 cycles; 4 to 12 poles for 50 cycles, and 4 to 6 poles for 25 cycles.

High-speed motors are standardized in various horsepower ratings of from 250 to 5000 for speeds of 3600 to 514 rpm for 60 cycles; 3000 to 500 for 50 cycles, and 1500 to 500 for 25 cycles.

Engine-type motors cover horsepower ratings from 20 to 5000 in standard sizes at speeds of 450 to 180 rpm for 60 cycles and a suitable range for 50 and 25 cycles.

Exciter Sizes. For large 0.8-pf motors, exciter capacities for various speeds are fixed by standardization.

Normal Torques. These values for 60-cycle, general-purpose motors are listed below in percentage of rated full-load torque:

	Speed	Starting	Pull-in	Pull-out
	Rpm	%	%	%
1.0 pf to and including 200 hp	1800	110	110	150
	1200–514	110	110	175
0.8 pf to and including 150 hp	1800	125	125	200
	1200–514	125	125	250

Starting torque is for rated voltage applied to the motor terminals.

The pull-out torque is with rated voltage and normal excitation applied.

It will be recalled in dealing with pulling-into-step phenomena that the ability to synchronize was dependent upon the inertia load. Hence, to determine the pull-in torque for purposes of standardization and comparison, it is necessary to use an expected or normal WR^2 value for calculation. N.E.M.A. Standards list such values for all speeds and ratings. They are for external load, and do not include the motor rotating parts.

Normal WR^2 of Load Exclusive of Motor WR^2. More complete tables are available, but only one rating will be given.

Horsepower = 200

Rpm	1800	1200	900	720	600	514
WR^2	51	115	205	320	460	627.5

The above items are not at all complete, but give a partial idea of some of the characteristics called for by standardizing groups.

CHAPTER XLIII

THE ROTOR AS A TORSIONAL PENDULUM

356. Chapter Outline.

Hunting.
The Rotor As a Torsional Pendulum.
The Fundamental Equation.
Effect of Adding Loads of Various Types.
  Analysis of Case I with Steady Load Added.
    Non-oscillating or Aperiodic.
    Oscillating.
  Evaluation of Constants for the Equations.
    The Polar Mass Moment of Inertia.
    Damping Constant.
    "Field-elasticity" Constant and Synchronizing Torque.

357. An Introductory Statement Pertaining to the Study of Hunting in Synchronous Machines. When a synchronous motor is operating under a constant torque demand, the torque developed is equal at all times to the torque "consumed," and the speed is absolutely constant. The rotor is then displaced from its no-load running position by the torque angle α as previously explained. If the torque demand is suddenly reduced, the excess torque results in acceleration of the motor. This acceleration reduces the torque angle and the developed torque, but because of the momentum the rotor cannot at once take up the stable position necessary for the equalization of the two torques. The "overshooting" of the rotor results in a decrease, or even reversal, in the developed torque, retardation of the rotor, and an excessive torque angle followed by acceleration and a repetition of the entire cycle. The important point is that any sudden change in load may be attended by an oscillation, and this oscillation will take place at a definite frequency. The elastic effect of the flux in the air gap, the flux being "stretched" by the load through the torque angle, and the mass of the rotating members, are the elements necessary to give a natural period of oscillation to the rotor as a torsional pendulum.

Owing to this elastic effect of the flux lines, any displacement of the rotor from its no-load position results in a force on the rotor, propor-

tional and opposite to the displacement. This is known as the *synchronizing torque,* produced by the current and flux of the motor.

The determination of the natural period of oscillation of the rotor as a torsional pendulum is important in any motor which drives a load of pulsating torque demand or which has pulsations in its supply. When pulsations arise from load variation or from variations in the source they may result in what is known as "forced" oscillations. If their period agrees closely with that of the natural period of the rotor oscillations, then the rotor will receive impulses in time harmony, acting to amplify each succeeding swing. If, on the other hand, the forced impulses occur at a greater or lesser period than that of the natural oscillation, at times the impulse will act against the oscillation and will result in reduced instead of augmented amplitude.

It is customary to assume that, if the forced period is within 20 per cent of the natural period, cumulative oscillations will occur and the machine will pull out of step unless powerfully damped.

The oscillating period and the torque angle swing with load change are both influenced by the moment of inertia (or WR^2) of the rotating masses and by the "synchronizing torque." We will find that the synchronizing torque will depend upon the excitation, and hence the natural period of oscillation will be a function of the excitation or power factor at which the machine is operated.

In addition, a damping winding in the pole faces as previously mentioned will influence the period and transient torque angle.

Any thorough quantitative analysis of hunting in a synchronous machine will have to be comprehensive enough to include all these factors. In the chapters which follow, the linear differential equation of the second order, expressing the items acting on the rotor as a torsional pendulum, will be analyzed under various conditions. This presentation is by no means complete or entirely accurate, owing to various simplifying assumptions which will be made. This serves instead as an introduction to the fundamentals of a subject which can be found treated in greater detail in the technical literature covering not only synchronous motors but also the parallel operation of alternators and the whole question of power system stability.

358. Analysis of the Rotor As a Torsional Pendulum. When a synchronous machine operates on a system the rating of which is large compared with its own capacity, it behaves as a torsional pendulum. Three forces act upon such a pendulum:

(*a*) The torque due to the acceleration of its own mass. This is positive when the speed is increasing and may be expressed by

$$J\frac{d^2\alpha_m}{dt^2}$$

where J = the polar mass moment of inertia of the rotating parts, consisting of rings or concentrated masses. For the method of calculating this value, see Article 362

α_m = the instantaneous value of the angle of oscillation measured in mechanical radians

α = the instantaneous value of the angle of oscillation measured in electrical radians

$$\alpha_m = \frac{\alpha}{\frac{P}{2}}$$

α_{m0} = the constant value of α_m due to load torque T existing at the instant ΔT is applied

α'_m = the variable part of α_m due to the sudden load change ΔT

$\alpha_m = \alpha_{m0} + \alpha'_m$

(*b*) The torque due to damping action. This is approximately proportional to the time rate of change of the angle of oscillation. Its value is positive when α_m is increasing. Accordingly, this torque may be expressed as

$$k_1 \frac{d\alpha_m}{dt}$$

To determine k_1, see Article 363.

(*c*) The torque produced by the current and flux in the machine as previously described. This torque has been shown to be proportional to

$$\frac{VE}{Z_s} \sin \alpha$$

for non-salient-pole machines, and to

$$\frac{VE}{X_d} \sin \alpha + \frac{V^2(X_d - X_q)}{2X_dX_q} \sin 2\alpha$$

for salient-pole machines by the more exact two-reaction theory. We have also pointed out one expression for synchronizing power, denoted by P_r (Article 331). Unfortunately, from these expressions, the torque is not proportional to α, but has the form of one or more sine functions of the torque angle which cannot be used in this analysis. They lead to elliptical functions and elliptical integrals which are too involved for this presentation. Hence we will set up an expression for torque in terms of $k_2\alpha_m$ instead of $k_2 \sin \alpha_m$ or $k_2 \sin \alpha_m + k'_2 \sin 2\alpha_m$. Since the terms in the final expression are chosen to represent torque in pounds-feet, k_2 will express pounds-feet per mechanical radian of torque angle. Except for units, it is of the same nature as the synchronizing power P_r already

defined. The substitution of α_m for sin α_m results in little error for small values of torque angle, and hence it is necessary to point out that, in the following analyses, the work applies less accurately for large oscillating angles.

A graphical method of estimating k_2 will be shown later. The method of obtaining this will be the only distinction required in these analyses between salient- and non-salient-pole machines.

In addition to these three forces, we may have present either a load or a disturbing force which is constant or which varies as some known function of time. Here the following cases will be considered:[1]

Case I. A steady load of torque ΔT is thrown on or off suddenly at the instant $t = 0$.

Case II. A pulsating torque is thrown on or off at the instant $t = 0$.

In addition to these problems, that of determining the natural period of oscillation of an undamped machine will be outlined with sufficient detail for solution, but without all the theoretical ramifications which present themselves.

359. Analysis of Case I. By combining the elements operative upon the synchronous motor, Case I can be expressed by the following equation:[2]

$$J\frac{d^2\alpha_m}{dt^2} + k_1\frac{d\alpha_m}{dt} + k_2\alpha_m = T + \Delta T \qquad [593]$$

[1] Some additional cases are discussed in Steinmetz, "Theory and Calculation of Electrical Apparatus," Chapter XVIII; "Theory and Calculation of Electric Circuits," Chapter XI. Both are published by McGraw-Hill Book Co.

See also:

E. Arnold and J. L. LaCour, "Die synchronen Wechselstrom Maschinen," Julius Springer, Berlin.

H. Mauduit, "Machines électriques."

Miles Walker, "Specification and Design of Dynamo-electric Machinery," Longmans, Green and Co., New York.

An article by Dr. Rosenberg, *J.I.E.E.* (British), Vol. 42, 1916.

[2] This equation is of the same form as that for a circuit consisting of an inductance, capacitance, and resistance in series with a battery and switch.

$$E = L\frac{di}{dt} + iR + \frac{Q}{C}$$

Or, with

$$i = \frac{dQ}{dt}$$

$$E = L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C}$$

For oscillograms of such circuits, see Magnusson's "Electric Transients," Chapter V, McGraw-Hill Book Co.

All these symbols have been defined. Briefly, however, the first term represents the torque due to rotor and load inertia; the second term represents the torque due to the effect of dampers; and the third represents that due to the elastic deformation of the field flux.

Because of the linear form of this equation, the solution, for the steady load torque T and added torque ΔT, can be considered with both effects together, or separately, and the latter results added. ΔT is taken as positive if load is added; negative, if it is removed. The most general solution of such an equation for Case I is

$$\alpha_m = A\epsilon^{(-a+b)t} + B\epsilon^{(-a-b)t} + \frac{T + \Delta T}{k_2} \qquad [594]$$

A and B are constants of integration to be determined from the boundary conditions of the problem; $(-a + b)$ and $(-a - b)$ are the roots of an auxiliary equation which is written from equation 593 as

$$Jm^2 + k_1 m + k_2 = 0 \qquad [595]$$

It has the roots

$$\frac{k_1}{2J} \pm \sqrt{\left(\frac{k_1}{2J}\right)^2 - \frac{k_2}{J}} \qquad [596]$$

From this the abbreviations are chosen:

$$a = \frac{k_1}{2J} \qquad [597]$$

$$b = \sqrt{\left(\frac{k_1}{2J}\right)^2 - \frac{k_2}{J}} \qquad [598]$$

In order to express the solutions conveniently, it is necessary to distinguish three subcases. They are

(*a*) $\left(\frac{k_1}{2J}\right)^2 > \frac{k_2}{J}$ (non-oscillating or over-damped)

(*b*) $\left(\frac{k_1}{2J}\right)^2 < \frac{k_2}{J}$ (oscillating)

(*c*) $\left(\frac{k_1}{2J}\right)^2 = \frac{k_2}{J}$ (aperiodic, non-oscillating or borderline condition)

The significance of these three conditions can be seen from their influence on the expression for (*b*). Figure 282 gives an idea of the resulting curves.

(*a*) In this case the expression under the radical in equation 598 is positive and the roots are real. This gives a transient condition in which α_m goes to its final value without oscillation. It does not "overshoot" the final torque angle during the period of adjustment. This is possible only with heavy damping and small flywheel effect. Under certain conditions α_m may become large enough to cause the motor to

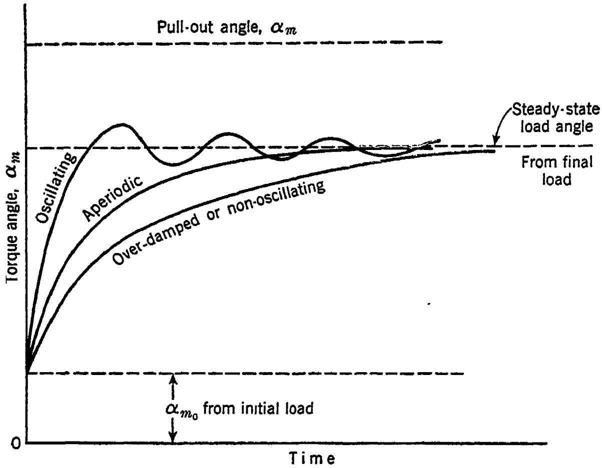


FIG. 282. Approximate forms of torque-angle transients with various degrees of damping and inertia.

break from step, but without the presence of an oscillation. It is not ordinarily achieved in practice.

(*b*) When $(k_1/2J)^2$ is less than k_2/J, the roots are imaginary and α_m goes to its final value after oscillation of the rotor. This is the condition for hunting and is most commonly present in practice. Because of the negative sign under the radical equation 598 must be written

$$jb = \sqrt{\frac{k_2}{J} - \left(\frac{k_1}{2J}\right)^2} \qquad [599]$$

(*c*) When $(k_1/2J)^2 = k_2/J$, the value of (*b*) reduces to zero, and the exponents of equation 594 are both $-a$. The solution of equation 593 then has the form

$$\alpha_m = (At + B)\epsilon^{-at} + \frac{T + \Delta T}{k_2} \qquad [600]$$

This case will not be considered. It represents the limiting non-oscillating condition, or so-called critical case, which is not ordinarily obtained or maintained in practice.

360. Constants of Integration: Case Ia. These constants are evaluated by noting that, when the disturbance occurs, α_m will have a value, say, α_{m0}, which is fixed by the machine load at the instant before the disturbance. It is assumed throughout that the torque being delivered to the load has remained constant at all times.[3] Then when t is zero, the exponents of ϵ become zero, and from equation 594:

$$\alpha_{m0} = A + B + \frac{T + \Delta T}{k_2} \qquad [601]$$

Since α_{m0} equals T/k_2, we can write from this

$$A + B = -\frac{\Delta T}{k_2} \qquad [602]$$

For a second equation involving A and B, we note that at the first instant after the disturbance the inertia must "carry" all ΔT. Therefore, at that instant,

$$J\frac{d^2\alpha'_m}{dt^2} = \Delta T \qquad [603]$$

or

$$\frac{d^2\alpha'_m}{dt^2} = \frac{\Delta T}{J}$$

Let us now obtain the second derivative of equation 594. Then

$$\frac{d^2\alpha'_m}{dt^2} = (-a + b)^2 A\epsilon^{(-a+b)t} + (a + b)^2 B\epsilon^{-(a+b)t} \qquad [604]$$

Note that equation 594 was written for the complete angle α_m (constant α_{m0} plus the variable α'_m) while equation 604 involves only the variable portion. This is in line with the statement previously made that the entire phenomenon can be considered, or any of its parts, with the component effects added. Here we prefer to assume that no initial load exists; hence α_{m0} and T are zero. Then equation 604 expresses the transient angular change only.

At the instant t equals 0, equation 604 becomes

$$\frac{d^2\alpha'_m}{dt^2} = (-a + b)^2 A + (a + b)^2 B \qquad [605]$$

[3] For a discussion of circumstances under which this is not the case, see Steinmetz, "Theory and Calculation of Electrical Apparatus," p. 293, McGraw-Hill Book Co.

Combine with equation 603 and we obtain

$$\frac{\Delta T}{J} = (-a + b)^2 A + (a + b)^2 B \qquad [606]$$

From equation 602:

$$-\frac{\Delta T}{k_2} = A + B \qquad [607]$$

These two equations are to be solved simultaneously for A and B, yielding

$$A = -\frac{\Delta T}{4ab}\left[\frac{(a + b)^2}{k_2} + \frac{1}{J}\right] \qquad [608]$$

$$B = +\frac{\Delta T}{4ab}\left[\frac{(-a + b)^2}{k_2} + \frac{1}{J}\right] \qquad [609]$$

Equations 608 and 609 are in terms of known machine constants. With A and B evaluated, they may be substituted in equation 594 to yield values of torque angle at any instant. If more convenient, the term T can be dropped from this equation, and the torque angle then obtained is α'_m. To this can be added α_{m0} equals T/k_2

361. Case I*b*: Oscillation. For the rotor to oscillate as a torsional pendulum with load change or similar disturbance it is necessary that the machine constants be such that

$$\left(\frac{k_1}{2J}\right)^2 < \frac{k_2}{J}$$

This is the case ordinarily. Then

$$a = \frac{k_1}{2J} \qquad [610]$$

$$jb = \sqrt{\frac{k_2}{J} - \left(\frac{k_1}{2J}\right)^2} \qquad [611]$$

The general equation 594 takes the form

$$\alpha'_m = \epsilon^{-at}(A\epsilon^{jbt} + B\epsilon^{-jbt}) + \frac{\Delta T}{k_2} \qquad [612]$$

This represents only the oscillating component superposed upon the initial fixed torque angle α_{m0}. The expression in parentheses expresses two oppositely rotating vectors, A and B, which diminish with time by the factor ϵ^{-at}.

Recalling the various methods of expressing vector rotation, it will be seen that equation 612 can be written in the more convenient form

$$\alpha'_m = \epsilon^{-at}[A(\cos bt + j \sin bt) + B(\cos bt - j \sin bt)] + \frac{\Delta T}{k_2}$$

$$= \epsilon^{-at}[(A + B) \cos bt + j(A - B) \sin bt] + \frac{\Delta T}{k_2} \qquad [613]$$

Let

$$(A + B) = A_1, j(A - B) = B_1 \quad \text{and} \quad \sqrt{A_1^2 + B_1^2} = C.$$

Then

$$\alpha'_m = \epsilon^{-at}[A_1 \cos bt + B_1 \sin bt] + \frac{\Delta T}{k_2} \qquad [614]$$

$$= \epsilon^{-at}C \sin (bt + \theta) + \frac{\Delta T}{k_2}$$

$$\theta = \tan^{-1} \frac{A_1}{B_1}$$

Our problem is to solve for the constants A and B, or A_1 and B_1, from the boundary conditions. When t equals 0, from equation 614:

$$\alpha'_m = 0 = A_1 + \frac{\Delta T}{k_2} \qquad [615]$$

Then

$$A_1 = -\frac{\Delta T}{k_2} \qquad [616]$$

For a second condition, apply the reasoning expressed with equation 603 and again we obtain

$$\frac{d^2\alpha'_m}{dt^2} = \frac{\Delta T}{J} \qquad [617]$$

For the final condition, obtain another expression involving A and B by taking the second derivative of equation 612, and simplify by the use of A_1 and B_1.

This is

$$\frac{d^2\alpha'_m}{dt^2} = (a^2 - b^2)A_1 - 2abB_1 \qquad [618]$$

Combine with equation 617:

$$\frac{\Delta T}{J} = (a^2 - b^2)A_1 - 2abB_1 \qquad [619]$$

Solve simultaneously with equation 616:

$$B_1 = -\frac{(a^2 - b^2)\dfrac{\Delta T}{k_2} - \dfrac{\Delta T}{J}}{2ab} \qquad [620]$$

and

$$A_1 = -\frac{\Delta T}{k_2} \quad \text{(from equation 616)} \qquad [621]$$

These values, determined from known machine constants, are substituted in equation 614 to obtain instantaneous values of the torque angle. To these must be added α_{m0}. Or if more convenient, the term C can be used directly with equation 614. This will be

$$C = \pm \frac{\Delta T}{k_2}\sqrt{\left[\frac{k_2 + J(a^2 - b^2)}{2abJ}\right]^2 + 1} \qquad [622]$$

and

$$\theta = \tan^{-1}\frac{2abJ}{J(a^2 - b^2) + k_2} \qquad [623]$$

This solution fails if b equals 0.

This completes the analyses to be given in detail for Case I. Before going further, the physical significance of J, k_1, and k_2 will be studied with particular reference to methods of evaluating them.

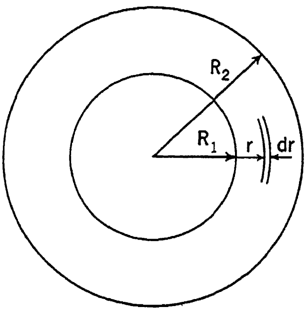


FIG. 283.

362. Rotor Dimensions and Polar Mass Moment of Inertia. Examination of the fundamental formula pertaining to the rotor as a torsional pendulum shows that three physical values of the machine must be determined. The first of these is the polar mass moment of inertia, previously used for retardation runs. (See Alternators, Article 92.)

Consider the rotor as being made up of two parts: (1) a circular mass such as the pole yoke or flywheel rim, and (2) the radial parts such as spider legs or spokes and the poles with their copper, etc.

To find the polar mass moment of inertia of a ring, the following values must be known:

l = axial length of ring in feet
g = 32.16 ft per sec per sec
W = weight of ring material in pounds per cubic foot

$\omega = 2\pi \times$ rps
$t =$ time in seconds
R_1 and $R_2 =$ radii in feet as shown in Fig. 283.
$r =$ the variable as shown

The moment at any point r is then

$$r^2\left[\frac{2\pi r l W(dr)}{g}\right] \text{ pound-feet}^2 \qquad [624]$$

The moment for all points of a ring:

$$\frac{2\pi l W}{g}\int_{R_1}^{R_2} r^3 dr = \frac{2\pi l W}{4g}(R_2^4 - R_1^4) \qquad [625]$$

To this may be added $\sum \frac{W_T}{g} r_1^2$ for the poles or other concentrated masses, where

$W_T =$ the total weight of the mass considered
$r_1 \approx$ the distance from the center of gravity of the mass to the center of the shaft, in feet

The polar mass moment of inertia for a rotor of several rings and other parts can then be expressed

$$J = \sum \frac{2\pi l W}{4g}(R_2^4 - R_1^4) + \sum \frac{W_T}{g} r_1^2 \qquad [626]$$

363. The Damping-torque Constant. The expression $k_1(d\alpha_m)/(dt)$ expresses the damping effect in terms of pound-feet. Hence the constant k_1 must be in terms of pound-feet per mechanical radian per second. This damping effort arises from the squirrel-cage bars in the pole faces, cutting through the air-gap flux when it moves during periods of load change or hunting, and thereby inducing a voltage in them. This causes a current to flow, which in turn reacts with the gap flux such as to oppose the motion or give a damping torque. Because of the low frequency of this current, it is opposed chiefly by the resistance of the bars and rings, the reactance being negligible.

In evaluating k_1 one can take into account the air-gap flux and the rate of cutting to determine the voltage induced per bar. If a portion of the end-ring resistance is assigned to each bar, the effective current of the bars can be determined by Ohm's law. This together with the gap flux yields a calculation for torque, somewhat as that expressed for induction motors. Since this is essentially a design calculation it will not be in-

cluded here. However, a comparatively simple calculation based on test data will be shown.

The damping torque in pound-feet per mechanical radian per second is a function of the torque developed by the cage, running as an induction motor, with a slip of s and with the gap flux corresponding to synchronous-motor operation. At the lower part of the induction-motor speed-torque curve the relationship is almost a straight line. Assuming it is such, the procedure might be somewhat as follows:

If synchronous speed for a certain motor is 900 rpm, this corresponds to 15 rps or 94.2 mechanical radians per second. Let us assume that tests show that when running at the correct air-gap flux the torque developed is 980 lb-ft at a slip of 4.5 per cent. This corresponds to a torque of

$$\frac{980}{4.5} \quad \text{or} \quad 218 \text{ lb-ft for } 1\% \text{ slip} \quad \text{(straight line)}$$

One per cent slip corresponds to 0.01 times 94.2 or 0.942 mechanical radian per second. Hence the torque developed per mechanical radian per second is

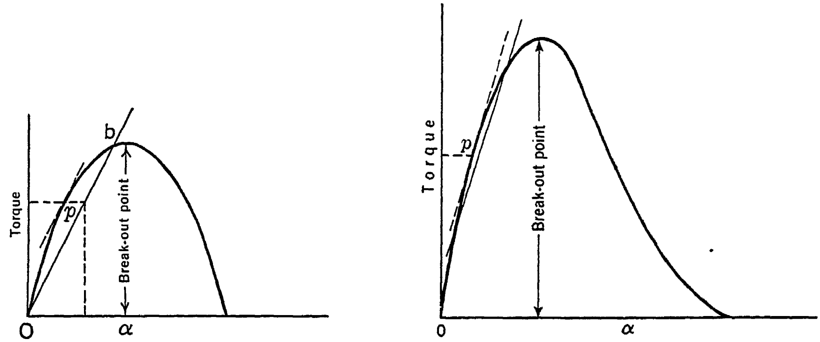
$$\frac{218}{0.942} \quad \text{or} \quad 231 = k_1$$

The above value is based on *average* torque, and hence if "cogging" occurs in the motor, it will not express the instantaneous value of torque at the particular position at which the synchronous motor is oscillating and the damping winding is called into play. This is particularly true in those cases in which the bars or end rings of the damping winding are not continuous about the entire rotor periphery, but are in the salient poles only. Approximate corrections for this can be applied to the calculations made from design data, and also to the above method calculated from a test point on the speed-torque curve. In the latter case the cogging results in a superposed variation on the average torque as a second harmonic function of the torque angle.

364. "Field-elasticity" Constant. The torque produced by the current and flux in the usual synchronous motor action is expressed by $k_2\alpha_m$. Since α_m is the torque angle in mechanical radians, it follows that k_2 represents the pound-feet of torque change per mechanical radian. The torque developed per unit of angular displacement is a familiar term, being the $dP/d\alpha$ or P_r already calculated, but in different terms. (See Articles 328, 335, and 336.) Hence by taking the derivative of the expression for power developed, with respect to the torque angle, a means is obtained for evaluating k_2. Conversion must be made from power to

torque, and from electrical degrees to mechanical radians. Graphically this rigidity factor or stability factor is the tangent to the curve at the torque angle of calculation.

Refer to Fig. 284. Note in either *a* or *b* that the dashed line would represent the slope of the curve, or the stability factor at some point *p*. If, on the other hand, the torque angle, over which the hunting occurs,



(*a*) Cylindrical rotor

(*b*) Salient-pole rotor

Fig. 284.

varies over a considerable range, it may be more logical to draw the straight lines as shown, from which an average slope can be used to determine k_2. Such a calculation gives a mean value over the range selected by the location of the line approximating the curve.

For non-salient-pole machines, the sinusoidal curve for torque angle versus torque is used for the graphical construction as shown in Fig. 284*a*. For salient-pole machines by the two-reaction theory, the torque curve of Fig. 284*b* is more correct, containing the second-harmonic component.

This distinction between the two torque curves is the only manner in which the saliency of the poles comes into these analyses.

CHAPTER XLIV

NATURAL PERIOD OF OSCILLATION. PERIODIC TORQUE.

365. Chapter Outline.

Natural Period of Oscillation.
Without a Damper.
With a Damper Winding.
Examples.
Case II. Periodic Torque.
Example.

366. Effects without Dampers. The natural period of oscillation of the rotor as a torsional pendulum is of importance in determining stability and will be investigated here. The cases with and without damping windings will be considered. Actually some damping effect is brought about in the pole faces, even though no damping winding is present, but such effect will be neglected.

The first obvious change brought into the analyses results from k_1 being equal to zero. The exponent a is then zero, and the non-oscillating conditions of Cases Ia and c, wherein $(k_1/2J)^2$ is greater than k_2/J, are obviously impossible.

For Case Ib we have shown that

$$\alpha'_m = \epsilon^{-at}(A\epsilon^{jbt} + B\epsilon^{-jbt} + \frac{\Delta T}{k_2} \qquad [627]$$

and

$$B_1 = \frac{-(a^2 - b^2)\dfrac{\Delta T}{k_2} - \dfrac{\Delta T}{J}}{2ab} \qquad [628]$$

With a equal to zero, B_1 also becomes zero, and the torque angle is expressed

$$\alpha'_m = A_1 \cos bt + \frac{\Delta T}{k_2} \qquad [629]$$

Since we know (equation 616) that

$$A_1 = -\frac{\Delta T}{k_2}$$

it follows that

$$\alpha'_m = \frac{\Delta T}{k_2}(1 - \cos bt) \qquad [630]$$

Now, as the cosine term can vary between the limits of $+1$ and -1, the amplitude of torque pulsation will be from

$$\alpha'_m = 0 \quad \text{to} \quad \alpha'_m = 2\frac{\Delta T}{k_2}$$

These values of torque angle are to be added to any initial angle α_{m0} which may have existed at the time when the motor was loaded by ΔT.

The angular velocity of the rotating vector, represented by equation 630, is obviously b. This is the instantaneous angular velocity of hunting.

$$bt = 2\pi$$

Hence the time for one complete cycle of hunting is

$$t = \frac{2\pi}{b} = 2\pi\sqrt{\frac{J}{k_2}} \text{ seconds} \qquad [631]$$

and the frequency of hunting is

$$f' = \frac{1}{2\pi}\sqrt{\frac{k_2}{J}} \qquad [632]$$

367. Period of Oscillation with Dampers. Refer back to equation 612 for the torque-angle formula with damping present. The term ϵ^{-at} provides the decrement by which the oscillations are reduced, a being the decrement or the attenuation constant. The vectors A_1 and B_1 are oppositely rotated by the operators ϵ^{+jbt} and ϵ^{-jbt}, respectively. Then b is a measure of the instantaneous speed and the instantaneous angular velocity of hunting.

$$bt = 2\pi$$

Hence the time for one complete cycle of hunting is

$$t = \frac{2\pi}{\sqrt{\frac{k_2}{J} - \left(\frac{k_1}{2J}\right)^2}} \qquad [633]$$

368. Examples. The theory developed in the previous paragraphs will be applied to the salient-pole synchronous motor already described in Article 336.

The motor rating and data calculated are as follows:

200 hp	60 cycles	440 volts	three-phase Y-connected
Unity pf	8 poles	900 rpm	

I = 210 amperes

r_e = 0.0285 ohm per phase

X_d = 1.27 ohms per phase

X_q = 0.774 ohm per phase

Full-load torque angle = 33° 13′

Maximum torque angle (pull-out point) = 69° 40′

To Determine the Value of k_2. Note that the stability factor P_r has already been determined for this motor, for unity pf at full load, as 217.5 kw per electrical radian. Transferred to pound-feet of torque per mechanical radian, this was found to be 6880, although this is high, being based on input rather than internal value. However, as a first approximation, $k_2 \approx 6880$.

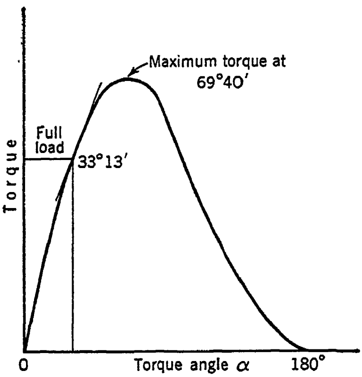


FIG. 285.

Refer to Fig. 285 giving the torque versus torque-angle characteristic for this motor. The value of k_2 or P_r represents the slope of the curve at the full-load point.

To Determine the Value of J. The WR^2 value for the motor is known to be 1380 lb-ft^2. Hence

$$J = \frac{1380}{32.16} \quad \text{or} \quad 42.9$$

The Natural Frequency of Oscillation.

$$f = \frac{1}{2\pi}\sqrt{\frac{k_2}{J}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{6880}{42.9}} \quad \text{or} \quad 2.02 \text{ cycles per second}$$

N.E.M.A. Standards recommend that the natural frequency of oscillation be determined as follows:

$$F = \frac{35{,}200}{\text{rpm}}\sqrt{\frac{P_r \times f}{WR^2}} \qquad [634]$$

F = the frequency in cycles per minute

P_r = synchronizing power as previously defined

Then

$$F = \frac{35{,}200}{900}\sqrt{\frac{217.5 \times 60}{1380}} \quad \text{or} \quad 120$$

120 cycles per minute equals 2.0 per second, checking with the previous method.

Limits of Torque Angle When Oscillating. Assume that the motor is operating with a load torque of 500 lb-ft when an additional load of 650 lb-ft of torque is suddenly applied. With damping neglected, what will be the limits of torque-angle oscillation? Assume that k_2 maintains the average value of 6880 lb-ft per mechanical radian over the resulting oscillation. From equation 630 and the limits shown for that equation,

$$\alpha'_m = 0$$

$$\text{to} \quad \alpha'_m = 2\frac{650}{6880} \quad \text{or} \quad 0.19 \text{ mechanical radian}$$

The initial torque angle is

$$\frac{T}{k_2} = \frac{500}{6880} \quad \text{or} \quad 0.0729 \text{ mechanical radian}$$

The oscillation, then, is between the limits of 0.0729 and 0.0729 plus 0.19 or 0.2629 mechanical radian, occurring at a frequency of 2 per second.

369. Second Example. Damping. Up to this point no damping effect has been considered. We will deal with the same motor, using a slight change in excitation, resulting in an average value of k_2 (as obtained from a graphical construction such as Fig. 284) equal to 5450 lb-ft per mechanical radian. The cage in the face of the poles is such as to yield a value of k_1 equal to 212 lb-ft per mechanical radian per second. (See Article 363 for method.)

(*a*) To calculate the new period of oscillation,

$$\left(\frac{k_1}{2J}\right)^2 = \left(\frac{212}{2 \times 42.9}\right)^2 \quad \text{or} \quad 6.1$$

$$\frac{k_2}{J} = \frac{5450}{42.9} \quad \text{or} \quad 127$$

From equation 597:

$$a = \frac{212}{2 \times 42.9} \quad \text{or} \quad 2.48$$

From equation 599:

$$b = \sqrt{127 - 2.48^2} \quad \text{or} \quad 11.0$$

Since $(k_1/2J)^2 < k_2/J$, the rotor will oscillate when load is applied. Its frequency (from equation 633) will be 1.75 cycles per second. This change is brought about both by modification of k_2 and the effect of k_1. With no damping, the value of k_2 would result in a frequency of 1.80 cycles per second.

(*b*) Assume that the damping is raised (lowered resistance of the bars and end rings of the pole cages) until k_1 equals 1100. An original load of 300 lb-ft of torque (T) is increased to 1360 lb-ft ($T + \Delta T$). Determine the equation of the torque angle. Then:

$$\left(\frac{k_1}{2J}\right)^2 = 165.0; \quad \frac{k_2}{J} = 127; \quad a = 12.83; \quad b = 6.13$$

From equation 608:

$$A = -0.3005$$

From equation 609:

$$B = 0.1055$$

As checked in equation 598, b is real, and hence there are no oscillations when this load is increased.

The final equation 594 is then, for the entire value,

$$\alpha_m = -0.3005\,\epsilon^{-6.7t} + 0.1055\,\epsilon^{-18.96t} + \frac{1360}{5450}$$

$$= 0.25 \text{ mechanical radian at } \quad t = \infty$$

$$= -0.3005 + 0.1055 + 0.25 \quad \text{or} \quad 0.055 \quad \text{at} \quad t = 0$$

Thus the initial torque angle is 0.055 radian and the final value, reached without oscillation, is 0.25 mechanical radian. The curve is shown as 1 on Fig. 286. Such extreme damping, as previously mentioned, is not to be expected in practice.

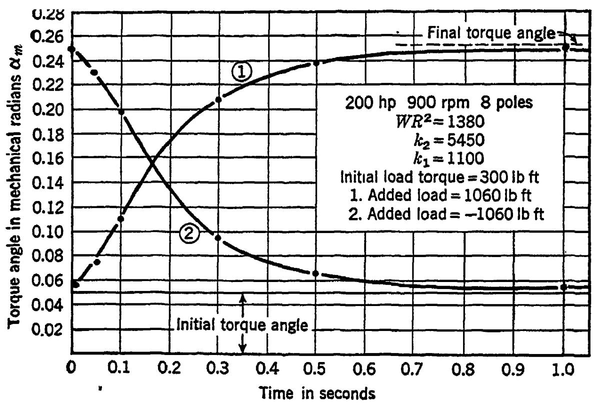


FIG. 286.

If the load of 1060 lb-ft is suddenly removed, the torque-angle change is illustrated by curve 2 on this figure, and the equation becomes

$$\alpha_m = 0.3005\,\epsilon^{-6.7t} - 0.1055\,\epsilon^{-18.96t} + \frac{300}{5450}$$

370. Analysis of Case II. Periodic Torque. If a load requiring a constant average torque ΔT, with an alternating component $T_m \sin(\omega t + \phi)$, is suddenly applied to a synchronous motor, the fundamental equation of motion becomes

$$J\frac{d^2\alpha'_m}{dt^2} + k_1\frac{d\alpha'_m}{dt} + k_2\alpha'_m = \Delta T + T_m \sin(\omega t + \phi) \qquad [635]$$

ΔT = the average load torque increment. If the load is removed, both ΔT and T_m are negative. Both are in pound-feet.

T_m = the maximum value of the disturbing torque, above or below the average

ϕ = the time angle at which the applied mechanical load is thrown on. At $t = 0$ the torque is fixed by the value of ϕ. Physically, if we consider a compressor as comprising the motor load, the particular position in its stroke at which the load was thrown on would determine ϕ.

α'_m = the variable part of the torque angle due to the increment. as previously defined

An initial steady load may have been present, giving a torque angle α_{m0}. Note that equation 635 considers only the added torque. As this equation is written, only the fundamental component of the disturbing torque is considered. But, because the equation is linear, the alternating-torque term may represent other torque harmonics, whose effects may be investigated independently. The total torque angle would then be the sun

$$\alpha_m = \alpha_{m0} + \alpha'_m = \alpha_{m0} + \alpha_{mT} + \alpha_{m1} + \alpha_{m2} \cdots$$

α_{mT} = the increment of α_m due to ΔT

α_{m1} = the increment due to the fundamental torque

α_{m2} = the increment due to the second harmonic, etc.

For non-sinusoidal waves of load torque each harmonic will have its own value of ϕ.

371. Solution. Subcases. The solution of equation 635 takes the form

$$\alpha'_m = A\epsilon^{(-a+b)t} + B^{(-a-b)t} + \frac{\Delta T}{k_2} + \frac{T_m \sin(\omega t + \phi - \theta)}{\sqrt{(k_2 - \omega^2 J)^2 - \omega^2 {k_1}^2}} \qquad [636]$$

where

$$\theta = \tan^{-1} \frac{k_1 \omega}{k_2 - \omega^2 J} \qquad [637]$$

As before, the first two terms represent the transient disturbance, and the last two terms represent the steady-state behavior. The last term is a "steady-state" component in the sense that it is periodically repeating, although it indicates a varying torque angle.

The transient terms fall naturally into three cases:

(*a*) $$\left(\frac{k_1}{2J}\right)^2 > \frac{k_2}{J} \quad \text{(non-oscillating)}$$

(*b*) $$\left(\frac{k_1}{2J}\right)^2 < \frac{k_2}{J} \quad \text{(oscillating)}$$

(*c*) $$\left(\frac{k_1}{2J}\right)^2 = \frac{k_2}{J} \quad \text{(aperiodic)}$$

For the transient oscillatory case (*b*), the exponents in the transient terms are complex, so that these terms represent oppositely rotating vectors with diminishing amplitude. Hence

$$\alpha'_m = \epsilon^{-at}(A_1 \cos bt + B_1 \sin bt) + \frac{T_m \sin(\omega t + \phi - \theta)}{\sqrt{(k_2 - \omega^2 J)^2 + k_1^2\omega^2}} + \frac{\Delta T}{k_2} \quad [638]$$

Equation 638 shows that two frequencies of mechanical oscillation are present:

1. Free oscillations: $f_1 = \dfrac{b}{2\pi}$ cycles per second

2. Forced oscillations: $f_2 = \dfrac{\omega}{2\pi}$ cycles per second

If the damping action is very small so that it need not be considered, B_1 drops out and equation 638 becomes:

$$\alpha'_m = A_1 \cos bt + \frac{\Delta T}{k_2} + \frac{T_m \sin(\omega t + \phi)}{k_2 - \omega^2 J} \quad [639]$$

To apply equation 638 to an actual problem, the constants A_1 and B_1 must be determined. This can be done by noting that at the instant when the oscillating load is thrown on, t equals 0 and α'_m equals 0. Equation 638 then becomes

$$\alpha'_m = 0 = A_1 + \frac{T_m \sin(\phi - \theta)}{\sqrt{(k_2 - \omega^2 J)^2 + k_1^2\,\omega^2}} + \frac{\Delta T}{k_2} \quad [640]$$

This determines A_1. The second derivative of α'_m with respect to t gives the acceleration. At the time t equals 0, the torque which must be carried by the inertia of the moving parts is ΔT plus $T_m \sin \phi$ and this must be equal to $J(d^2\alpha'_m/dt^2)$. From this follows a relation containing A_1 and B_1, and hence B_1.

The final value of α_m is the sum of α'_m, as found, and any initial value α_{m0}. Commercially, the problem is to keep α_m sufficiently below the break-out point. N.E.M.A. Standards prescribe that, when synchronous motors drive reciprocating loads, "the combined installation shall have sufficient inertia ⋯ to limit the variations in motor armature current to a value not exceeding 66 per cent of full-load current." Although such variation is to be determined by oscillograph, the method shown here for determining torque angles could be used to determine armature current at the extremes of the torque-angle pulsations, and hence the variation.

372. Example of Case II. A small synchronous motor is connected to a double-acting compressor. The following data apply to the motor:

Phases	3	Poles	6
Cycles	60	Horsepower	15
Voltage	220	Amperes	34.2

J for motor and load = 0.972, polar mass moment of inertia of rotating parts
k_1 (damping constant) = 15.0 lb-ft per mechanical radian per second
Maximum torque angle = 0.523 mechanical radian
Compressor speed is 200 rpm
The load pulsation is represented by Fig. 287

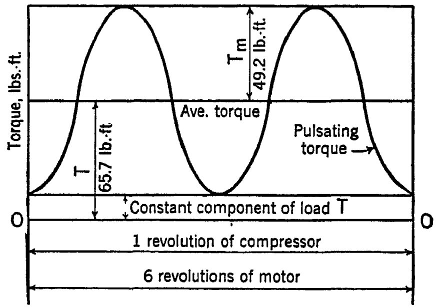


FIG. 287.

The following constants are calculated (solution not shown):

k_2 = 667 lb-ft per mechanical radian
$a = 7.72$
$b = 25$
$A_1 = -0.0581$ and -0.1389
$B_1 = +0.206$ and -0.268

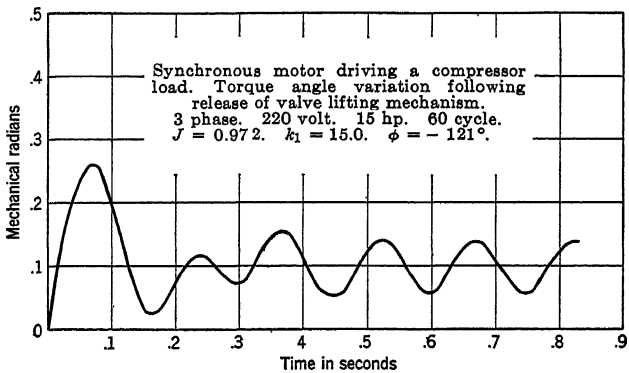


FIG. 288.

When the load is thrown on at the instant in which ϕ equals $-121°$, the torque angle equation 638 becomes

$$\alpha_m = \epsilon^{-7.72t}(-0.0581 \cos 25t + 0.206 \sin 25t) + 0.0985 + 0.0404 \sin (\omega t - 90°)$$

This is shown in Fig. 288.

When ϕ is $+59°$, the transient is as shown in Fig. 289. After 0.5 sec, the pulsation is due to the nature of the load, the transient oscillation having practically died

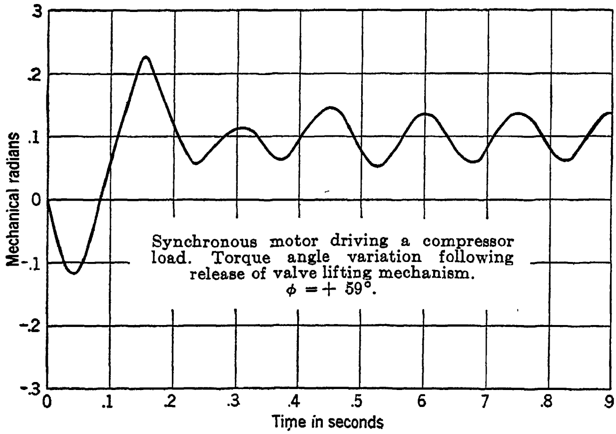


FIG. 289.

out. It is recommended that this continued pulsation be not greater than 3.5 electrical degrees, plus or minus, to limit mechanical vibrations.[1] Previously cited N.E.M.A. Standards achieve this limit by current pulsation rather than by torque angle.

[1] R. E. Doherty, "Flywheel Effect for Synchronous Motors Connected to Reciprocating Compressors," *Gen. Elec. Rev.*, p. 653, August, 1920.

ALTERNATORS IN PARALLEL

CHAPTER XLV

SYNCHRONIZING. LOAD DIVISION BETWEEN PARALLELED UNITS

373. Chapter Outline.

Synchronizing.
  Methods of Indicating Synchronism.
  Connections.
Division of Load between Paralleled Units.
  Effect of Change in Excitation.
  Effect of Change in Torque.

374. Synchronizing. The interconnection of large generating stations and the use of several a-c generators to supply the load of one central station involve the parallel operation of the generating units. In order to operate one generator in parallel with another or in parallel with a bus system supplied by other sources, a number of conditions must be fulfilled in connecting the oncoming machine to the line. This chapter will deal with the process of "synchronizing," that is, getting a machine "in rhythm" with the others, the conditions necessary for successful parallel operation, and the methods of properly dividing the load between the several units.

For a d-c generator to be connected in parallel with another it is necessary that it be brought up to speed, its voltage adjusted until it equals that of the other machine (or the bus to which it is to be paralleled), and its polarity checked so that the leads of the oncoming generator will be connected to ones of like sign on the other machine. The main switch connecting the machines can then be closed. On a permanent setup it is not necessary to check the polarities except for making the connections at the time of installation. Of course, if for any reason a machine builds up with the wrong polarity, correction must be made.

With two d-c machines in parallel, the load can be divided between them in various proportions by adjustment of their excitations. It is not necessary that the speeds of the two machines be the same.

With alternators the problem is slightly more complicated. Before they can be switched together it is necessary that their terminal voltages be equal, of the same frequency, and in phase with each other. Hence, in the process of synchronizing an a-c generator its speed must be exactly equal to that of the other machines, if they have the same number of poles; otherwise the speeds must differ in the proper proportion to give equal frequencies. Once in parallel no speed deviation is permissible, nor under ordinary conditions will it be possible. With

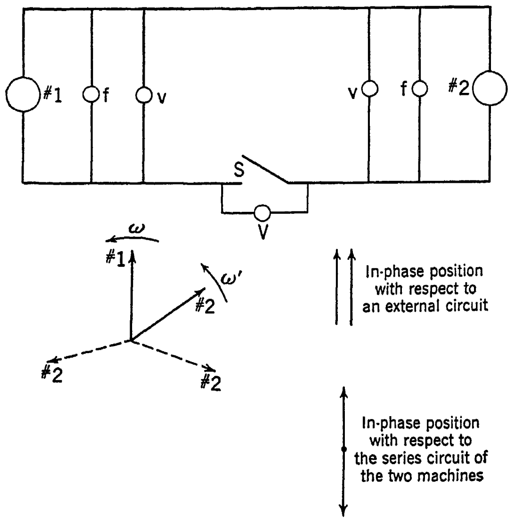


FIG. 290.

the two machines building up the same terminal voltage at exactly the same frequency, the phase relationships may still be incorrect. Refer to Fig. 290, showing two single-phase alternators. With one line closed, connecting the two machines, the voltmeter across the other open switch indicates the difference in phase position between the two voltages. Such a phase may be anywhere from 0° to 180°. Obviously if the difference in potential across the switch is other than zero it cannot safely be closed without a large rush of current between the two machines.

If the speed of the oncoming machine is adjusted slightly, the vector representing its voltage will rotate at a different speed from the other and the vectors will pass each other. In other words, the machines will be coming in and out of phase with a frequency depending upon the difference in frequency of the two machines. By adjusting the speeds, the period of swing, in and out of phase, can be made very slow. This swing will be indicated by the voltmeter across the open switch. At

the instant the voltmeter reads zero, or slightly before, the switch can be closed, as the machines are then in phase. For reasons which we will see later the machines will continue to operate in phase, rotating at identical speeds.

The process of synchronizing then involves the following steps:

(*a*) Obtaining the correct voltage.

(*b*) Obtaining the correct frequency, or one extremely close to the correct value.

(*c*) Using some method of indicating relative phase positions of the machine.

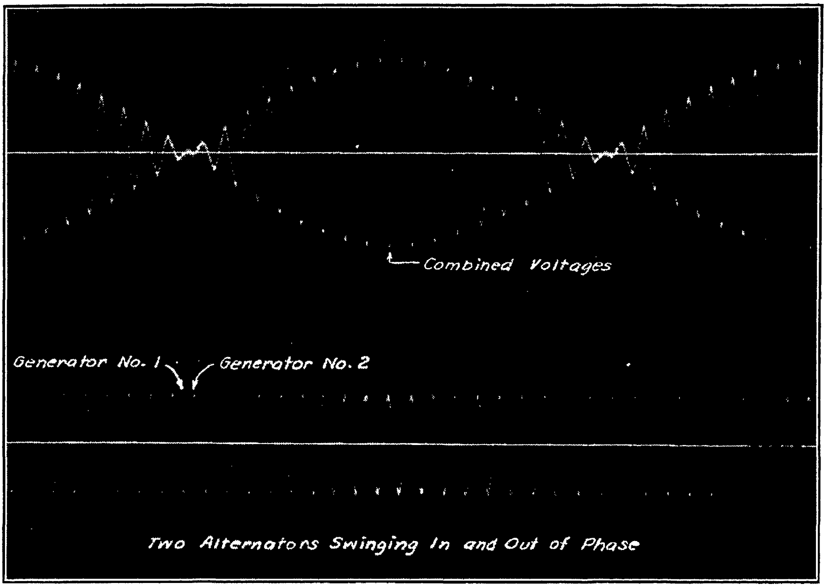


FIG. 291.

375. Methods of Indicating Synchronism. Instead of using a voltmeter to indicate the swing of the alternators in and out of synchronism, lamps can be used, connected across the switches as shown in Fig. 292*a*. In this case the lamps will flicker, being alternately bright and dark. The number of flickers per second is a measure of the difference in speed between the two machines. When the lamps are dark, synchronism is indicated. The voltage on the lamps, when the two machines are out of synchronism, is double that of the voltage of one machine.

If the lamps are connected across the alternator switch as shown in Fig. 292*b*, synchronism is indicated when the lamps are bright. The

reason for this is obvious from the polarity indications on the diagram.

In order to indicate the point of synchronism more accurately than it can be judged on flickering lamps, several types of synchronizing devices known as *synchronoscopes* or *synchroscopes* have been developed. A common form utilizes a split-phase arrangement to give a rotating

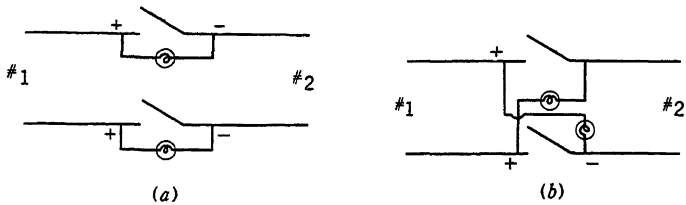


FIG. 292. Connections for indicating synchronism by lamps. The "bright" lamp connection removes the danger of improperly closing the switches if the lamp burns out.

field which is excited from the other generator which is to be synchronized. The reaction of these fields on each other produces rotation if the frequencies of the two machines differ; the pointer connected to the rotor then indicates the relative phase positions of the machines. The direction of rotation indicates whether the oncoming machine is too fast or too slow.

As it is obviously impossible to use lamps or direct-connected synchroscopes on any other except low voltages, small transformers are

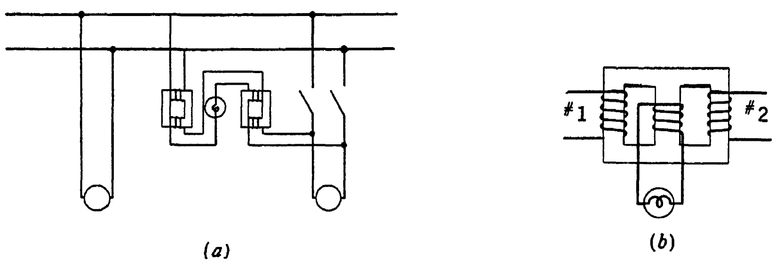


FIG. 293.

necessary for reducing the voltage applied to the synchronism indicator. A typical diagram is shown in Fig. 293*a*.

376. Synchronizing Three-phase Alternators. When synchronizing three-phase alternators of unknown phase rotation, three lamps, connected for either dark or bright synchronizing, can be used not only for determining when the correct phase position is reached, but also to

indicate if the phase rotations of each are identical. Figure 294 reveals that one phase of a three-phase machine may have its correct position as shown by an indicator, but closing the third lead would short-circuit a potential equivalent to the length cc' on the diagram. Under such conditions with the switches a and b closed, the lamp at c would stay bright, indicating the wrong phase sequence on the machines. A reversal of two leads on one of the machines will correct this.

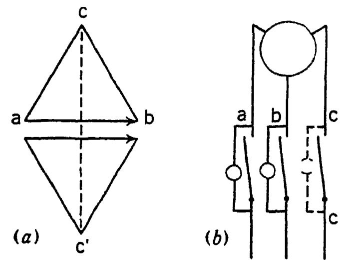


FIG. 294. Connections for "dark" lamps, three-phase. The vector diagram indicates opposite phase sequences.

Figure 294b shows the connection for indicating synchronism when the lamps are dark. A combination of two "crossed" and one "straight" lamp connection can be used, in which case the lamps will glow in rotation, indicating by the direction whether the generator is running too fast or too slow.

377. Circulating Current. When the machine has been synchronized and connected to another alternator, it "floats" idly on the line, provided that the torque supplied to its shaft is just equivalent to the losses. If additional torque is supplied to the oncoming machine after synchronizing, it will step ahead by a slight angle and hence relieve the first alternator of some of its load.

If the alternator "floats" on the line, all the ammeters and wattmeters connected in its leads will read approximately zero if the excitation is 100 per cent. Our initial concern is with making this machine share a part of the electrical load, but we will investigate first the effect of increasing its excitation. To avoid confusion, the voltage considered will be taken at

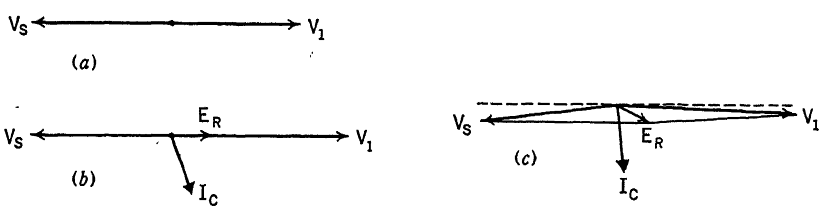


FIG. 295.

the point of parallel connection, say, on the common bus, and any impedance of the leads between machine and bus will be so small as to be neglected, or else will be included with the machine impedances.

The vector diagram representing bus voltage and individual alternator voltage is shown in Fig. 295. These two voltages are in phase

opposition when considered with respect to the series circuits of their armature windings.

Suppose next that we increase the field of the alternator so that E_1 would be greater than the system voltage E_s. The unbalance in voltage which results is shown in Fig. 295*b* as E_R, and it causes a circulating current I_c to flow between the two armatures.

$$I_c = \frac{E_R}{2(r_e + jx_s)} \tag{641}$$

Or, in case of different impedances of the two machines,

$$I_c = \frac{E_R}{(r_e + r'_e) + j(x_s + x'_s)} \tag{642}$$

This current is limited by the impedances of both armatures, and furthermore it lags its productive voltage by an angle, arc tan x_s/r_e. Owing to armature reaction and the impedance drop brought about by the circulating current, the terminal voltages V_s and V_1 are equalized. That is, the current lags the output voltage of machine 1 with the increased excitation, and reduces its voltage. As this same current is leading the other voltage, it helps magnetize the field of that machine and raises the voltages until the two are equalized.

The current I_c is not in time quadrature with either of the voltages, owing to the resistance of the armature circuits. Its position indicates generator action with machine 1 and motor action with the bus supply. The additional torque of this motor action plus the constant torque supply of the prime mover accelerates the generator or generators supplying the bus and causes the phase positions indicated by Fig. 295*c*. Keep in mind that the change in speed which resulted was but for an instant and resulted only in a shift in phase, not an increased number of revolutions per minute.

As a result of the new phase positions, E_R, the resultant of E_s and E_1 will take up the position as shown, with the circulating current again lagging it by the internal impedance angles. In this case, I_c has a projection on both E_s and E_1 such as to represent a condition of system equilibrium. It is this projection of the current on the voltage vectors which represents a slight amount of power; yet this power as I^2R loss is all that can be obtained on the oncoming alternator by adjustment of the field rheostat, and it does not represent useful output. As the circulating current causes copper losses in the armature windings, the input to the generators must be increased. This increase in input represents slightly increased load on the prime movers, and as they have a small

speed regulation, the change in excitation which brought about the circulating current results in an extremely slight reduction in speed.

378. Effect of Varying the Torque of the Prime Mover. If the torque of one of the prime movers is increased (by giving it more steam in the case of steam drive) the alternator connected to it will swing ahead in phase with respect to the other alternator or alternators connected to the common bus. That is, as this alternator is receiving more power than it delivers, it will accelerate. Consider in Fig. 296 that E_1 has been accelerated in this manner. It may be assumed that E_1 was equal to the system voltage E_s before the change in phase was brought about.

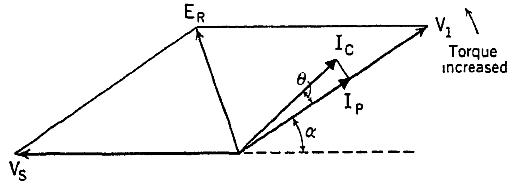


Fig. 296.

Although alternator 1 is accelerated, its voltage will remain constant, for the change in speed was only for the instant necessary to bring E_1 to its new position. The resultant of these two voltages is then E_R, which again causes the lagging current I_c. In this case, however, by reason of the position of I_c, it has a large projection on the voltage E_1. This projection is represented as I_p, and the product of I_p and E represents the generator power developed by this machine. The generator is now loaded, and an equivalent load is removed from the other units with which this machine was paralleled. Considering that the system has not yet come into equilibrium, the initial increased power fed into the prime mover continues to increase the lead of E_1, resulting in increased I_c and increased generator power until the alternator output is equivalent to the increased power of the prime mover. If the generator output exceeds the load being supplied by the system, then the current I_c causes the other machines to be driven as synchronous motors.

CHAPTER XLVI

PARALLEL OPERATION

379. Chapter Outline.

Influence of Speed-governing Mechanism on the Load Division.
Self-synchronizing Action of Alternators in Parallel.
Control of Active and Reactive Power.

380. Influence of Governors on Load Division. In the previous chapter we pointed out that the load is divided between the alternators working in parallel by torque adjustment of the prime movers. In common

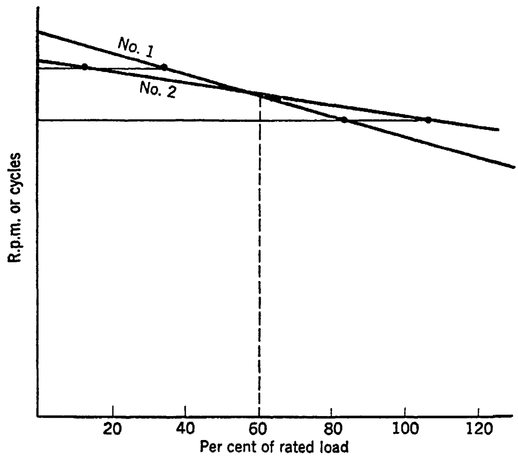


FIG. 297.

practice this adjustment is brought about by the speed-governor mechanism on the engines. It is evident, therefore, that, if the total load is to be shared among a number of units over a whole range of different outputs, it is necessary that the engine governors must function so that the speed regulation is the same for all units. In Fig. 297 are shown the speed curves for two units in which the condition mentioned above is not met.

At a load of 60 per cent the output is properly divided between the two machines. At decreased loads machine 1 carries a greater share of the load; at loads above 60 per cent machine 2 carries an increased share.

This condition can be eliminated only by increasing the speed regulation of machine 2 until the curves are parallel. Then if the governors are adjusted for equal speed at no load the two speed-regulation curves will coincide.

Similar speed characteristics are not so important if the demand on the generators does not vary over a great range. Normally this demand on many generating stations supplying a system "base load" may be constant.[1]

Engine governors may affect the operation of the alternators also by their sensitiveness to load response. If a governor is not properly damped it may alternately tend to speed up and then retard the speed, resulting in an oscillation on the electrical system. Inasmuch as the torque *output* from a reciprocating engine varies with different positions of the crank arm, and the torque *demand* may be constant over the entire revolution, the engine speed varies slightly from instant to instant. Too sensitive a governor would tend to emphasize such cyclic variation and promote hunting in the generators. The speed regulation lines of Fig. 297 should more properly be replaced by speed regulation "bands," for in practice the usual speed-governing device is not so sensitive but that a change in load of 1 per cent or so may result in no actuation of the governing mechanism.

It is obvious from the examination of the speed-regulation curves that for stable operation of alternators in parallel their prime movers must have drooping speed characteristics. Usually a speed regulation of 3 or 4 per cent is suitable to give good load division stability. A regulation of only 1 per cent would signify that an extremely small change in speed, either momentarily or sustained, would result in a great change in load division.

381. Self-synchronizing Action of Two Equal Alternators. As the following analysis will show, two alternators operating in parallel tend to maintain synchronism inherently.

The vector diagram of two equal alternators operating in parallel with the total output divided equally between them is shown in Fig. 298*a*. The equal terminal voltages are shown as V_1 and V_2 for the respective machines, and the total current delivered to the load is I_1 plus I_2 as scalar quantities, $I_1 - I_2$ when referred to V_1 and $I_2 - I_1$ when referred to V_2, as vectors.

The resultant of V_1 and V_2 (or E_1 and E_2) is zero under the condition

[1] For a discussion of the problems introduced by interconnection of large systems with regulated frequency for time-keeping, see H. E. Warren, "Synchronous Electric Time Service," *Trans. A.I.E.E.*, Vol. 51, p. 546, June, 1932. Note also Mr. P. Sporn's discussion following this paper.

shown and no circulating current flows between the two machines. If E_1 differs from E_2 either (*a*) in magnitude or (*b*) in phase position, a circulating current will flow, but the effect of this current will depend upon its cause as described in the previous chapter.

(*a*) If E_1 is greater than E_2, a resultant voltage as shown in Fig. 298*b* causes the circulating current I_c. This current is out of phase with the voltage which causes it, by the internal impedance angle β. Since the

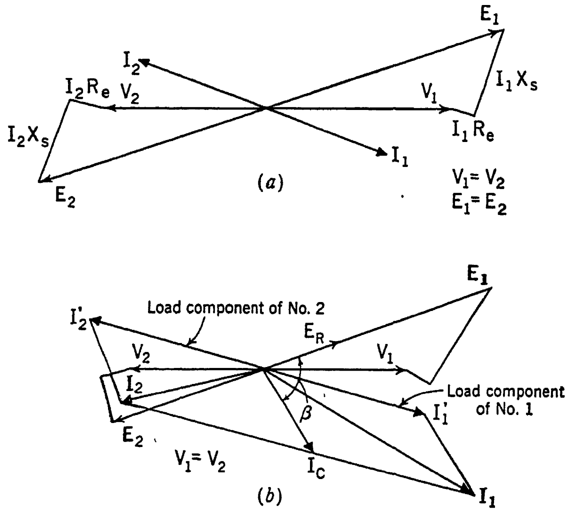


FIG. 298. (*a*) Two equal alternators operating in parallel. (*b*) Two equal alternators operating in parallel with the excitation of No. 1 larger than that of No. 2.

synchronous reactance of the armature is high as compared with its effective resistance, β is usually from 85 to 90°. The current I_c is a lagging component of the total armature current of the first machine and, through its influence on armature reaction, demagnetizes the field and reduces the terminal voltage. At the same time the current I_c is a leading component of the total current of machine 2. In this case the total machine current I_2 leads its voltage, and V_2 differs from E_2 by a relatively small amount. Thus it can be seen that any increase in excitation on machine 1 which causes E_1 to be greater than E_2 starts a series of actions which result in equalization of the terminal voltages. Any such circulating current has no power component with respect to either machine (if the small I^2R loss it causes is neglected) and is not effective in changing the division of load between them.

(*b*) On the other hand, if the two alternators are displaced in phase from exact synchronism, the circulating current which results gives an

apparent exchange of energy tending to restore synchronism. Refer to Fig. 299. It is assumed that generator 1 has "pulled ahead" slightly so that E_1 is no longer exactly opposite from E_2 with respect to their own series circuit. For simplicity, however, it will be assumed that each machine has the same excitation so that E_1 equals E_2. As a result of their difference in phase, $E_1 + E_2 = E_R$. A circulating current, caused by E_R, flows between the two machines. As before, I_c lags the voltage which causes it by the internal impedance angle β.

I_c has a projection upon the voltage E_1, and with respect to that machine it represents generator action. Owing to its relative position

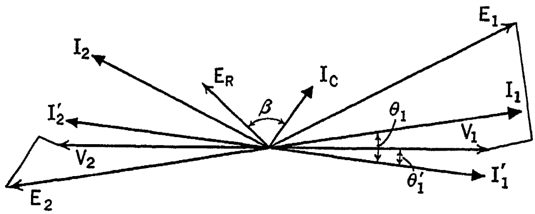


FIG. 299. Vector diagram of two equal alternators. Machine 1 has swung ahead of its true synchronous position so that E_1 is not 180° from E_2.

with E_2 it represents motor action for machine 2 and results in an accelerating torque. As E_1 was leading its true synchronous position, the generator action represented by the circulation current tends to retard it. The instantaneous acceleration of machine 2 and the retardation of 1 result in a final position for E_1 and E_2 in opposition. The readjustment to this position may be accompanied by more or less hunting. The important point to be brought out here is that the operation of two alternators in parallel would be impossible without a circulating current to cause this readjustment by the apparent interchange of energy between them. Of course it is not an independent current but shows up as a component of the total in each machine. Thus, during the time of phase displacement as shown on Fig. 299, when the circulating current is I_c, the total current of machine 1 is I_c plus I'_1 or I_1; that of machine 2 is I'_2 plus I_c or I_2. I'_1 and I'_2 are the respective load component currents of the two machines.

The circulating current I_c is called the synchronizing current because the power component which it represents is approximately equal to the synchronizing power. The product of voltage, synchronizing current, and cosine of the angle between them will be, for machine 1:

$$E_1 I_c \cos (90° - \beta - \tfrac{1}{2}\alpha) = \text{synchronous power supplied by 1} \qquad [643]$$

where β = the internal impedance angle.

α = the angle by which E_1 is shifted from its true synchronous position. (Equivalent to the torque angle in a synchronous motor.)

It must be kept in mind that equation 643 expresses the power due only to the synchronizing current and excludes that due to the load component.

A similar expression for the power of machine 2 due to the synchronizing current is:

$$+E_2I_c \cos (90° + \beta - \tfrac{1}{2}\alpha) \qquad [644]$$

The cosine term will give this expression a minus sign, in line with the convention for motor power. The difference in the two is

$$E_1I_c \cos (90° - \beta - \tfrac{1}{2}\alpha) - [E_2I_c \cos (90° + \beta - \tfrac{1}{2}\alpha)] \qquad [645]$$

Since $E_1 + E_2 = E_R$, this difference is

$$E_RI_c \cos \beta \qquad [646]$$

The circulating current has a projection ($I_c \cos \beta$) on the voltage E_R only through the effect of resistance in the armature circuit. Hence it can be seen that

$$E_RI_c \cos \beta = I_c^2R \quad \text{(in both armatures)}^2 \qquad [647]$$

The conclusion to be reached is that the synchronizing power of a generator equals $EI_c \cos (90° - \beta - \frac{1}{2}\alpha)$ and that this power arising from the circulating current I_c is all effective in bringing a generator back into step except for the small amount dissipated as heat loss in the armature winding.

382. Effect of Reactance. It has been pointed out that, when E_1 is not equal to E_2, a circulating current is caused to flow between the two machines operated in parallel even when both carry equal loads. Examination of Fig. 298*b* shows that the current arising from this cause is largely reactive. Now if the windings and leads of two alternators *having no inductance* were connected in parallel, I_c (Fig. 299) would be in phase with E_R and would have equal positive projections upon both E_1 and E_2. Hence without reactance the machines could not develop

[2] This need not be exactly equal to the copper loss caused by the circulating current under all conditions inasmuch as:

(*a*) Copper loss by load current $I'_1 = (I'_1)^2R$.

(*b*) Copper loss by actual armature current when I_c is present $= I_1^2R$.

The difference between (*a*) and (*b*) is the copper loss caused by I_c and need not be I_c^2R because $I'_1 + I_c$ equals I_1 only by vector addition.

motor and generator power, respectively, to restore synchronism; parallel operation would be impossible.

It can be shown that, when the voltages E_1 and E_2 are in their proper phase relationship of 180°, the maximum synchronizing power will occur on a machine in which $R_e = X_s$ or $\beta = 45°$. As E_1 swings out of phase with E_2 (or α increases) the maximum synchronizing power occurs when X_s is less than R_e. Practically, the maximum value of synchronizing torque is rarely necessary to maintain synchronism between two alternators, and other design factors necessitate values of X_s much larger than R_e. In addition, such a small value of X_s would make the machine very "stiffly coupled," resulting in severe strain when being synchronized, or when self-synchronizing, and would cause excessive values of circulating current.

383. Résumé of Method. The analysis that has been followed herein for parallel operation considered two alternators with the adjustments brought about between them for various conditions of operation. Equalizing action considered the effect of a circulating current between the two machines. This current has no existence, other than as a load component.

A second method of analysis, for instance, explaining the self-synchronizing action, could be built up entirely upon torque-angle considerations. We know that, for a fixed excitation and terminal voltage, no change can be brought about in the power developed by a machine (either in a positive or negative sense) without an appropriate change in torque angle. Hence any change tending to pull one alternator out of step must be followed by a suitable change in torque angle if the new condition is to be maintained. The absence of the additional power represented by such a torque-angle change would only result in a tendency to restore stability, provided the torque disturbance or voltage disturbance is not too great.

Torque-angle analyses are useful in studies of synchronizing torque, and in studies of the stability of individual machines when short-circuits or large load changes occur suddenly on the system.

384. Control of Active and Reactive Power between Generators. When two or more alternators are operating in parallel (with any load distribution), raising or lowering the excitation on one machine will produce a like effect on the common terminal voltage of the whole group. The phenomena are studied most easily by considering only a single pair of equal machines. The action then is much the same as with a synchronous generator and motor. By this means, the pf on a heavily loaded base-load generator may be improved; that is, by the aid of a partially loaded or idling generator or motor. This reduces the field excita-

tion that would otherwise be needed on the base-load generator. In a sense, the lightly loaded machine assists in exciting the heavily loaded one.

The armature copper loss is equal to the sum of active-current loss ($I_p^2R_e$) plus reactive-current loss ($I_r^2R_e$). If, now, the lightly loaded machine is over-excited to the point where it raises the power factor of the base generator to, say, unity, its copper loss will be $I_r^2R_e$ while that of the base generator will be $I_p^2R_e$. Thus the total copper loss will be the same as before, and electrically there is no gain from this procedure.

When it is considered, however, that a second generator may be required to take care of short overloads and peaks, the use of the base-load generator as described, operating at high efficiency and full load, may result in better operating economy than the division of the load between the various units, each operating at reduced load and lower efficiency.

385. Hunting of Alternators. When two alternators are operating in parallel, any instantaneous reduction in the angular velocity of one machine causes a change in load division between them and a circulating current. The circulating current acts as an additional load on one machine and lightens the load on the other. This retards the former and permits the latter to accelerate until the two are once more in the proper relative phase positions where no circulating current flows if the excitations have been equal. The change to correct phase position cannot be accomplished without some "overshooting" on the part of the rotors, accompanied by a retardation, with a repetition of the entire cycle. That is, the alternators hunt, their actions being exactly equivalent to those of synchronous motors under similar conditions. The period of the swing agrees with the natural oscillating period of the rotor as a torsional pendulum.

If the prime mover of one of the alternators is a reciprocating engine, its torque output will pulsate. If this pulsation has a "forced" frequency within 20 per cent of that of the natural oscillating frequency of the alternator rotor, the oscillation following any load change will be cumulative and the machines may be thrown out of synchronism. The whole problem of stability of alternators operating in parallel deals fundamentally with the theory already developed under Synchronous Motors.

The so-called synchronizing power tending to restore the correct phase position is proportional to the displacement. That is,

For a cylindrical rotor (see Article 331):

$$\frac{V_tE_0}{z}\cos\left(\alpha - \tan^{-1}\frac{R_e}{X_s}\right)$$

For a salient-pole rotor (see Article 335):

$$\frac{V_t E_0}{x_d} \cos \alpha + \frac{V_t^2(x_d - x_q)}{x_d x_q} \cos 2\alpha$$

And for small angular variation, in radians:

$$k_2 \alpha_m \quad \text{(see Article 364)}$$

It is customary to specify, for alternators to be operated in parallel, the allowable torque-angle variation. Machines driven by internal-combustion engines must have large flywheels or heavy damping windings to prevent excessive oscillation.[3]

386. Parallel Operation of Generators through Transmission Lines. The impedance of a line between two generators adds directly to the sum of the synchronous impedances of the machines. This increase in impedance acts to reduce the synchronizing current and power, so that they are less strongly held in synchronism. They become more sensitive to torque and voltage disturbances of large value, beyond the ability of the softer coupling that results. Also, the resultant reduced synchronizing power is more likely to lead to disconnections of individual units of apparatus and shutdowns from the operation of automatically reclosing-type circuit breakers.

In addition to the consideration that adding line impedances modifies the self-synchronizing action by a change in the constants of the synchronizing power equation, the line drop reduces the voltage. Since synchronizing power varies directly as V^2, the effect of high impedance lines may result in instability of operation.

[3] For the influence of wave shape, see C. P. Steinmetz, "Parallel Running of Alternators," *Elec. World*, Vol. 23, p. 285, 1894.

E. Rosenberg, "Parallel Operation of Alternators," *J.I.E.E.*, p. 524, 1908–1909.

SYNCHRONOUS CONVERTERS

CHAPTER XLVII

CHARACTERISTICS OF THE SYNCHRONOUS CONVERTER

387. Chapter Outline.

Rectifiers.
  The Synchronous Converter.
    Application.
    Characteristics.

388. Introduction. Methods of Rectifying Alternating Current. It is frequently necessary, for industrial purposes, to convert an alternating current into direct current. A number of different devices are available for such work, the type to use on any application depending upon the amount of power necessary and the voltage at which it is to be used.

Common rectifying devices are [1]

(*a*) Electrolytic rectifiers (including the copper-oxide rectifier).
(*b*) Thermionic rectifier.
(*c*) Mechanical rectifiers.
(*d*) Motor-converter or cascade converter.
(*e*) Mercury-vapor rectifier.
(*f*) Motor-generator set.
(*g*) Synchronous converter.

Only the last three of these rectifiers will be described in the chapters to follow.

389. Applications of Synchronous Converters. The synchronous converter is used frequently in substations as a d-c source for railways and for d-c distribution networks in metropolitan areas. It is also widely used in electrochemical plants and factories in general which require a

[1] Electrolytic rectifiers: *Technological Papers of the Bureau of Standards*, Vol. 18, No. 265, 1924. *Gen. Elec. Rev.*, pp. 35, 101, 192, 248, 1913; p. 429, 1920. *Elec. J.*, p. 257, 1914; p. 226, 1917. *Elec. World*, Vol. 74, p. 937, *A.I.E.E. Trans.*, p. 357, 1927.

Mechanical Rectifiers: *Elec. World*, Vol. 51, p. 352; Vol. 58, p. 638; Vol. 64, p. 856; Vol. 68, pp. 409, 636. *Elec. J.*, Vol. 13, p. 9, 1916. *Gen. Elec. Rev.*, p. 27, 1923,

d-c supply. A few of the advantages of the converter as contrasted with the motor-generator set include reduced floor space and cost, larger capacity for a given bulk, and increased efficiency. The motor-generator set finds its chief advantage in its greater flexibility of voltage range and voltage control and in the fact that it can be made more readily immune to damage from short circuits.

In the past dozen years the steel-tank mercury-vapor rectifier has made large inroads in the railway substation and electochemical fields.

390. The Synchronous Converter. Figure 300 shows the schematic diagram of the armature of a d-c dynamo. The current in any coil or

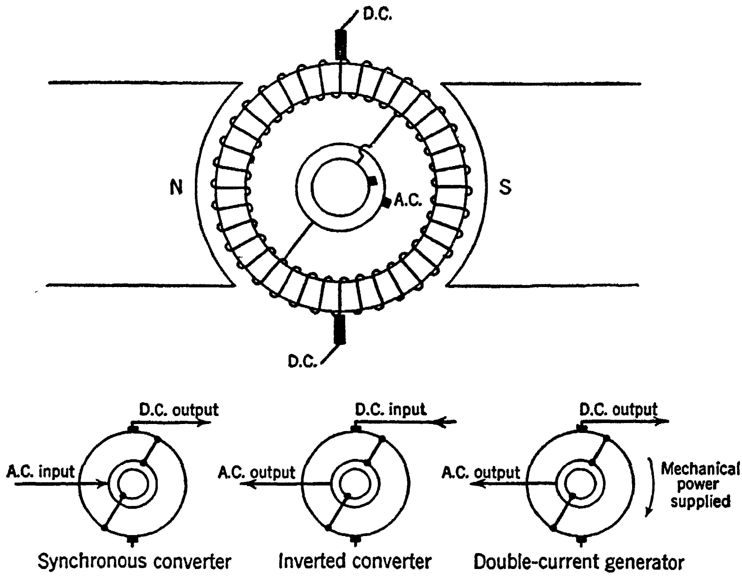


Fig. 300.

conductor on this armature is not an alternating one in the usual sense, but it does reverse in direction each time the coil ends pass under a brush. The commutator of such an armature acts as a rectifying device so that the current or voltage in the external circuit is unidirectional. If, in addition, slip rings and brushes are provided for making connection between certain terminals of the moving coils and the external circuit, an alternating emf can be obtained as the armature rotates in its magnetic field.

If power is supplied to the d-c side of such a machine, it runs as a d-c motor in the usual manner. If alternating current is taken from the slip rings, the machine is then an *inverted converter,* transforming

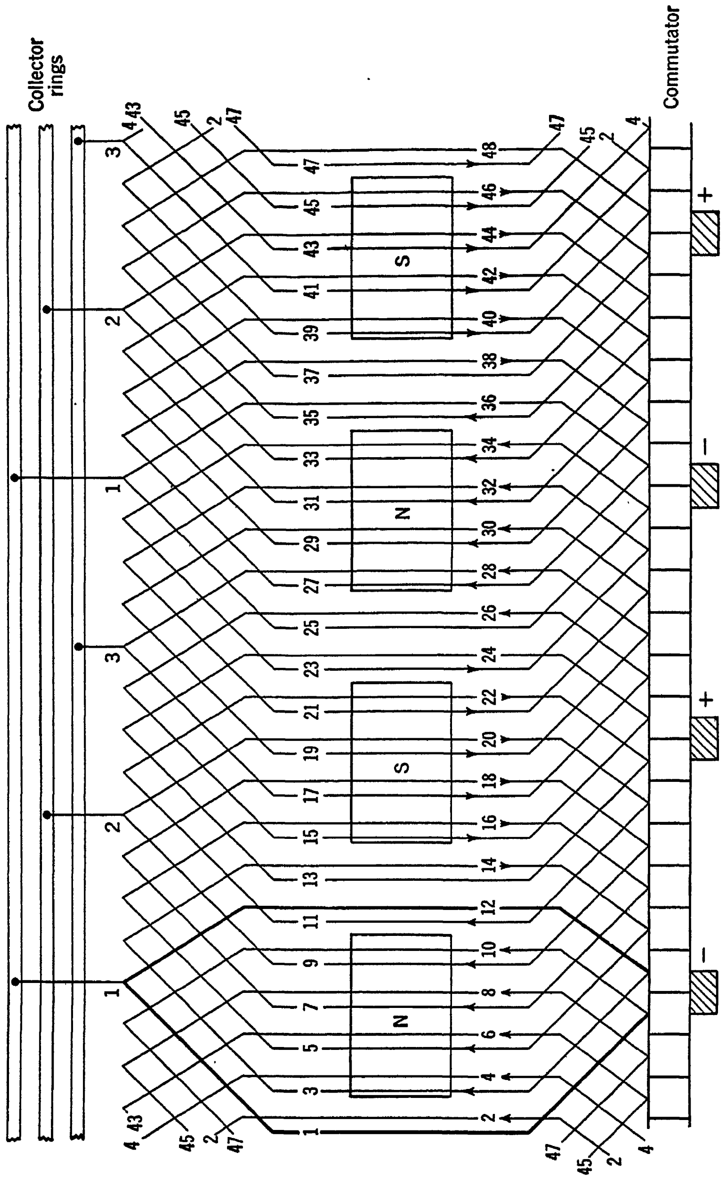


FIG. 301. Three-phase converter winding.

direct current to alternating. On the other hand, if alternating current is supplied to the slip rings and the d-c field is excited, the armature rotates as a synchronous motor and the brushes on the commutator are then the terminals of a d-c supply. Such a machine is then called a synchronous converter. The same armature winding is used for both the alternating and direct currents, and in both the cases pointed out above, the coils will carry the resultant of these two currents. Now, since the alternating current furnishes motor action, and the direct current generator action (or vice versa), the currents flow in prevailingly opposite directions, and so the instantaneous values of currents in the coils are the differences of the direct and alternating components. This results in peculiar wave shapes of coil current and unusual heating effects. They are different for different coils and for different numbers of phases, and, of course, for various loads and pf's.

The single-phase converter is not often used inasmuch as it is larger, heavier, less efficient, and shows an abnormal tendency to hunt. Instead, an increase in the number of phases greatly decreases the heating effect and increases the relative ratings of converters. Most larger converters are designed for six-phase operation. A few very large machines have been built to operate twelve-phase. A connection diagram showing the taps of a three-phase converter is given in Fig. 301.

391. Voltage Ratios. For convenience in indicating voltage relationships a ring-wound armature will be considered as shown in Fig. 302.

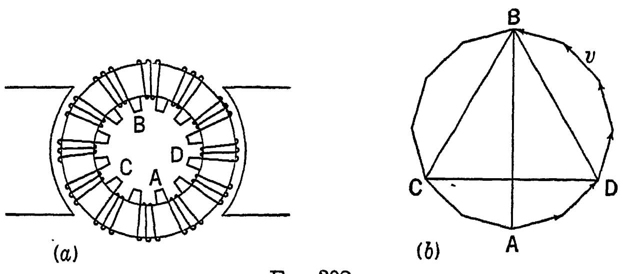


FIG. 302.

This winding has twelve coils. Assume that the flux distribution in the air gap is sinusoidal, determined by the pole-shoe shape and the pole pitch. Then the voltage built up in each coil as the armature rotates will be a sine wave. Since each coil is displaced 30° from its neighbor, the voltage built up in each will reach its maximum 30° in time from the maximum of the adjacent coil. Assume that the flux density and speed are such that each coil generates v volts, crest value. Then, since the voltage of the entire winding can be represented as the closed polygon of

Fig. 302*b*, the voltage between points diametrically opposite will be proportional to the length $\overline{AB}$.

Next assume that this winding is tapped with commutator leads in the usual manner, and the armature is rotated by an outside source. Under this condition, the voltage read across the d-c brushes will also be $\overline{AB}$. This was the crest value of the a-c voltage. When the armature has rotated through 90°, the points A and B will each lie under the center of the respective poles, and the difference in potential across them will be zero. That is, the alternating voltage has gone to zero on its cycle, but the voltage between the d-c brushes has remained constant, proportional to $\overline{AB}$, owing to the symmetry of the winding with respect to the brushes. Hence we can see that if the d-c voltage is of the same magnitude as the a-c peak voltage, the effective ratio will be

$$\frac{AB}{\sqrt{2}} = V_{ac}$$

or

$$V_{dc} = \sqrt{2}V_{ac}$$

This holds only for voltage taps which are diametrically opposite and depends, of course, on the ratio of effective to peak value. A non-sine wave would result in a different ratio.

Consider next that the winding is tapped at the points C and D. Under this condition the voltage read across points B and C will be equal in magnitude and have a phase position indicated by the chord $\overline{BC}$. So also with the voltages $\overline{CD}$ and $\overline{DB}$.

Brushes connected to the slip rings corresponding to taps B, C, and D become terminals for a three-phase a-c supply. The voltage obtained has a peak value corresponding to the length of chord $\overline{BC}$ when compared to the single-phase voltage which could be obtained from $\overline{AB}$. It happens that $\overline{BC}$ is 0.866 times $\overline{AB}$ and therefore:

$$V_{ac} = \frac{V_{dc}}{\sqrt{2}} \times 0.866$$

$$V_{ac} = 0.612V_{dc}$$

This is a fixed ratio. Any change in flux or speed will change the voltage built up in each coil or across the brushes, but a corresponding change is brought about on both a-c and d-c sides.

The assumption has been made in the above analysis that both direct and alternating current were to be taken from the machine and its motive power came from a separate source. Such machines are sometimes used, and are then called *double-current generators*. In the case of the

synchronous converter to be studied in more detail in the pages which follow, the machine is assumed to run as a synchronous motor, operating at the same time as a d-c generator. The action is practically the same; the analysis of the voltage ratios still holds. In the converter, however, the a-c voltage built up by generator action is opposite in direction to that applied, and acts as a counter emf. These two are practically equal and opposite, although the slight difference upsets the theoretical voltage ratios by a few per cent.

392. Operating Characteristics. The first point to be noted in regard to synchronous-converter operation is the constant voltage ratio between the a-c input and the d-c output as mentioned above. This shows that d-c voltage control on the converter must come through an adjustment of the supply potential. The various methods will be discussed later.

Because the armature current of a loaded converter is the resultant of a direct and an alternating current, its effective value may be comparatively small. This implies small heat losses or increased capacity. It also results in comparatively small armature reaction and hence good voltage regulation, and good commutation under normal conditions. Because the converter is equivalent to a synchronous motor on the a-c supply, field adjustment can be used for pf control. A reduced excitation gives a lagging component of current and lower pf. Such a lagging current tends to magnetize the field (in motor action) and brings the resultant air-gap flux about up to its normal value, thus resulting in practically the same counter emf and d-c voltage as was generated at any other pf. More important, however, the lagging component of current may be so displaced as to give a new resultant armature current which produces an excessive heating effect. This limits the range of pf at which the converter is operated, usually to within 0.95 lag or lead.[2] Still, converters are useful for pf correction, chiefly because any load of high pf produces beneficial results on a system otherwise operating at low pf.

In the pages which follow, synchronous-converter characteristics will be discussed in more detail. As could be imagined from the knowledge of synchronous motors, important characteristics and items of operation will include: starting methods, synchronizing, and hunting. Then, in addition, certain points are peculiar to the converter alone, such as unusual heating effects, the influence of pf on heating, the effect of the number of phases, voltage regulation, and efficiency.

[2] Because of the desirability of operating large converters at pf's close to unity, the use of pf indicators has been largely superseded by reactive-factor indicators. The latter instrument is much more sensitive to changes in power factor in the neighborhood of unity.

CHAPTER XLVIII

VOLTAGE AND CURRENT RATIOS

393. Chapter Outline.[1]

Voltage Ratios in the Synchronous Converter.
  Effect of Wave Shape.
Current Ratios.
Armature Current.
  Wave Shape.
  Heating Effect.

394. Voltage Ratios. The following paragraphs will present in a more comprehensive manner an analysis of the voltage built up in the individual coils of a converter armature and the ratios of a-c to d-c voltages for different numbers of phases.

Figure 303*a* represents a 2-pole converter in which the flux is assumed to be sinusoidally distributed around the air gap. The winding is uniformly distributed.

Consider the conductor c which is displaced from the center of the pole or the center of the flux wave by an angle α. Suppose that, when passing across the pole axis, any conductor cuts flux at a rate such as to produce E_m volts. Then each conductor as it comes to the position of c will have generated in it a voltage

$$e = E_m \cos \alpha \qquad [648]$$

If there are z conductors in series between the brushes, the number of conductors per radian will be z/π. The voltage built up in these conduc-

[1] For the derivation of many formulas pertaining to converters, and discussions of characteristics, see Dr. A. S. McAllister, "Alternating Current Motors," McGraw-Hill Book Co.

R. R. Lawrence, "Principles of Alternating Current Machines," McGraw-Hill Book Co.

V. Karapetoff, "Experimental Electrical Engineering," Vol. II, John Wiley & Sons, Inc.

D. C. and J. P. Jackson, "Alternating Currents and Alternating Current Machinery," The Macmillan Co., 1928.

C. P. Steinmetz, "Electrical Engineering," McGraw-Hill Book Co.

tors, i.e., the voltage between brushes, will then be the sum of the instantaneous values of all the conductor voltages or

$$E_{dc} = \frac{z}{\pi} \int_{-\pi/2}^{\pi/2} E_m \cos \alpha d\alpha$$

$$= \frac{2zE_m}{\pi} \qquad [649]$$

This will be the steady value of d-c potential across the brushes. Suppose next that the armature winding is tapped at two places, diametrically opposite, and leads are brought out from the taps, through slip rings to the external circuit. When these taps are under the brushes, the instantaneous voltage will be exactly the same as that read across the brushes. When the armature has revolved so that the taps are in the position shown in Fig. 303*b* the difference in potential between them is zero. Hence the slip-ring voltage has gone from a maximum to zero in one-fourth revolution. The flux distribution has been assumed to be sinusoidal, so also will the a-c voltage be sinusoidal. Since the maximum value of the a-c voltage is the same as the d-c value, then the ratio of effective a-c to d-c voltages must be 0.707.

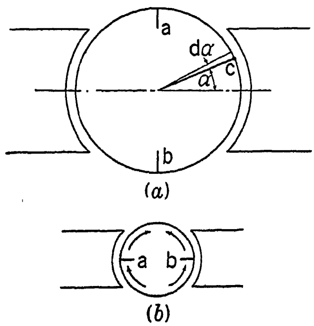


FIG. 303.

This analysis can be generalized for any number of taps on the winding. Let there be n taps or n slip rings. These are the total taps for a 2-pole machine, or for more poles n becomes the number of taps per pole pair. The angle between two taps will be $2\pi/n$ electrical radians. The maximum voltage between will be found by integration as before:

$$E_{m_{ac}} = \frac{z}{\pi} \int_{-\frac{1}{2}2\pi/n}^{+\frac{1}{2}2\pi/n} E_m \cos \alpha d\alpha$$

$$= \frac{2zE_m}{\pi} \sin \frac{\pi}{n}, \qquad [650]$$

The effective value between taps will be

$$E_{ac} = \frac{2zE_m}{\sqrt{2}\pi} \sin \frac{\pi}{n} \qquad [651]$$

The ratio of voltages of an n tapped converter will then be

$$\frac{E_{ac}}{E_{dc}} = \frac{\dfrac{2zE_m}{\sqrt{2}\pi}\sin\dfrac{\pi}{n}}{2zE_m} \qquad [652]$$

Then for a three-phase converter with three taps per pole pair the ratio becomes

$$\frac{E_{ac}}{E_{dc}} = \frac{\sin\dfrac{\pi}{3}}{\sqrt{2}}$$
$$= 0.612$$

Other values are shown in Table XXI.

TABLE XXI

RATIOS OF E_{ac} TO E_{dc}

$\dfrac{\text{Pole arc}}{\text{Pole pitch}} =$	0.80	0.75	0.70	Sine	0.65	0.60	0.55
1 phase	0.67	0.69	0.71	0.707	0.73	0.75	0.77
3 phase	0.59	0.60	0.62	0.612	0.64	0.66	0.675
4 phase	0.48	0.49	0.50	0.500	0.52	0.53	0.55
6 phase	0.34	0.347	0.354	0.354	0.367	0.377	0.387
12 phase	0.177	0.182	0.185	0.185	0.192	0.197	0.204

395. Effect of Wave Shape. If the air-gap flux is distributed other than sinusoidally in space, the ratio of voltages previously given will be different. The d-c voltage will not change so long as the total flux remains the same. The problem of determining ratios, such as those shown in Table XXI, can be solved by laying out a flux-distribution plot considering the pole arc and shape of the pole shoe, finally yielding a wave shape for E_{ac}. The effective value follows either (*a*) through point-by-point graphical construction, or (*b*) through the use of calculus. The waves for E_{ac} may be resolved into their harmonic components by Fourier's analysis.[2] If a sufficient number of components are taken the results will be very close to those of (*a*) or (*b*).

[2] T. T. Hambleton and L. V. Bewley, "The Synchronous Converter. Theory and Calculations," *A.I.E.E. Trans.*, Vol. 46, p. 60, 1927.

A list of factors affecting the voltage ratio at no-load include:

Flux distribution.
Flux shift relative to brush position, or vice versa.
Flux pulsation due to variation in reluctance from slot position.
Brush width.
Winding pitch and distribution.

Under load the additional items upsetting the theoretical ratio are:

Flux pulsation caused by armature reaction pulsating mmf.
Brush contact resistance.
Armature impedance.

For commutating-pole converters, Hambleton and Bewley state that the majority of machines show a no-load diametral ratio of 0.725, and from resistance losses, full-load ratios of 0.74 to 0.75.

396. Current Ratios. External circuit. In a synchronous converter of unity pf and no losses, the average a-c input equals the average d-c output. Then for single-phase, since

$$V_{ac} = 0.707V_{dc} \qquad [653]$$

and

$$V_{ac}I_{ac} = V_{dc}I_{dc} \qquad [654]$$

$$I_{ac} = \frac{V_{dc}}{0.707V_{dc}} I_{dc} \qquad [655]$$

or

$$I_{ac} = 1.414I_{dc} \qquad [656]$$

These are effective values.

For three-phase, an ideal converter shows the following ratio:

$$\sqrt{3}V_{ac}I_{ac} = V_{dc}I_{dc}$$

But

$$V_{ac} = 0.612V_{dc}$$

Therefore

$$I_{ac} = \frac{V_{dc}}{\sqrt{3} \times 0.612V_{dc}} I_{dc}$$

or

$$I_{ac} = 0.943I_{dc}$$

These relationships can be generalized by the following equation which also considers the efficiency and the pf. These are line currents, not armature conductor currents.

$$\frac{I_{ac}}{I_{dc}} = 2 \sin\frac{\pi}{n} \cdot \frac{V_{dc}}{V_{ac}} \frac{1}{n \times \text{efficiency} \times \text{power factor}} \qquad [657]$$

Or approximately, by using the ratio of voltages,

$$\frac{I_{ac}}{I_{dc}} = \frac{2\sqrt{2}}{n \times \text{efficiency} \times \text{power factor}} \qquad [658]$$

where n equals the number of slip rings.

To determine the coil current, or the current flowing through the armature windings, requires a consideration of the winding type and the number of parallel circuits. In general, all d-c windings are of the closed type. Hence it would be impossible to have a Y connection of windings between the slip rings of a three-phase converter since the ends of such a winding are only closed through the load circuit. A combination of Y and delta is possible, however.

Consider an armature winding for a 2-pole converter with three slip rings. There will be two parallel paths for the direct current. For the alternating current the line value will be the product of the mesh amperes and the square root of three.

Example. A three-phase, 2-pole converter has a d-c output of 100 amperes. Neglecting losses, the line current on the a-c side will be

$$\begin{aligned} I_{ac} &= 0.943 \times I_{dc} \\ &= 94.3 \text{ amperes} \end{aligned}$$

The alternating current in the windings will then be

$$\frac{94.3}{\sqrt{3}} = 54.6 \text{ amperes}$$

This example considered a 2-pole converter. In general there will usually be as many parallel paths per phase as there are pole pairs, and the following relationship can be shown to exist:

$$I_{\text{a-c coil}} = \frac{I_{\text{a-c line}}}{\left(2 \sin \frac{\pi}{n}\right) \times \frac{P}{2}} \qquad [659]$$

Example. Assume that the converter of the previous example had four poles instead of two. What would be the alternating current in the coils on the above assumptions of winding type?

Since there are two pole pairs, two parallel paths will result for each phase. Hence the coil or conductor current will be 54.6 divided by 2, or 27.3 amperes. Or, by applying equation 659,

$$\begin{aligned} I_{\text{a-c coil}} &= \frac{94.3}{\left(2 \sin \frac{\pi}{3}\right) \times 2} \\ &= 27.3 \text{ amperes} \end{aligned}$$

These values are tabulated in Table XXII.

Since a converter is operating simultaneously as a motor and a generator, the current in the individual coils will be the resultant of a generator current and an alternating current required for "motoring." Generator action requires that the current and the induced voltage flow through the armature in the same direction; motor action is accompanied by a flow of current in the opposite direction to the induced emf. Hence, the actual current in the coils of a converter, being the resultant of the two, will in most instances be the difference of their instantaneous values because of the relative directions. This implies that the copper loss of a machine operated as a converter will be much less than that of the same machine operated as a d-c generator. We will find that this is especially true for an increased number of phases, but that the inverse is true for a single-phase converter. Since the limit of output on machines is heating and commutation, the efficiency, due to decreased heating, or the output for the same heating, is increased in polyphase converters over the usual values of d-c machines.

TABLE XXII

CURRENT RATIOS. $\frac{I_{ac}}{I_{dc}}$ LINE TO LINE.* TWO POLES

$\frac{\text{Pole arc}}{\text{Pole pitch}} =$	0.80	0.75	0.70	Sine	0.65	0.60	0.55
1 phase	1.50	1.45	1.41	1.41	1.37	1.33	1.30
3 phase	1.00	0.97	0.94	0.94	0.915	0.89	0.87
4 phase	0.75	0.73	0.71	0.71	0.69	0.67	0.65
6 phase	0.50	0.48	0.47	0.47	0.46	0.44	0.43
12 phase	0.25	0.24	0.24	0.24	0.23	0.22	0.22

$\frac{I_{ac}\text{ (effective) in winding *}}{I_{dc}\text{ in line}}$ Two Poles

$\frac{\text{Pole arc}}{\text{Pole pitch}} =$	0.80	0.75	0.70	Sine	0.65	0.60	0.55
1 phase	0.75	0.725	0.707	0.707	0.685	0.665	0.650
3 phase	0.565	0.550	0.545	0.545	0.520	0.570	0.495
4 phase	0.525	0.575	0.500	0.500	0.485	0.470	0.460
6 phase	0.490	0.480	0.470	0.470	0.455	0.445	0.430
12 phase	0.480	0.460	0.455	0.455	0.435	0.425	0.410

* For unity pf. If pf is not unity, then I_{ac} represents the power component.

397. Wave Shapes of Armature Current. In order to determine quantitatively the effect of the number of phases upon the heating, it is necessary that either graphical or mathematical expressions be found for the instantaneous coil current. Consider the single-phase converter of Fig. 304 in which the flux is assumed to be sinusoidal and to line up with the pole centers. The brushes are on the geometric neutral axis, and the tap coils are *a* and *b*. The voltage across the taps *a* and *b* is a maximum at the instant shown, and if the pf is unity, the alternating current is also a maximum. The conductor or coil *c* is midway between the taps *a* and *b*, and since the voltage of *c* is of the same phase position as the total voltage across *ab*, both its current and voltage are in the opposite direction to the direct current of that conductor. The resultant current in coil *c* is shown at point 1 in Fig. 305. The alternating current is the dotted line, the direct current is the dot-dash line, and the solid line is the resultant. Note that the coil *c* will not commutate until it has rotated through 90° from the position shown. When it does, both the alternating and direct currents will be zero. If these currents are combined point by point around the air gap with rotation of the armature, a resultant coil current curve is obtained as shown by the solid line of Fig. 305*a*. Obviously this current will have a smaller rms value than either the alternating or the direct current if either were acting alone.[3] The assumption is made that the current reverses instantaneously during commutation.

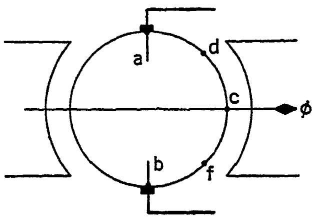


FIG. 304.

The resultant current in each of the conductors does not follow the same locus. The wave shape depends upon the position of the individual conductors with reference to the a-c taps and also upon the pf. Consider the conductor *d* located 45 electrical degrees behind *c*. Since this conductor is in series with *c* it will carry the same alternating current, but because of its position it will not commutate until the armature has rotated through 45 electrical degrees after *c* has commutated. The required position of this d-c wave is shown in Fig. 305*b*. The alternating current is in the same position as before, and the resultant current in conductor *d* follows the solid line. This current wave will have a different rms value from that of conductor *c*. Consideration of the instantaneous values of the current in conductor *f*, 45° ahead of *c*, shows the wave of Fig. 305*c*. This illustrates the statement previously made, that

[3] Nicholas Stahl, "The Heating of Synchronous Converters," *Elec. World*, Vol. 58, p. 1060.

the heating in the individual conductors will differ, the hotter ones being those nearest the taps and the cooler ones in the center. The heat

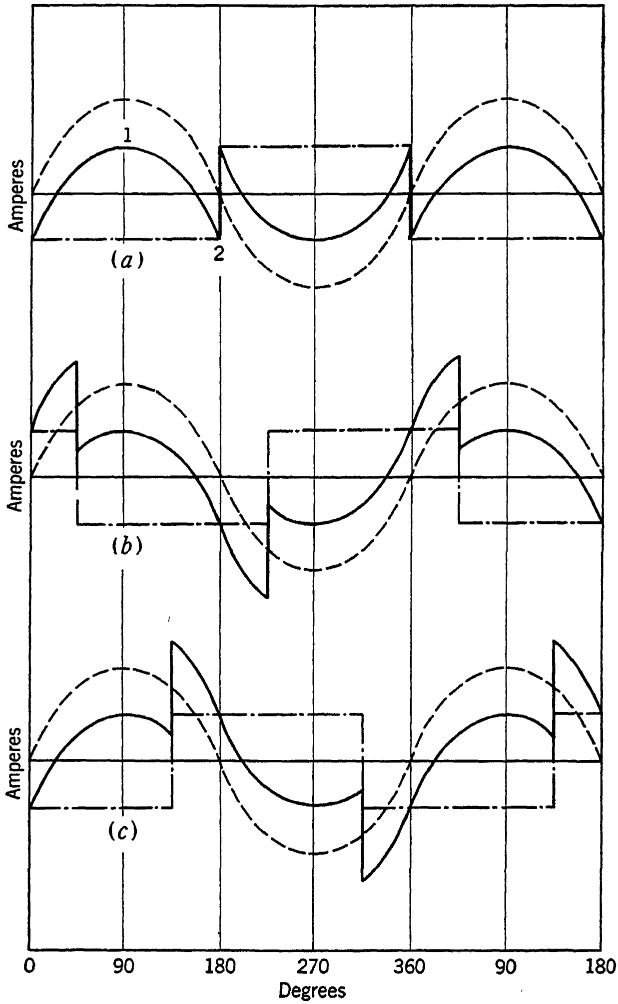


FIG. 305. Current wave shapes for various converter coils.

generated in coils, symmetrically located with respect to the taps, are equal at unity pf.

The above analysis applies to operation at unity pf. Suppose that the current is lagging the voltage by 45°, owing to decreased excitation. Note conductor c on Fig. 304. The voltage in this conductor is a maxi-

mum at the instant shown, but the alternating current will not be a maximum until the armature has rotated through 45°. But the direct current commutates at the same position in any coil, at any pf. Hence the a-c and d-c curves of conductor c will be displaced from each other by 45°. Now these waves are so displaced in coil d and give the resultant current of Fig. 305b. Hence, at a pf of 70.7 per cent, lagging, the current locus in c is the same as the current locus of a conductor 45 electrical degrees behind it. We can generalize by saying that, with lagging current, of pf angle θ, the current flowing in any conductor is the same as that flowing in a conductor at unity pf, θ degrees ahead. And also, with leading currents of pf angle θ, the current in any conductor is of the same shape as that flowing in a conductor at unity power factor, θ degrees behind. This of course is based on the assumption that the maximum value of the alternating current has been kept constant regardless of the pf.

We will find that the total heating will be increased greatly by operation at any power factor of less than unity, increased out of all proportion to that caused simply by the additional current required to give the same power component with lagging or leading currents. Because single-phase converters are rarely if ever used except in very small sizes, this analysis will be continued on a six-phase converter.

398. Heating Effects of Armature Current. Six-phase Converter. Figure 306 shows the armature connections for a six-phase converter.

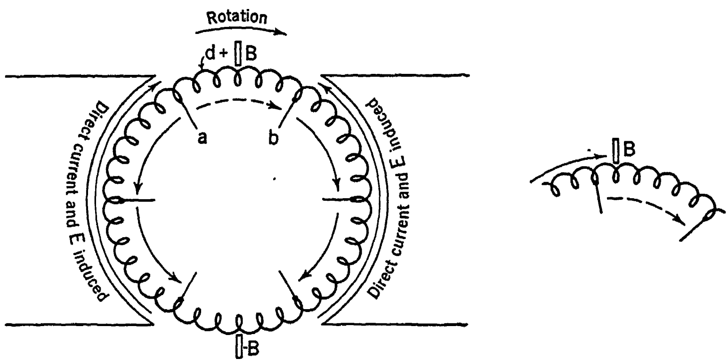


FIG. 306.

The armature is shown ring wound for convenience in following the analysis. The pole flux is assumed to be sinusoidally distributed around the air gap. The brushes BB are on the geometric neutral. The slip rings are not shown, but they are connected to the armature winding at the positions indicated by the taps. The directions of the direct currents

and the induced voltages are indicated by the arrows. Assume that zero time is indicated when the phase between taps a and b is centered under the positive brush. At this instant the voltage in this phase is zero and

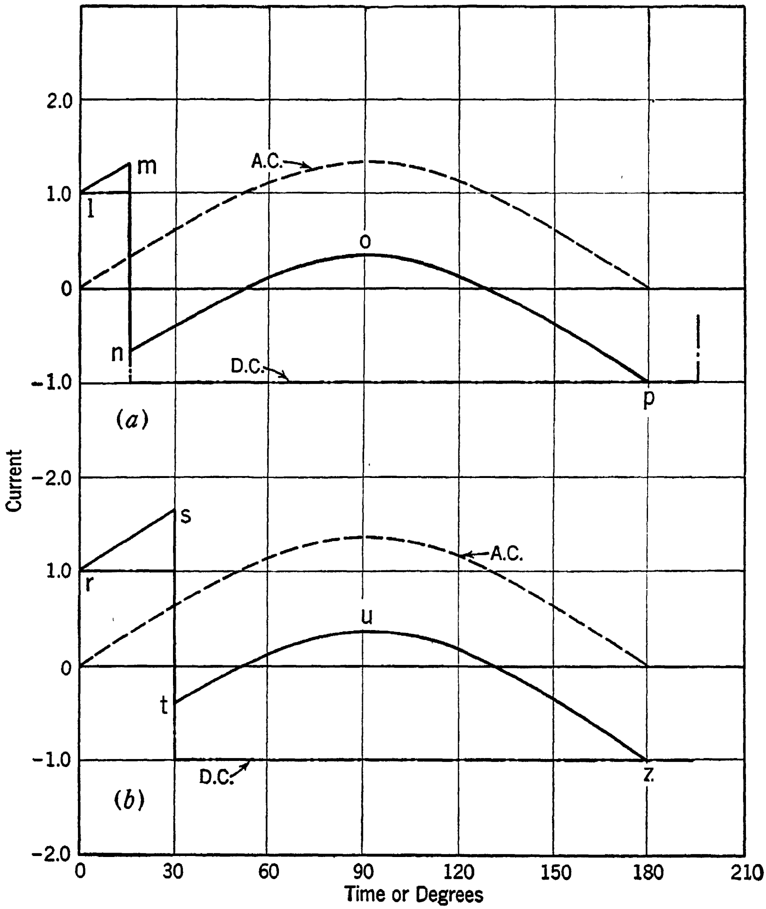


Fig. 307.

for unity pf so is the alternating current zero. Next consider that the armature is rotated in the direction indicated. The input, alternating current, must flow in the opposite direction to the induced emf as the input represents motor action. At the same time, the direct current is flowing from a to the positive brush, and so the coils between a and the positive brush will carry a direct current and an alternating current which is just starting to flow in the same direction.

The conductor connected to the tap *a* will commutate 30° in time from the position shown in Fig. 306. The alternating current in this phase is zero at the instant shown. Hence the blocked representation of the direct current on Fig. 307*b* and the dotted line of alternating current can be located with reference to each other. The resultant current shape for rotation through 180° is shown as curve *rstuz*. This is for the tap conductor *a*.

Next consider a conductor *d*, 15° from *a*. This conductor will commutate 15° in time after the position shown. The alternating current in

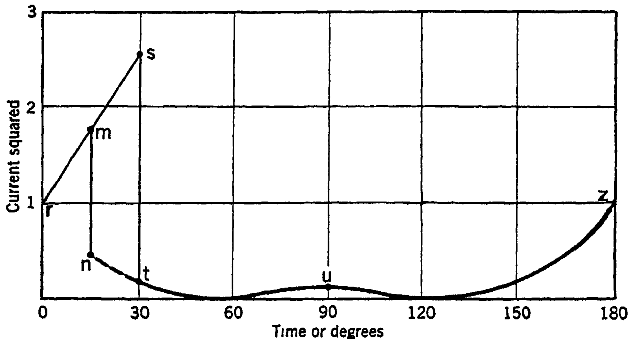


FIG. 308.

this conductor (as well as in the tap conductor *a*) is zero at the initial instant. The resultant of the direct current and the alternating current is indicated in Fig. 307*a*. It follows *lm* (commutates) *nop* and is similar, of course, through the next half of the revolution.

The heat in any conductor is a function of the current squared. Therefore, if the current curve is squared, point by point, the heat generated in the conductor can be indicated from time to time in its revolution. Figure 308 shows such curves of current squared. For tap conductor *a*, the heat generated through one-half revolution is given by curve *rstuz*. For conductor *d* the heat generated instantaneously is given by curve *rmntuz*. The area under these curves represents the total heat generated during one-half cycle. Hence it can be seen that tap conductor *a* is hotter than conductor *d* by the area *mstn*.

399. Mathematical Expression for Heating. The graphical method pointed out above for determining the heating effects in the various coils can be generalized mathematically by the expression derived below. Assume:

One phase of a converter has a spread of ϕ degrees.

The center coil or conductor is located $\phi/2$ degrees from each tap coil.

Let C = any coil, α degrees from the center coil, and β degrees from the brush axis, midway between poles. Both are measured in the direction of rotation.

θ = the angle between generated voltage and alternating current in the center coil

i_{ac} = the alternating current in the coil or conductor

i_{dc} = the direct current in the coil or conductor

$2i_{dc}$ = the current per brush

The current in the conductor c is then

$$\sqrt{2}i_{ac} \sin (\beta - \alpha - \theta) - i_{dc} \tag{660}$$

The heating is proportional to the rms value, which for this current is

$$I^2_{\text{cond}} = \frac{1}{\pi}\int_0^\pi [\sqrt{2}i_{ac} \sin (\beta - \alpha - \theta) - i_{dc}]^2 d\beta \tag{661}$$

The ratio of line currents can be used to give the following ratio for coil currents (equation 658):

$$i_{ac} = i_{dc} \frac{2\sqrt{2}}{n \times (\text{efficiency}) \times (\text{pf}) \sin \dfrac{\pi}{n}}$$

where n equals the number of taps, or the number per pair of poles.

By substituting in equation 661:

$$I^2_{\text{cond}} = \frac{i^2_{dc}}{\pi}\int_0^\pi \left[\frac{4 \sin (\beta - \alpha - \theta)}{n \times \text{pf} \times \text{efficiency} \sin \dfrac{\pi}{n}} - 1\right]^2 d\beta$$

$$= i^2_{dc}\left[\frac{8}{n^2 \times (\text{pf})^2 \times (\text{efficiency})^2 \sin^2 \dfrac{\pi}{n}} - \frac{16 \cos (\alpha + \theta)}{n \times \text{pf} \times \text{efficiency}\ \pi \sin \dfrac{\pi}{n}} + 1\right] \tag{662}$$

The heat generated in any conductor is proportional to the value of this equation. Note that the only variables are α and θ, which affect the second term; all others are constant, and the heating in individual conductors will vary with cos $(\alpha + \theta)$. When plotted, the equation yields curves similar to those of Fig. 310. This equation is also useful in

determining the ratio of armature copper losses of a converter to those in the same machine when used as a d-c generator. This ratio is

$$R = \frac{n}{2\pi}\int_{-\pi/n}^{\pi/n}\left[\frac{8}{n^2 \times (\text{pf})^2 \times (\text{efficiency})^2 \sin^2 \frac{\pi}{n}} - \frac{16 \cos(\alpha + \theta)}{n \times \text{pf} \times \text{efficiency}\ \pi \sin \frac{\pi}{n}} + 1\right] d\alpha \qquad [663]$$

This integration provides the summation for all conductors between the taps of one phase belt. The entire expression is the ratio of *averages*. Then

$$R = \frac{8}{n^2 \times (\text{pf})^2 \times (\text{efficiency})^2 \sin^2 \frac{\pi}{n}} - \frac{16}{\text{efficiency} \times \pi^2} + 1 \qquad [664]$$

The ratio of comparative outputs depends upon the square root of the copper loss ratio. Hence,

$$\text{Output of } n\text{-phase converter} = \frac{\text{output of d-c generator}}{\sqrt{R}} \qquad [665]$$

400. Effect of Power Factor on Heating. If the pf is unity, the current and voltage in the converter can be represented simply as the two vectors of Fig. 309*a*. The current components in a certain conductor such as *d* in Fig. 306 can be represented as the curve of Fig. 309*b* as previously shown. If the field excitation is reduced so that the current lags by an angle θ from its former position, the new current is OI in Fig. 309*c*. This is the equivalent of adding a reactive current HI to the initial power component, and this current can be added to those already present in Fig. 309*b*. As the reactive current is 90° out of phase with the power component, the wave 1–1′ will be at zero when the other is a maximum. This curve in Fig. 309*d* represents lag. If the field were over-excited instead, the resulting leading current would be 180° from the component shown. This procedure can be followed for any pf. A reactive component of the correct magnitude is added to the current waves and the instantaneous current and heating values can be determined for any position.

The total heat generated, when supplying a given d-c load, will be greatly increased by low pf's. Furthermore, the distribution of heat in the various coils will be changed. This can also be seen from equation 662.

The heat generated in conductor *a* as it goes through one-half revolution has been shown in Fig. 308. The average heat in this conductor can be determined from the area under its curve. Similarly the heat

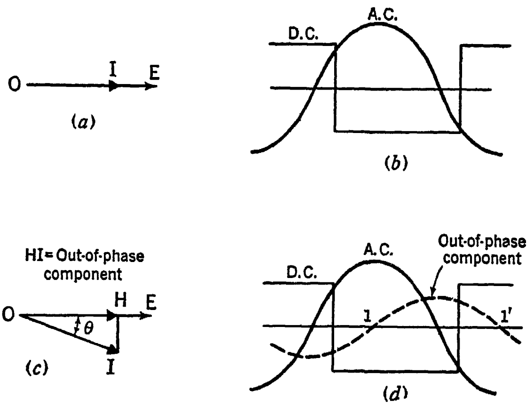


FIG. 309. Effect of operation at other than unity pf.

generated in any other conductor can be so determined. Then the heat generated in each conductor can be plotted against its position relative to the taps. Such a curve for a six-phase machine with one phase belt covering 60 electrical degrees is shown in Fig. 310. Note the influence of operation at other than unity power factor and the resulting shift

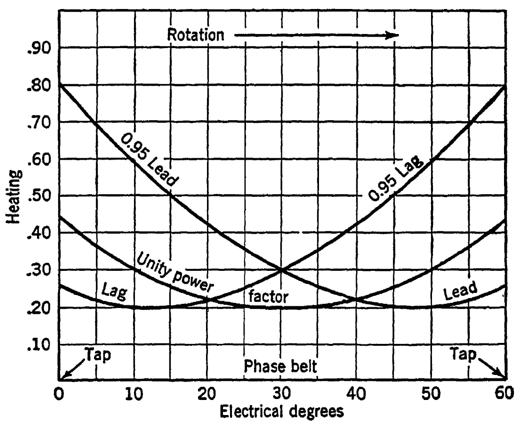


FIG. 310.

in the positions of maximum heating. In any case, the maximum heat is generated in the tap coils. On these curves the heat generated with direct current alone is used as unity.

401. Effect of Changing the Number of Phases. If analyses are carried through for converters with different numbers of taps, a great difference in the heating effects will be indicated. That is, a converter tapped for six-phase operation will display a different heating and heat distribution between coils from one tapped for single- or for three-phase inputs. In Fig. 310 the variation in heat generated from center coils to tap coils for six-phase unity pf operation was shown to involve a change of 20 to 44 per cent of the heat generated if direct current only were used. Or the relative change is

$$\tfrac{44}{20} = 2.2 \text{ times}$$

That is, 2.2 times as much heat is generated in the tap coil as that of the center coil, under conditions of unity pf. Results for other pf's and numbers of phases are shown in Table XXIII.

TABLE XXIII

RATIOS OF MAXIMUM TO MINIMUM LOSSES IN CONDUCTORS

Phases	Unity Power Factor	0.90 Power Factor
1	6.6	7.4
3	5.3	8.1
4	3.6	6.8
6	2.2	4.9
12	1.3	2.8

The actual temperatures do not vary in these proportions because of the increased heat radiation and conduction from the hotter parts of the windings.

TABLE XXIV *

RELATIVE OUTPUTS OF A-C MACHINES COMPARED TO D-C AS UNITY

(Efficiency = 100 per cent)

Phases	Synchronous Converters		Ordinary A-c Generators	
	100% pf	90% pf	100% pf	90% pf
1	0.848	0.73	0.7071	0.64
3	1.338	1.10	0.9186	0.83
4	1.627	1.28	1.000	0.90
6	1.937	1.46	1.0610	0.96
12	2.185	1.58		0.98
Infinity	2.291	1.63	1.1105	1.006

* From McAllister, "Alternating Current Motors," Third Edition, p. 165.

CHAPTER XLIX

VOLTAGE REGULATION AND CONTROL

402. Chapter Outline.

Voltage Regulation.[1]
  Armature Reaction.
Voltage Control.
  Synchronous Boosters.
  Control of Supply Voltage.
  Control of Wave Shape.

403. Armature Reaction. When a synchronous converter is loaded on its d-c end so that an output current flows in the external circuit, an equivalent increase occurs in the a-c input. This current produces local impedance drops in the armature circuit and also an armature reaction. For convenience the armature reaction can be divided into three components.

(*a*) A d-c reaction acting midway between the poles, similar in nature to the cross-field armature reaction in any d-c machine.
(*b*) An a-c reaction midway between the poles which is opposite in direction to that of item *a*.
(*c*) An a-c reaction along the pole axis.

Considering that (*a*) results from generator action and (*b*) from motor action, it can readily be imagined that the resultant of these two would practically cancel each other.

As for the third component: In a synchronous motor, if the excitation is too low to cause the generated voltage to be about equal and opposite to the applied stator voltage, the armature current takes up the position necessary to cause the armature ampere turns to strengthen the field poles. The air-gap flux is then caused by the vector resultant of field and armature ampere turns. Similarly, if the d-c field is over-excited, the leading current, through armature reaction, weakens the air-gap flux until the counter emf and applied voltages are almost equal. This

[1] F. D. Newbury, "Voltage Variation in Rotary Converters," *Elec. J.*, Vol. V, p. 617, 1908.

is also true in the synchronous converter. The armature-reaction component along the pole axis results from too strong or too weak an excitation and yields an approximately constant main flux. This causes the d-c voltage output to remain approximately constant, neglecting the effect of local reactance and local resistance drops in the winding.

Strictly speaking, another component of armature reaction is present as a result of that part of the current necessary to supply the rotational losses. As these losses are practically constant over the whole range of output, and as they are small, the power current needed to supply them remains constant and its effect can be neglected. Such a current must be in phase with the generated voltage, in which case the effect of armature reaction is to strengthen and weaken the pole tips, i.e., it is a transverse reaction.

On account of the smallness of the last component of reaction—the near canceling of the first and second components, and the effect of maintaining constant air-gap flux brought about by the third—armature reaction in synchronous converters is not a serious factor in producing bad voltage regulation. Because of this, the ratio of armature ampere turns to field ampere turns can be greater than would be possible in ordinary d-c generators.

404. Example of Armature-reaction Calculations. The theories underlying the determination of armature-reaction ampere turns have been presented at some length under Alternators. Few added items are necessary to apply the various methods to converters. The *example* given below shows by what amount the cross-effect of the a-c reaction fails to cancel the d-c reaction.

Consider a three-phase, 2-pole converter with a full-pitch armature winding as illustrated schematically in Fig. 311. Each phase belt covers 120°.

Let $I_{dc} = 20$ amperes

$Z_{\text{armature}} = 120$ conductors (only a few are shown).

$P = 2$

Then:

$$i_{dc} = \frac{I_{dc}}{P}$$

$$i_{ac_{\max}} = \frac{4}{n \sin \frac{\pi}{n}} \cdot \frac{I_{dc}}{P} \text{ (for 100 per cent efficiency and pf)}$$

The d-c ampere turns (armature) per pole:

$$\frac{NI}{P} = \frac{Zi_{dc}}{2P} \tag{666}$$

$$= \frac{120 \times 10}{4} \quad \text{or} \quad 300$$

The a-c ampere turns per pole:

The same general methods presented under Alternators can be used here. Only three alternatives will be shown:

1. The ampere turns for each phase are found for the instantaneous currents at some assumed time, and the vector resultant is found. Note the vector diagrams to the right of the armature layout. At the instant shown in (*a*) the current in phase II

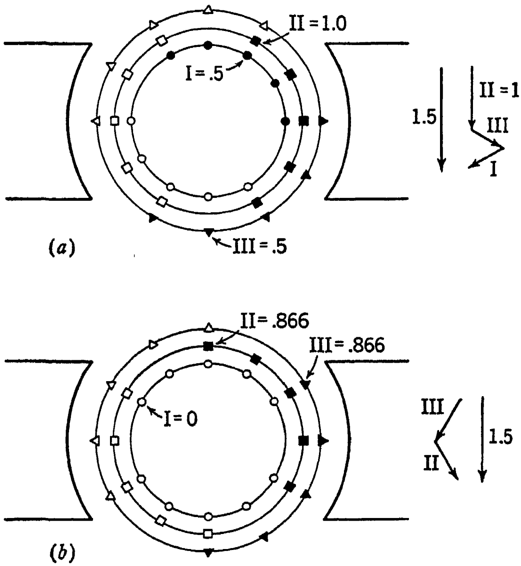


FIG. 311. Schematic representation of a three-phase converter winding, showing directions and values of the armature mmf built up by the alternating current. Two different positions of rotation are shown.

is a maximum; the currents in phases I and III are each half maximum. Considering that the ampere turns they represent are displaced from each other, the vector sum is shown to be

$$\text{Crest } NI = 1.5 \times (\text{crest } NI \text{ for one phase}) \qquad [667]$$

The same result is obtained in (*b*) when the armature is in a different position such that phase I is carrying zero current. For the *example:*

$$i_{ac_{\max}} = \frac{4}{3 \times \sin \frac{\pi}{3}} \times \frac{20}{2} \quad \text{or} \quad 15.4 \text{ amperes}$$

The turns per phase per pole:

$$\frac{\text{Conductors}}{2 \times \text{poles} \times \text{phases}} = \frac{120}{2 \times 2 \times 3} \quad \text{or} \quad 10$$

Then from equation 667:

$$NI_{ac_{\text{per pole}}} = 1.5 \times 15.4 \times 10 \quad \text{or} \quad 231$$

2. The fundamental sine components are used instead of the actual values. In this case, the amplitude of the fundamental component of the crest ampere turns, per turn of full pitch, is

$$\frac{4}{\pi} \times (1 \text{ turn}) \times 15.4 \text{ amperes} = 19.6 \text{ ampere turns}$$

The distribution factor is

$$k_d = \frac{\text{chord of } 120°}{\text{arc } \dfrac{2\pi}{3}} \tag{668}$$

$$= \frac{1.732}{\dfrac{2\pi}{3}} \quad \text{or} \quad 0.826$$

$$\text{Crest ampere turns per pole (a-c)} = 1.5 \times (19.6 \times 10) \times 0.826 \quad \text{or} \quad 243$$

3. "Stepped" curve method. The actual wave shapes of ampere turns are constructed for the two positions of the armature shown in Fig. 311. The average area of the two is found and replaced by a sine wave of equal area. The calculations will not be shown for this method but the results follow:

$$\text{Average } NI, \text{ position } a = 154$$

$$\text{Average } NI, \text{ position } b = 144.5$$

$$\text{Average for both} = 149.25$$

This corresponds to $\pi/2 \times 149.25 = 234$ crest ampere turns per pole, for the sine wave of equal value.

Then by these three methods the ampere turns of (a-c) armature reaction are 231, 243, 234.

Note the representation of armature reaction in Fig. 312.

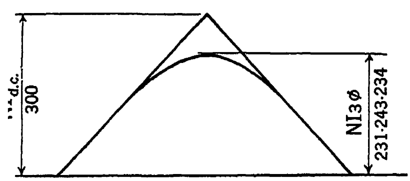


FIG. 312.

The armature reaction effects of the a-c and d-c, listed as components (*a*) and (*b*) of Article 403, fail to cancel by 69, 57, or 66 ampere turns, depending upon which method of calculation is used. With the d-c effect as a basis, this is a difference of 23, 19, or 22 per cent. Because of this fact, the ampere turns and bulk of the interpole windings are much smaller than in d-c generators or motors of comparable rating.

These same methods are applicable to all other polyphase converters. The equation for method 2 can be generalized as follows: It applies to uniformly distributed windings with not too few slots.

$$A_{\text{crest}} = \frac{m}{2} \cdot \frac{\sqrt{2} \cdot 2}{\pi} Z_1 I_{ac} k_d k_p \qquad [669]$$

where m = the number of phases
Z_1 = the number of conductors per pole per phase
I_{ac} = the effective alternating current per conductor

This equation yields a crest value of a-c ampere turns per pole. Equation 666 gives the effect of armature reaction from the direct current, applicable to any polyphase converter, and the two can be used in any case to obtain values for comparison.

405. Voltage Control.[2]

Because the ratio of a-c to d-c voltages is fixed in any converter, few methods of voltage control are possible without the aid of external devices. Except when series reactance is present, a change in the excitation results merely in leading or lagging components in the current drawn from the line, which compensate for the changed excitation. All the practical methods for controlling the d-c voltage output are based on control of the a-c supply.

These methods employ:

(*a*) Synchronous booster.
(*b*) Induction voltage regulator.
(*c*) Tap-changers on the supply transformers.
(*d*) Series reactances in the supply lines.

Two other schemes are possible:

(*a*) Control of the d-c output by a booster. The comparative cost of this method militates against its use.

(*b*) Change in the ratio of a-c to d-c voltages by change in the wave shape. Inasmuch as the fixed ratios are based on the sine wave effective value of 0.707 times the peak, a change in wave shape would vary the ratio. This is brought about by a split-pole converter.

406. Synchronous Booster. If a special alternator is properly arranged on the shaft of the converter, its voltage may be used to boost or lower that applied to the converter. Such an alternator must be wound for

[2] J. L. McK. Yardley, "Synchronous Booster Rotary Converters," *Elec. J.*, Vol. 11, p. 267, 1914.

T. F. Barton and T. T. Hambleton, "Developments in Conversion Apparatus for Edison Systems," *A.I.E.E. Trans.*, Vol. 40, p. 663, 1921.

the same number of poles and the same number of phases as the main machine. Its voltages must be directly in phase with those of the converter, although reversing the excitation on the field poles of this booster will then cause its voltage to be in opposition to that supplied at its terminals. In the first case the booster acts as a generator and is driven by the converter; in the latter case it is a motor and helps drive the converter.

Such an arrangement is relatively expensive but it has the advantage of flexibility. The d-c voltage can be varied over a considerable range

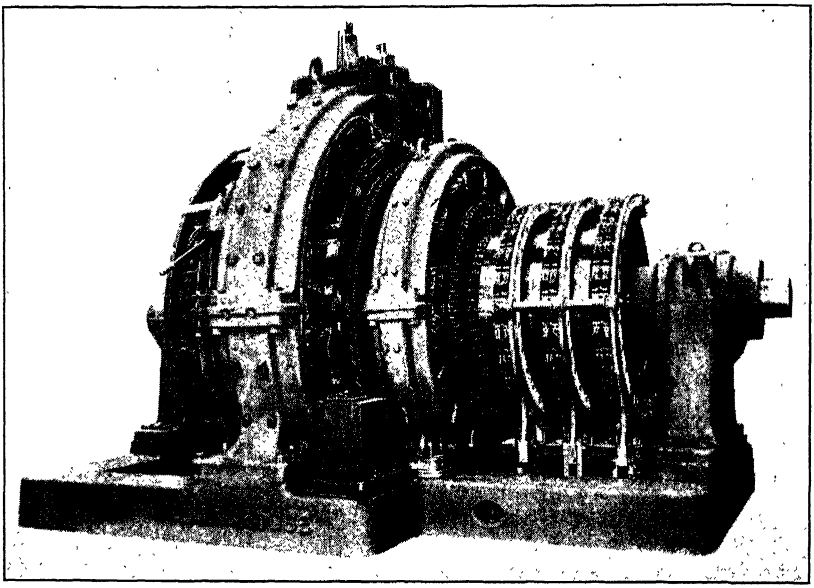


FIG. 313. Booster-type synchronous converter. (*Westinghouse Electric and Mfg. Co.*)

by adjustment of the field of the auxiliary booster poles. Standard designs permit voltage changes of 5, 10, or 20 per cent above or below the nominal rating. Special windings are frequently necessary for the commutation poles.

407. Induction Voltage Regulator. This plan for voltage control makes use of an induction regulator connected between the supply transformers and the converter slip rings. By varying the regulator rotor position its voltage can be made to add or subtract from that of the supply and result in increased or decreased voltage applied to the converter. This method is comparatively complicated and expensive and has never had a wide application.

A more economical scheme for voltage variation employs tap-changing transformers which vary the voltage ratio in steps. Such an arrangement is much used but does not yield a nicety of voltage adjustment.

408. Reactance in the Supply Lines and Field Control. A d-c generator may be made to have a rising voltage characteristic with load increase by the use of a series field. Such a series winding, added to the poles of a synchronous converter, would cause, on increased d-c output, an increase in excitation. This would result either in less lagging or more leading

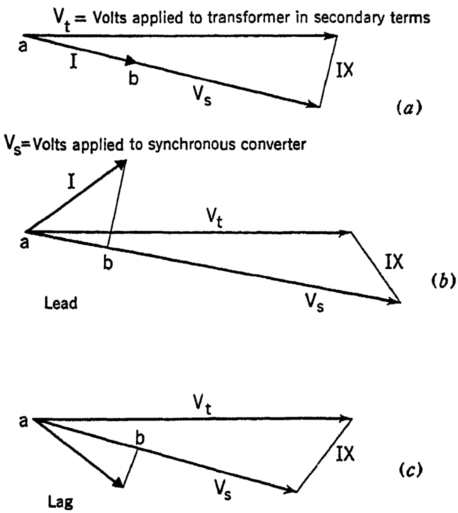


FIG. 314.

reactive current, depending upon the initial excitation. This reactive current would bring about a flux change compensating for the increased excitation. If, however, this reactive current is made to flow through reactance coils in the supply leads, the drop across these coils can be made to vary the terminal voltage applied to the converter and thus the d-c output voltage. This compounding can be made just enough to compensate for the internal drops of the converter and supply transformers, thus yielding constant d-c output voltage, or it can be increased to the extent necessary to give voltage control by series or shunt field adjustment. In some cases the leakage reactance of the supply transformers and line is enough to give sufficient voltage variation without the use of additional series reactors.

In Fig. 314 are shown the vector diagrams of a synchronous converter connected through series reactors to the supply transformers. V_t is the voltage applied to the transformers (one phase) in secondary terms. X

represents the sum of the leakage reactance of the transformers and the series reactors.

If the field is adjusted so that the current and voltage are in phase at the converter, i.e., the armature operates at unity pf, then the vector relationships will be as shown in Fig. 314*a*. The voltage, V_s plus the IX drop, applied to the converter, equals the output of the supply transformers. The resistance drop is small and is neglected here for simplicity. If the field is adjusted so that the converter current leads the applied converter voltage as shown in *b* of the figure, the direction of the IX drop is changed so that, with constant transformer voltage, the applied voltage at the slip rings is increased to the new value of V_s. In this case it is assumed that the d-c output is such that the in-phase component *ab* of the input current remains constant.

The condition for reduced excitation, resulting in lagging current, is shown in *c* with the resultant converter voltage of V_s.

In the diagrams shown, V_s has varied over a considerable range. Actually the regulation of the converter itself will result in a d-c voltage, different from V_s by other than the fixed ratio of a-c to d-c and give a final change in d-c voltage somewhat less than that indicated by the variation of V_s.

The chief disadvantage of this method lies in the dependence of voltage control on pf. The two cannot be varied independently. Hence, for a large change in voltage, the resultant change in phase angle gives excessive heating in some of the armature coils. The method has the advantages of simplicity. It was one of the early methods used for voltage control (patented in 1896) and was superseded by the induction voltage regulator, adjustable ratio transformer, and booster converter methods. These latter give a control of 10 to 20 per cent, but for installations in which voltage change of over 5 per cent is not required, the series reactor method with series or shunt field control is still useful, particularly with small converters.[3]

409. Voltage Control by Control of the Wave Shape. The voltage built up in a d-c generator depends upon the total flux cut, and is not affected by the distribution of that flux. The voltage induced in the coils of an a-c machine not only is influenced by the total flux but also depends upon its distribution. In a synchronous converter, since the d-c and a-c voltages both result from the same air-gap flux, the so-called fixed ratio of voltages can be varied by variation of the flux space distribution. This will vary the effective value of the a-c voltage. The ratio is

[3] For an outline of the procedure in calculation and design, see Arnold and La Cour, "Wechselstromtechnik," Vol. 4, "Die synchronen Maschinen," Julius Springer. Berlin.

affected also by changes in line wave shape with changes in load conditions.

The earliest method of varying the flux wave made use of a field pole split into three parts. Each of these three parts could be independently excited. If the outer sections are under-excited, the d-c voltage is a minimum. If the center section is under-excited the d-c voltage is intermediate, and if all sections are fully excited the d-c voltage is a maximum. The resulting a-c wave shape is different under each condition, being influenced also by the tap position. Such an arrangement could be used to give a voltage control of several per cent. It gave rise to

(*a*) Three part pole (*b*) Regulating pole

FIG. 315.

third-harmonic components of the voltage wave, but proper connections of the supply transformers can be used to eliminate any difficulty from them.

A second form of split-pole converter varies the voltage in the same manner as would be achieved through brush shift on a d-c generator. For good commutation, however, it is impracticable to shift the brushes very far. Instead, the field flux is shifted relative to the brushes by means of a regulating pole, smaller than the main poles and provided with an independent winding. Normally only the main poles are excited. Increasing the flux on the auxiliary pole to the same polarity as the adjacent pole is, in effect, a brush shift which increases the d-c voltage. Reversing the polarity of the auxiliary or regulating pole gives the minimum voltage.

These methods are of theoretical interest but have long been obsolete, owing largely to the objectionable wave forms which resulted. They caused telephone interference. The method was not popular with operators.

CHAPTER L

EFFICIENCY. VOLTAGE REGULATION

410. Chapter Outline.

Efficiency and Losses.
Regulation.
  Effect of the Wattless Current.
  Effect of the Brush Shift.

411. Efficiency. The capacity of a synchronous converter is limited by heating, regulation, and commutation, although in properly designed machines the last two items seldom cause trouble under normal conditions. This capacity can be expressed in terms of the rating of the same machine when operated as a d-c generator. (See Table XXIV.) Such a comparison is based on armature heating alone. In addition the armature-core eddy-current and hysteresis losses will be the same in either case and the shunt-field copper loss will be reduced when built to operate as a converter. Now since the output can be increased markedly with the same (or slightly decreased) losses, the efficiency of a converter will be considerably increased over that of the same machine operated as a d-c generator alone.

The influence of the number of phases on the capacity results in a like change on the efficiency. Hence the efficiency of a six-phase converter may be 2 or 3 per cent greater than that of a three-phase machine with the same d-c rating. The efficiency is, of course, subject to considerable decrease through operation at other than unity pf. In such a case the coils nearest the taps become hottest and the limit of output is determined by the maximum local heating rather than the average.

412. Vector Diagram. The Potier diagram can be applied to synchronous converters as shown below. The procedure will be outlined for different conditions.

(*a*) The input is on the a-c side, and the diagram is plotted for those quantities. Assume a zero brush shift. The cross ampere turns of the direct and alternating currents are assumed to be equal and opposite. For a specified load and power factor, the excitation is to be determined.

Refer to Fig. 316*a*. The current is lagging.

Lay off V, the applied voltage.

Behind V by the pf angle desired, lay off the current I. This can be the alternating current per conductor.

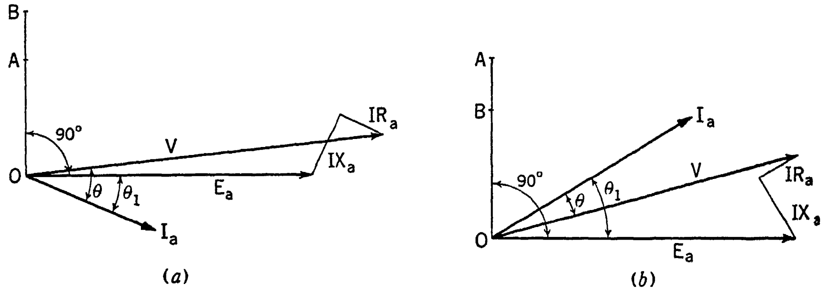


FIG. 316. Potier vector diagrams.

IR_a is the resistance drop. The ratio of armature copper loss in an n-ring converter to loss as a d-c generator was derived in equation 664. This ratio is R. Then for a loss of $I_{dc}^2 \cdot R_a$ watts as a d-c generator, the loss will be $I_{dc}^2 \cdot R_a(R)$ as a converter. Hence the resistance drop of $I_{dc} \cdot R_a$ as a d-c generator results in a resistance drop of $I_{dc} \cdot R_a\sqrt{R}$ as a converter. This drop, however, is not constant, but pulsates with different positions of the armature.

IX_a is the leakage reactance drop. The value of X_a can be calculated,[1] but for many purposes it may be assumed such that the full-load reactance drop is about 5 per cent of the applied a-c voltage.

FIG. 317. The average ordinate of the shaded area is k.

The solution of this diagram fixes the generated voltage, E_a.

For lagging currents, the ampere turns needed to produce E_a are OB. Of these, OA represents the field ampere turns and AB represents the magnetizing ampere turns due to the lagging wattless current. Denote this effect as A_w.

$$A_w = 0.707 \frac{Z}{2P} I_{ac} k \sin \theta_1 \qquad [670]$$

$$\text{where } k = \frac{\sin (\psi\ 90°)}{1.57\psi}. \quad \text{(See Fig. 317.)} \qquad [671]$$

[1] Arnold and La Cour, Vol. IV.

ψ = the ratio of pole arc to pole pitch

ψ	0.50	0.55	0.60	0.65	0.70	0.75	0.80
k	0.90	0.88	0.855	0.835	0.810	0.783	0.75

Z = the total armature conductors
P = the number of poles
I_{ac} = the alternating current per conductor

(*b*) Conditions as in (*a*) except that the load and field excitation are known, to find the pf: This is best done by plotting the current as a function of power factor for the given load. The diagram can then be drawn and solved as before.

(*c*) Conditions as in (*a*) except for leading current (Fig. 316*b*).

OA = the ampere turns in the field
AB = the demagnetization ampere turns of armature reaction due to the leading wattless current. They are calculated as in (*a*).

The other quantities are the same as before.

(*d*) Conditions as in (*a*) except that the brushes are shifted α degrees. If shifted in the direction of rotation the field flux is weakened.

$$NI_{\text{demag}} = \frac{2\alpha}{180}\frac{Z}{2P}I_{cdc} \qquad [672]$$

where I_{cdc} = the direct current per armature conductor.

In this case the field ampere turns OA of Fig. 316 (*a* or *b*) must be increased by NI of equation 672.

If the brushes are shifted against the direction of rotation the field is strengthened a corresponding amount. The field ampere turns OA must then be decreased by NI of equation (672). In each case the value of E_a fixes the generated d-c voltage.

CHAPTER LI

DIVERS TOPICS ON SYNCHRONOUS CONVERTERS

413. Chapter Outline.

Divers Topics on Synchronous Converters.
- Hunting and Flash-over.
- The Inverted Converter.
- Sixty-cycle versus 25-cycle Converters.

Transformer Connections.
- Three-phase to Six-phase.

Starting Methods for Synchronous Converters.
- From the D-c Side.
- By Means of an Auxiliary Motor.
- From the A-c Side.

Parallel Operation of Synchronous Converters.

414. Hunting and Flash-over. Inasmuch as the rotary converter is essentially a synchronous motor from its driving end, the conditions causing hunting in motors will produce similar effects here. Periodic changes in the d-c demand are not likely to produce hunting. It arises chiefly from the supply or from the effect of other pulsating load demands on the supply generator.

When hunting occurs in a synchronous converter its rotor will oscillate about its mean-torque-angle position. This is accompanied by a change in current and power on the a-c side with but little attendant change in the d-c output. As a result, each change in current produces a change in armature reaction and a resulting pulsation and shift of the air-gap flux across the pole faces.[1] Unusually heavy damping windings are provided in the pole faces to limit this displacement. As the flux

[1] Some of the first synchronous converters built were used with the Niagara Falls installation in 1894. They were observed to hunt badly under certain conditions. An investigation showed the presence of a shifting flux over the pole face and the practicability of a damping winding to supply the counteracting force. Thus damping windings or "amortisseurs" came into use about that time. They were developed originally for converters rather than synchronous motors.

See F. D. Newbury, "The Engineering Evolution of Electrical Apparatus," *Elec. J.*, Vol. 12, p. 27, 1915.

oscillates across the pole faces, so also does the neutral commutating zone oscillate. Vicious sparking may occur at the brushes, which is accentuated by the presence of commutation poles. These poles, in the interpolar space, provide a path of low reluctance and permit of wider flux sweep. The worst result from such sparking may be "flash-over" of the commutator. This may also result from suddenly throwing on or off the load of a synchronous converter, for under this condition the large swing of the rotor from its former load torque angle is accompanied by a corresponding flux sweep. This flux sweep causes the voltage between neighboring commutator bars to rise at certain points.

"Flashing" pertains to an arc set up through conducting vaporized material between brush sets or between brush sets and frame. If the load on a converter is excessive, as during a short circuit, or the field is shifted so as to cause excessive sparking at the brushes, the heat generated may vaporize the material and start an arc which spreads over the commutator to a brush of opposite polarity. Arcs may also arise across commutator segments between the brushes. The difference in potential between two commutator segments is a function of the flux density in the air gap through which the connected conductors are cutting. Such a potential gradient from segment to segment is not uniform because of non-uniformity of flux distribution. Any distortion of the field, such as accompanies hunting, may raise the voltage between adjacent bars so as to cause a spark to form between them, followed by a short-circuiting power arc which may spread over the entire commutator and severely damage it. Such action is aided of course by minute particles of dust, dirt, or finely ground conducting material which may accumulate between the segments.

To minimize the chance of flash-over, arcing barriers or shields are sometimes used between brushes. When an arc starts no circuit breaker can operate fast enough to open the line before the flash-over is complete.[2]

[2] B. G. Lamme, "Physical Limitations in D. C. Commutating Machinery," *A.I.E.E. Trans.*, Vol. 34, p. 1739, 1915.

This can also be found in B. G. Lamme, "Elec. Engineering Papers," published by the Westinghouse Electric & Mfg. Co.

F. D. Newbury, "Hunting of Rotary Converters," *Elec. J.*, Vol. I, p. 275, 1904.

J. J. Linebaugh, "Short Circuit Protection for Direct-current Substations," *A.I.E.E. Trans.*, Vol. 39, Part 1, p. 617, 1920.

Also in the same volume: Marvin W. Smith, "Flashing of 60 Cycle Synchronous Converters and Some Suggested Remedies," p. 631.

E. B. Shand, "Analytical Investigation of the Causes of Flashing of Synchronous Converters," *A.I.E.E. Trans.*, Vol. 41, p. 108, 1922.

415. The Inverted Converter. It is possible to reverse the functions of the converter and operate it with a d-c input, collecting alternating current from the slip rings. Such a converter is then said to be operating "inverted." In ordinary converter action any change in excitation produces a change in pf, but the motor action continues synchronously. When inverted, however, a change in excitation gives the usual effects found in d-c motors. That is, weakening the field increases the speed. Now since the output is alternating current, the increase of an electrical load at lagging currents, causes (by armature reaction) a demagnetizing effect on the field. The result is increased speed. Similarly, leading pf's give an increased excitation and decreased speed. Two bad results follow: (1) the change in speed results in a varying frequency at the load; (2) the increase of reactive (lagging) output may so weaken the field that the converter runs away. The latter action may result from cumulative speed action. That is, an inductive load across the lines weakens the generator field and speeds up the inverter. This increases the frequency and further increases the lagging component, resulting in still greater speeds. Inverters therefore must be protected by over-speed devices.

The response of an inverter to such load change depends greatly, of course, upon the ratio of armature ampere turns to field ampere turns. A low ratio results in better speed regulation. A second method of improving operation makes use of a separate exciter driven from the inverter shaft. The exciter iron is operated below the knee of its saturation curve so that its voltage gives a comparatively great response to any change in speed. Then when an inductive load is added to the a-c terminals, a slight increase in speed, through increased armature reaction, increases the excitation and gives a neutralizing effect, stabilizing operation with very little speed increase.

Inverted converters have found but little use industrially. Their efficiencies, current and voltage ratios, and gain in relative outputs are identical with those respective items previously derived for ordinary converters.

416. Sixty-cycle versus Twenty-five-cycle Converters. In synchronous machines the product of the number of poles and the speed is proportional to the frequency. When such an a-c machine must be equipped with a commutator, as is necessary on converters, the peripheral speed of the commutator is limited decidedly by the mechanical forces. On the other hand, operation at slow speeds requires more poles on 60-cycle than on 25-cycle sources. This reduces the number of commutator bars between brushes of opposite polarity and increases the number of volts per bar. As a result, 60-cycle machines, although satisfactory, are

inferior to lower-frequency types from the standpoint of momentary overload capacity without flashing, and ability to withstand rapid fluctuations in load.

417. Transformer Connections. To obtain standard voltages on the d-c output of the synchronous converter it is necessary that the a-c input be at other than standard a-c values. The conversion ratio depends upon the number of phases, and the heating effect decreases with an increase in phases. As a result, special transformers are required to step down the supply voltage to the unusual values necessary, and special connections are used to give an increased number of phases. Nearly

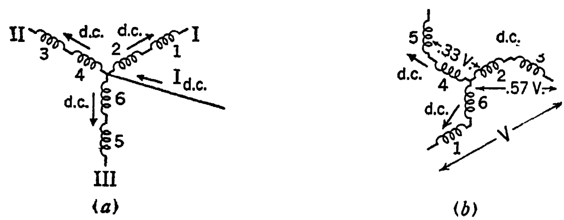


FIG. 318.

all medium- and large-sized synchronous converters are operated from a six-phase supply, converted from standard two- or three-phase sources. The transformers for achieving this are also useful to give reduced voltage by special taps when the converter is started from its a-c supply as an induction motor.

Six-phase transformer connections have been shown in Chapter XIX. In addition to the data discussed there, a special problem of some interest concerning a neutral point and unbalance is presented below.

Any transformer connection which gives a neutral point on its secondaries can be used to supply the neutral for the d-c side as well. This is due to the common armature winding for both the a-c and d-c sides. Hence a star or a double T provides such a point for use on Edison three-wire d-c distribution systems. If a three-phase converter is used and the secondaries are Y connected, detrimental effects result from unbalance on the d-c side. That is, if the load is not equally divided between the outside d-c lines and neutral, an unbalanced current flows back through the neutral and divides about equally through each transformer as shown in Fig. 318*a*. This gives detrimental saturation effects in the core. To eliminate this, one leg of the Y can be made up of windings from two different transformers. Examine the diagrams of Fig. 318. Suppose that the direct current through transformer I is one-third of the unbalanced current flowing back through the neutral. This current magnetizes the

iron core by the mmf of both secondaries. If the individual secondaries are connected as shown in *b* such an effect is eliminated. Transformer I, for example, with two secondary coils, 1 and 2, is now so connected that the direct current in 2 is in the opposite direction from the direct current in 1. The resultant mmf's then act against each other.

418. Starting Methods.[3] Synchronous converters can be started by the following methods:

(*a*) From the d-c side as a shunt motor.
(*b*) By means of an auxiliary a-c motor.
(*c*) From the a-c side as an induction motor.

Direct-current Starting. If the converter is operated in parallel with other units, or if direct current is available from some other source, the converter can be connected as a shunt motor and brought up to synchronous speed by that means. The a-c side is then synchronized with the line, like any a-c generator, and switched on. If the converter is provided with a series field this field must be short-circuited during the starting period. Normally connected to assist the main pole flux, when operated inverted or as a motor, this winding acts differentially. This weakens, or even reverses, the main field, if the machine is heavily compounded; such an effect is undesirable.

The supply transformers are considered as a part of the converter setup, and the leads from their secondaries are usually connected permanently to the slip rings. In such a case switching is done on the high-tension side when taking the converter out of operation or when synchronizing it. During the starting period the closed secondary circuits of the transformers act as short circuits across the d-c input and greatly increase the starting current. Some transformer connections are inherently worse in this respect than others.

Auxiliary-motor Starting. No special problem is presented by this method of starting. A small induction motor or commutator type of a-c motor can be used to bring the converter up to a speed somewhat above synchronism, after which it is allowed to coast and is synchronized with the a-c line as it reaches the correct speed. It is necessary that the induction motor have at least two less poles than the synchronous motor, otherwise its slip would prevent its bringing the converter into synchronism. The auxiliary induction motor starting method is used where reduced starting currents are an important factor.

419. Starting As an Induction Motor. As a synchronous converter is always equipped with a damping winding, it can be started as an

[3] F. D. Newbury and M. W. Smith, "Starting Rotary Converters," *Elec. J.*, Vol. 15, p. 24, 1918.

induction motor from the a-c side. A number of problems are presented by this method, however.

The usual starting procedure is to apply reduced voltage to the slip rings until the converter is about up to speed, then the shunt field circuit is closed and full voltage is applied.

Shunt Field. This circuit is usually open (although sometimes short-circuited) during the starting cycle. To prevent insulation breakdowns due to the high voltage induced therein as the armature flux sweeps by, it is customary to open the winding in several places by a sectionalizing switch.

Series Field. If the series field circuit is shunted, it forms a short circuit to the voltages induced in this winding during starting and should be opened during that operation.

Commutation When Starting As an Induction Motor. The polyphase currents cause an mmf which sets up revolving poles around the armature in a direction opposite to its rotation. This is similar to the effect of the primary of an induction motor. This sets up currents in the damper bars which react on the revolving flux to cause the torque accelerating the rotor. The revolving flux varies somewhat in amplitude as it moves from poles to interpolar space. The changing reluctance in its path distorts the wave form of the armature magnetizing current. If it were not for resistance, leakage reactance, and possible transients, the flux would remain constant.

The pole flux alternates with a frequency which varies from the supply value at zero speed, to zero at synchronism. This induces a transformer emf of variable frequency in the coils short-circuited at the brushes in the same way as in the a-c series or a-c repulsion motors. When commutation poles are present, the revolving armature mmf sets up in them an alternating flux of high value which remains stationary in space. This armature-induced interpole flux is also cut by the short-circuited coils as they pass underneath the commutating poles. The transformer emf due to alternation of the main pole flux and due to speed action on the commutating pole flux forms a resultant which with the accompanying circulating currents may cause severe sparking during the starting interval, unless some preventive means is used. To provide for this, commutating-pole converters are usually equipped with automatic or manually operated brush-lifting devices which keep the brushes away from the commutator while starting. To obtain direct current for excitation when the speed reaches synchronism, a special set of narrow brushes are provided which continue to ride on the commutator.

420. Polarity. When a converter is synchronized the polarity of the brushes has been fixed by the position of the armature mmf at the

instant the field was thrown on. This field is stationary in space at synchronism, but its position when brought to a stop can obviously be in one of two different directions with respect to the poles. As a result, the machine terminal which at some previous period of operation was positive may now be negative. If, however, the d-c field is excited from a separate source not subject in itself to change in polarity this uncertainty will not be present.

If the polarity at synchronism is incorrect it can be reversed by reversing the field switch, thereby causing the rotor to slip back a pole pitch. The field switch can then be reversed to its original position. It is possible that the armature reaction flux will be so strong as to maintain the same position even though the field is reversed. In this case the input switch to the converter must be opened momentarily to cause the rotor to slip back.

421. Parallel Operation.[4] Three methods of paralleling synchronous converters are possible. With the d-c outputs in parallel, these possibilities are:

(*a*) Paralleled a-c inputs, using the same transformer supply for two converters.

(*b*) Separate secondaries on the same transformers for each converter.

(*c*) Independent transformer supply for each converter.

In the first case the d-c load will divide between the two converters in the inverse ratios of their resistances and their counter emf's. The counter emf's can be adjusted by field change (with change in power factor), but the resistances, being made up of the armature circuit and brush contacts, are subject to wide variation under different operating conditions of temperature, load, etc. As a result, operation under this condition is likely to be very unstable, the load fluctuating from one machine to another for no apparent reason.

An increase in stability and flexibility can be gained by supplying each converter from a different set of secondaries on the same transformer cores. In this case there are no duplicated electrical connections between the d-c and a-c leads of each converter, but the magnetic link of the transformers prevents entire independence of voltage control for load adjustment from the a-c side. Such adjustment can be accomplished by using reactors in the supply leads of one converter and varying the excitation. By this means the proper reactance drop will lower the applied voltage in the converter and reduce its output. Such reactors increase

[4] F. D. Newbury, "Parallel Operation of Synchronous Converters," *Elec. J.*, Vol. 14, p. 274, 1914.

stability also. Without reactors, the most practical means for control of load division is to shift the brushes of the d-c commutator.

The most flexible system of connection employs an independent set of transformers for each converter. It is then possible to obtain a wider variation in voltage control by separate taps on the individual transformers, and operation can be made stable with dissimilar types of converters or transformer connections.

From consideration of the converters as d-c generators a number of conditions must be satisfied for successful parallel operation. If the voltage characteristics are drooping, the load can be made to divide properly over the entire range of operation, provided that the slopes of the regulation curves are correct for the respective ratings. Compound-wound converters, however, will not operate successfully in parallel without an equalizing connection from the series field of each paralleled converter. This connection is similar to that required on compound d-c generators.

Whenever a converter is operated in parallel with another source of direct current an additional hazard is introduced. This arises from the fact that should the a-c supply be interrupted the converter will operate as a motor obtaining power from the d-c bus. Under this condition the converter may reach a dangerously high speed as a d-c motor if the field is weak, as it would be if the converter had been operating with lagging current.

When a converter operates as a d-c motor, the reversal in current causes the series field to change its effect from cumulative to differential. This may cause an excessive increase in speed.

Protection is usually provided by means of (1) interlocking a-c and d-c circuit-breakers which open the d-c circuit in case of failure of the supply; (2) reverse current relays in the d-c circuit which actuate the d-c breaker in case of motor action; and (3) over-speed devices connected to the converter shaft which operate from centrifugal force to close a circuit opening the circuit breakers.

MERCURY-VAPOR RECTIFIERS

CHAPTER LII

THEORY, CONSTRUCTION, AND APPLICATIONS

422. Chapter Outline.[1]

Development and Theory.
Construction.
Auxiliary Apparatus.
Capacities.
Characteristics.

423. Introduction. In experimenting with the mercury-vapor lamp it was discovered that under suitable conditions mercury vapor possesses the property of carrying current in one direction only. If an electrode is sealed into an evacuated tube in which a supply of mercury vapor is maintained above a mercury pool, acting as another electrode, an alternating current can flow through such a circuit in one direction only. The flow is from the free electrode to the pool, and during the alternate period of reversed potential the vapor path acts as an insulator.

To obtain a unidirectional current from such a device which utilizes both halves of the sine wave, three electrodes are needed: one, the mercury pool, being called the cathode; the other two, active on alternate loops of the cycle, being called anodes.

The simplified diagram of connections for such a single-phase, full-wave rectifier is shown in Fig. 319. The alternating current is supplied through a transformer with a secondary midtap. Inasmuch as current can flow from the anode to the cathode, a direction of applied potential such as to make the point a positive on the secondary results in a flow from A_2 to C and hence through the load. This utilizes one-half of the secondary winding S_2. A reversal of emf on the other half cycle makes b positive and permits a flow of current through A_1, C, load and S_1. The

[1] Much of the material discussed in this section can be found in greater detail in a paper by O. K. Marti, "The Rectification of Alternating Currents," *Trans. A.I.E.E.*, Vol. 45, p. 668, 1926. See also the bibliography he gives and the partial one included with Chapter LIV of this text.

direction of current through the load has remained unchanged as indicated by the arrow. This full-wave rectification can be shown by the curves of Fig. 320. They disregard the effect of reactance in the supply

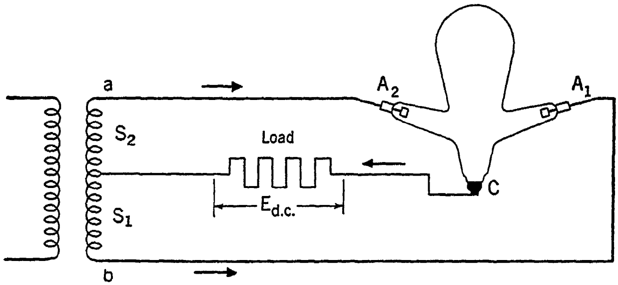


FIG. 319. Circuit of a single-phase, mercury-arc rectifier.

transformer and connecting leads. Such reactance causes distortions from the assumed sinusoidal loops. Actual wave shapes will be discussed later.

The polyphase rectifier is similar in its action to the single-phase type just described. A six-phase rectifier, for example, has six anodes each

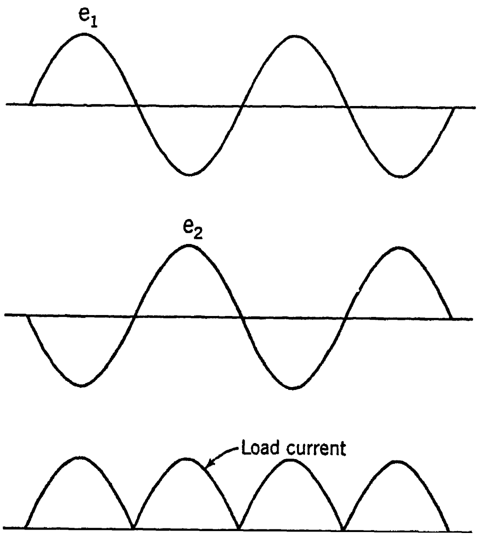


FIG. 320. Full-wave rectification, single phase.

connected to one phase of the supply. A simplified diagram of connections is shown in Fig. 321. In normal operation there is practically no interference between electrodes; as the potential rises on one of the

anodes due to cyclic change, all the load current is supplied through that anode. A decrease in instantaneous voltage on the active anode and the rise in potential of the next one cause the load current to be transferred from that of reduced potential to that with the highest instantaneous value. That is, the anode of highest potential always carries the load.

Any increase in number of phases serves to smooth out the ripples of the output (as increasing the number of commutator bars would do in a

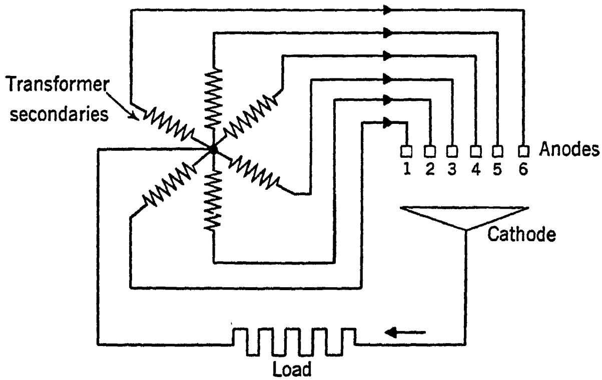


FIG. 321. Schematic diagram of a six-phase, mercury-arc rectifier.

d-c generator) and shorten the active period of each anode. Rectifiers have been built with as many as sixty phases.[2]

424. Construction and Items Modifying It. In the glass-bulb rectifiers, first used in the early nineteen hundreds, the chief difficulties of construction involved the sealing of the bulbs around the electrodes and the deleterious effect of the heat developed. Steel tanks were suggested to replace the glass bulbs, and experimental work began as early as 1905, work concerned chiefly with the provision of effective seals and insulators between the electrodes and the case. For the past twenty years power rectifiers of this type have been used, employing water jackets for cooling and maintaining satisfactory vacua by means of intermittently operated vacuum pumps.[3]

Figure 323 shows the cross-sectional view of the modern steel-tank rectifier. Note the baffles used to protect the anodes from the vapor blast and to prevent "flare-back."

[2] See Marti and Taylor, "Wave Shape of 30- and 60-phase Rectifier Groups," *Elec. Eng.*, April, 1940.

[3] Charles E. Woolgar, "Glass-bulb Mercury-arc Rectifiers for Traction Service," *Elec. Eng.*, August, 1941.

In operation a "hot spot" is maintained on the mercury pool. The passage of current from the anodes to the cathode maintains this spot and the supply of electrons passing out into the ionized space. (Such a movement of electrons from the spot is equivalent to a current flow into the cathode, according to our conventionalized concept of current direction.) A space-charge layer apparently exists around the cathode

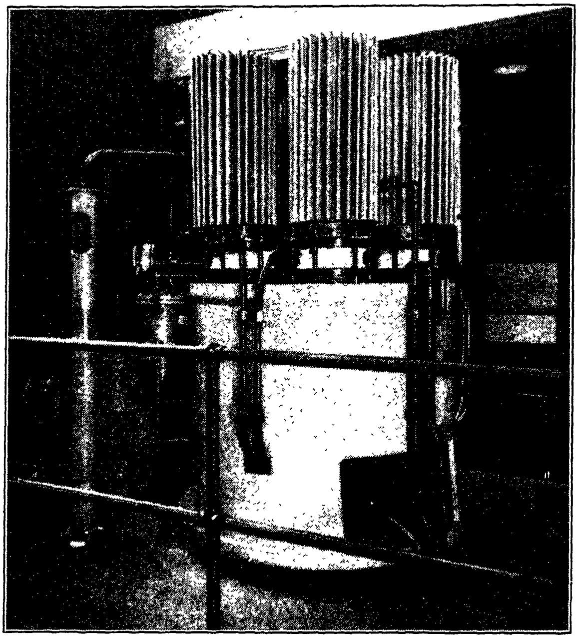


FIG. 322. Mercury-arc rectifier. (*Westinghouse Electric and Mfg. Co.*)

with a gradient of some millions of volts per inch. As this layer is extremely thin the actual voltage drop is about 15 volts.

One of the most important phenomena concerned with mercury-arc rectifiers is that known as "arc-back" or "flare-back." For reasons not thoroughly understood the rectifier may, at times, pass current in either direction. This causes rapid overheating and serious interruptions to service. Apparently the action takes place from the formation of cathode spots on the anode, and the result is similar to breakdown action of insulation. Such a breakdown is influenced chiefly by the presence of foreign gases in the tank, over-voltage, and the temperature and rectifier size. Modifications in design have reduced the tendency to arc-back, and proper construction of anode shields and baffles reduces the chances for condensed mercury to collect on the anodes.

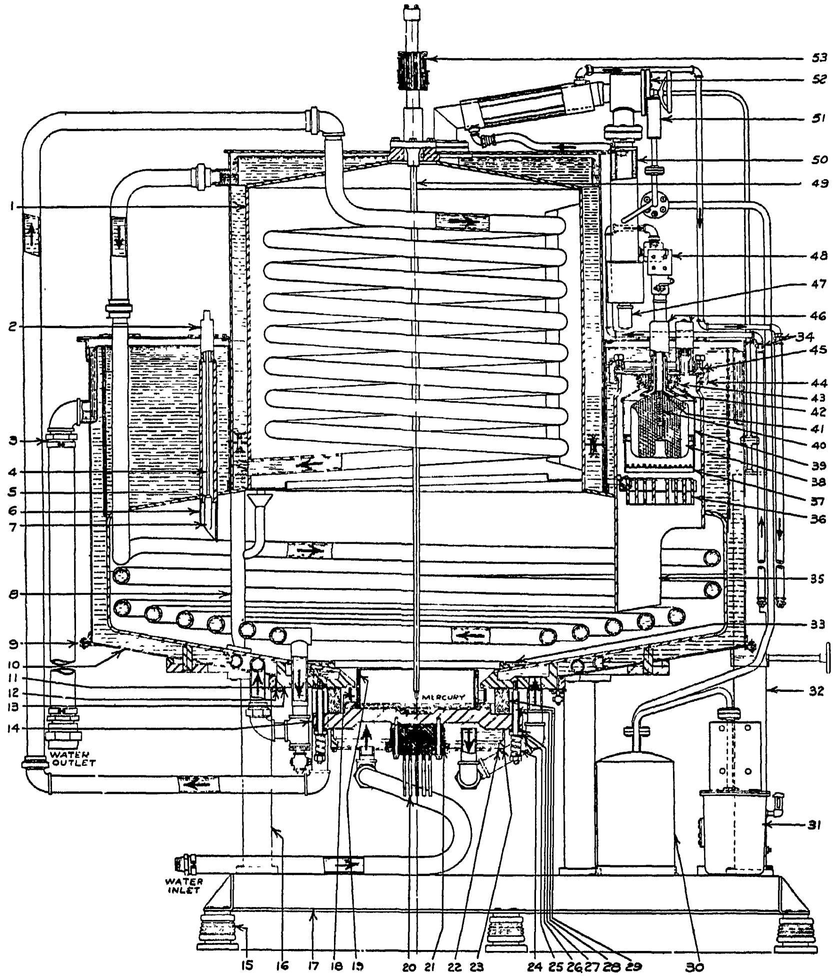


FIG. 323. Vertical section of a metallic-tank mercury-arc rectifier, and list of parts. (*The General Electric Co.*)

425. Starting. Auxiliary Apparatus. When the current through the rectifier reduces to 4 or 5 amperes, the cathode spot "goes out" and the rectifier action ceases. To prevent undesirable interruptions of operation at light loads a holding arc is maintained (from an auxiliary source) from the mercury pool to an extra electrode. In any case it is necessary to employ a starting circuit by which an auxiliary supply is used to set up an arc on the mercury surface. The resulting hot spot and mercury vapor then permit the passage of current from the main supply.

Auxiliary apparatus for the rectifier includes:

(*a*) A means of supplying power for starting and to insure continued operation at light loads.

(*b*) Vacuum pump and gauges.

(*c*) Cooling water supply and pumps. (Approximately one gallon of cooling water is required per minute for each 300 amperes output.)

(*d*) Reactors for the output circuit.

(*e*) Transformers for supplying the correct voltages and the correct number of phases.

(*f*) Regulation and protective apparatus.[4]

426. Sizes and Capacities. Steel-tank rectifiers have been built in capacities as high as 8400 kilowatts. Voltages are usually from 100 to 5000, although values as high as 30,000 volts are possible. Inasmuch as the drop in voltage from anode to cathode is about constant at approximately 15 or 20 volts, the loss this causes becomes relatively more important in low-voltage rectifiers.

Rectifier weights and sizes [5] vary from $3\frac{1}{2}$ to 9 lb per kw with overall tank heights of 90 to 130 in. Tank diameters are from 30 in. in the 200-kw size to 115 in. for 4000 kw.

427. Characteristics. The mercury-vapor rectifier has made large inroads in the railway and manufacturing substation field. Its advantages are:

Comparatively simple operation.
Readily made automatic in operation.
Maintenance and attention small.
Light weight and small floor space.
Comparatively noiseless.
High overload capacity.
Quick response to load demands.
High efficiency.

[4] See reference with footnote 1.

[5] "Standard Handbook for Electrical Engineers," Sixth Edition. Section 9, McGraw-Hill Book Co.

The last point deserves special attention. The full-load efficiency of a steel-tank rectifier of 1000-kw output (1500 volts) is about 95 per cent, although this figure depends somewhat on the voltage. A synchronous converter of the same rating would have an efficiency of about 92 per cent. This improvement in efficiency at full load is not so important, however, as the comparison at reduced load. At one-fourth load a synchronous converter may be from 80 to 85 per cent efficient as compared with about 93 for the rectifier. This is an especially favorable item for substations operating at a low load factor.

The drop in voltage with load increase is an important characteristic of the mercury-arc rectifier. At no load the voltage is considerably higher than the potential under light load. This initial drop is due to the fact that the anodes charge to the crest value of the alternating emf applied. A load current permits this potential to drop to the average value of the rectified wave. After this initial drop in the regulation curve the usual variation in voltage from, say, one-fourth to full load involves a drop of about 10 per cent. We will see that the number of phases influences the regulation also.

CHAPTER LIII

WAVE SHAPES. VOLTAGE AND CURRENT RATIOS

428. Chapter Outline.

Inductance in the Rectifier Circuit.
Output Voltage.
Wave Shape of Supply Current.
In the Anodes, the Transformer Secondaries and Primaries, and the Supply Lines.
Utility Factors.
Ripples in the Output Current.
Example of Calculations.

429. Effect of Inductance in the Output Circuit. Figure 320 shows the voltage and current waves for a single-phase, full-wave rectifier in which the load current and the voltage across the load, though unidirectional, were composed of sine loops. Such wave shapes disregard any inductive reactance in the transformer circuit or the supply leads. If, however, an inductance is added to the output circuit in series with the load, pulsations in the d-c output are choked out, and in the ideal case the d-c curve can be assumed to be a straight line. Since the output current in the single-phase rectifier is made up of two parts, half of which comes from each anode, the individual anode currents are blocked figures as shown in Fig. 324*b*. Now when current flows from anode 1, the voltage across the load and the inductance is the same as that impressed by the first half of the supply transformer. (See Fig. 319.) Then when anode 2 is operating the potential is that of the second half of the transformer; the resultant impressed potential is a series of sine loops. With the current output assumed non-pulsating, the *ir* drop of the load must remain constant. Hence to reconcile the conditions of pulsating potential but steady *ir* load drop, it is necessary that the pulsating component of the voltage be absorbed across the series inductance. This results in an average steady voltage across the load.

In a polyphase rectifier each anode in turn carries the load current. During cyclic change, as one anode reaches the highest potential it supplies the load current and maintains it until the potential on another

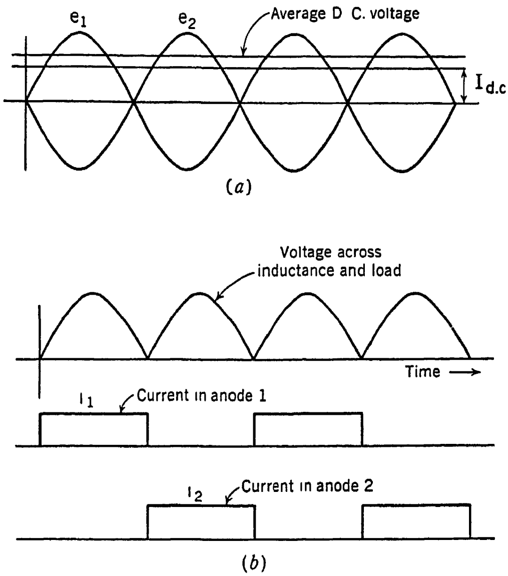


FIG. 324. Single-phase rectifier. Inductance in output circuit. (*a*) Single-phase, full-wave rectifier with current wave smoothed out by means of inductance in the output circuit. The loops e_1 and e_2 *above* the axis represent pulsating voltage applied to both inductance and load. The difference between these loops and the average d-c voltage (across load) represents the instantaneous value of voltage across the inductance. (*b*) Currents in anodes 1 and 2 are the component parts of the load current I_{dc} shown in (*a*).

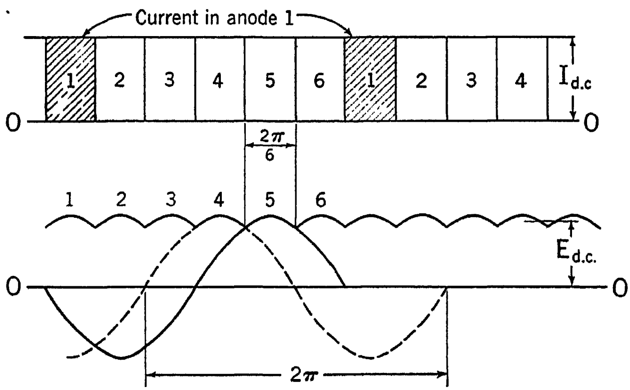


FIG. 325. Current and voltage waves; six-phase rectifier. An anode supplies the load current during the interval when its potential is highest.

electrode reaches a higher value. Hence during one cycle of the supply frequency each anode is operative once.

Figure 325 illustrates this condition for a six-phase rectifier; one anode is active for one-sixth of each cycle. Naturally such a blocked wave will have an rms value differing from ordinary alternating current, and since the period for which the current flows will depend upon the number of phases, both factors will influence the capacity of the supply transformers.

430. Output Voltage. Comparison of the curves of Figs. 324*b* and 325 indicates the influence of the number of phases on the output voltage wave shape. An expression for average d-c voltage output can be derived for any number of phases by consideration of the following relationships.

Let p equal the number of anodes, or the number of secondary windings with displaced potentials supplying the anodes. Then p also represents the number of phases except in the single-phase, full-wave rectifier.

Current flows to an individual anode during the period $2\pi/p$, and the average potential, E_{dc}, is the average of the emf wave during the period as shown in Fig. 325.

Then:

$$E_{dc} = E_{\max} \frac{p}{2\pi} \int_{-\pi/p}^{+\pi/p} \cos \theta d\theta$$

$$= E_{\max} \frac{p}{\pi} \sin \frac{\pi}{p} \qquad [673]$$

Values obtained from this equation are given in Table XXV.

TABLE XXV

Phases	E_{dc} as Percentage of E
1. Two anodes	90
3. Three anodes	117
6. Six anodes	135

431. Current Waves in the Anodes. The assumption of blocked-wave shapes is not entirely correct for reasons which will be explained later. Nevertheless such ideal cases are not so far in error but that useful studies of effective current values can be based thereon.

A blocked wave can be represented as a Fourier's series in which the number and magnitude of the various harmonics will be fixed by the

number of anodes or phases. The amplitude of the hth harmonic can be found from the expression:

$$i_h = \frac{1}{\pi}\int_{-\pi/p}^{+\pi/p} I_l \cos h\theta d\theta$$

$$= \frac{2I_l}{h\pi} \sin \frac{h\pi}{p} \qquad [674]$$

where I_l equals the load current.

Data for the following table were prepared from this equation.

TABLE XXVI

HARMONIC COMPONENTS OF ANODE-BLOCKED CURRENT WAVE

(As a multiplier of load current I_l)

Component of Blocked Wave	1 phase 2 anodes	3 phase 3 anodes	6 phase 6 anodes
Fundamental	0.637	0.552	0.318
2d harmonic	0	.276	.276
3d harmonic	.212	0	.212
4th harmonic	0	.138	.138
5th harmonic	.127	.110	.064
7th harmonic *	.091	.079	.045

* Sixth harmonic is zero in all except the quarter-phase rectifier.

The average value of the current per anode is simply the load current divided by the number of anodes or

$$I_{\text{average anode}} = \frac{I_l}{p} \qquad [675]$$

and the rms value is:

$$I_{\text{rms anode}} = \frac{I_l}{\sqrt{p}} \qquad [676]$$

Hence for two anodes the rms value of the anode current is $0.707I_l$; three anodes, $0.577I_l$; six anodes, $0.408I_l$; etc.

432. Transformers for Power Rectifiers. In order to calculate the capacity of transformers required to supply power rectifiers, Prince and Vogdes [1] make use of a utility factor, which is defined as the ratio of the permissible transformer output per phase for the rectifier to the output

[1] "Mercury Arc Rectifiers and Circuits," McGraw-Hill Book Co.

which would result in the same loss for sine waves. It can be readily understood that the rating of a transformer for rectifier service might differ from that for ordinary duty, owing to the shape of the current waves.

The base of the current block for any anode is a function of the number of anodes. To generalize, let us assume that the base is 2θ, and the amplitude is I_b. (See Fig. 326.) Let I_b vary as a function of θ in such a manner that the heating effect of the wave is constant. The value of the average voltage will not be independent of such a change, but will be greater with a decrease in θ.

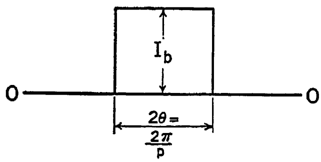


FIG. 326. Current block for an anode.

Then from equation 673, by substituting θ for π/p, the emf expression becomes

$$E_{dc} = E_m \frac{p}{\pi} \sin \frac{\pi}{p} = \frac{E_m \sin \theta}{\theta}$$

The secondary transformer onput per phase, when supplying rectifiers, is

$$W_{dc} = \frac{E_m I_{\text{load}} \sin \theta}{\pi} \qquad [677]$$

The copper loss per secondary transformer phase, when supplying rectifiers, is

$$\frac{I_{\text{load}}^2 R_2 \theta}{\pi} \qquad [678]$$

If a load of W_{ac} volt-amperes is applied to a transformer in an ordinary way, other than by a rectifier, the copper loss in the secondary is

$$\left(\frac{W_{ac}}{E}\right)^2 R_2$$

For equal losses on ordinary and rectifier loads, this loss must equal that of equation 678, from which $W_{ac} = EI_{\text{load}}\sqrt{\theta/\pi}$. Let $E = \frac{E_m}{\sqrt{2}}$, $\theta = \pi/p$, and take the ratio W_{dc} from equation 677 to W_{ac}. This gives

$$\frac{W_{dc}}{W_{ac}} = \sqrt{\frac{2}{\pi}} \frac{\sin \theta}{\sqrt{\theta}}$$

$$\frac{\sqrt{2p}}{\pi} \sin \frac{\pi}{p} \qquad [679]$$

This ratio is defined as the utility factor for the secondary. It represents the relative rectifier and non-rectifier loads that can be carried with the same heating.

If the d-c output consists of sine loops, i.e., no reactance is present in the output circuit, the utility factor reduces to the ratio of output kilowatts to kilovolt-amperes, or simply the pf.

433. Primary Waves. The voltage wave impressed on the primary of a rectifier supply transformer may be a sinusoidal function but the current wave is of blocked shape if inductance is used in the secondary circuit. The actual shape of the primary current wave is obviously governed largely by the number of anodes, the constants of the primary

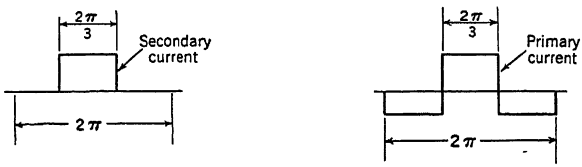


FIG. 327. Wave shapes of supply transformer windings. Secondaries, six-phase star; primaries, three-phase delta.

and primary supply circuits, and the type of connection used on the transformer. The d-c component carried by the secondaries would saturate the transformer core, but this may be avoided, if the number of anodes is even, by having one primary supply two secondaries and balancing out the d-c mmf. Hence the primaries will carry blocks of current of alternate signs and the heating effect will be $\sqrt{2}$ times greater than that of a primary carrying a pulsating current such as would result from half-wave rectification. Considering, however, that the transformer primary may be supplying two secondaries, the primary winding need have the capacity of only 0.707 times the rating of the two secondaries.

Simple representations of primary current and primary utility factors follow from the assumption that the current wave is a rectangular block and the applied potential is a sine wave.[2] Consider a six-phase rectifier supplied by transformers with Δ-connected primaries. The active period of one anode lasts for one-sixth of a cycle, and the base of the block is one-sixth of 2π, or $2\pi/6$. With the primary winding supplying two secondaries, a second rectangle of opposite sign will appear in the primary winding, giving current waves as represented in Fig. 327. In other words, the primary is active twice for each conductive period of one secondary.

[2] H. D. Brown and J. J. Smith, "Current and Voltage Wave Shape of Mercury Arc Rectifiers," *Trans. A.I.E.E.*, Vol. 52, p. 973, September and December, 1933.

The influence of transformer connections on wave shape can readily be seen by comparing this primary wave with one obtained from a three-phase rectifier. Assume that the transformers are connected Δ-Y. The conducting period for any one anode will now be one-third of the cycle, or $2\pi/3$ radians represents the base of the current rectangle. Since the number of secondaries is not even, so that one primary cannot be connected to supply two secondaries, the primary current wave will lack symmetry. Owing to the fact that, at the instant of conduction for one anode, the instantaneous values of currents through the other phases

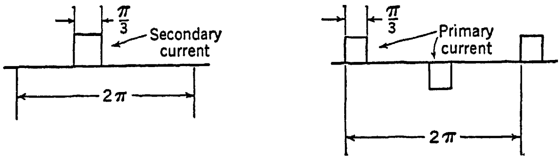


FIG. 328. Wave shapes of supply transformer windings. Secondaries, three-phase Y; primaries, three-phase delta.

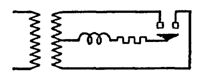
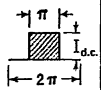
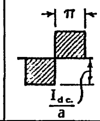
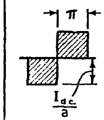
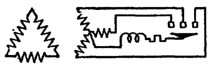
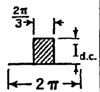
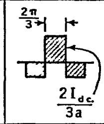
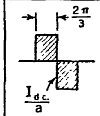
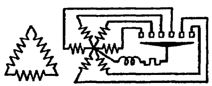
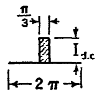
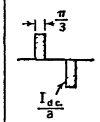
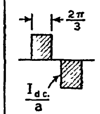
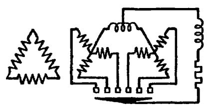
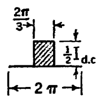
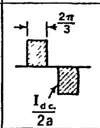
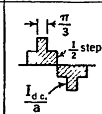
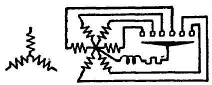
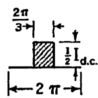
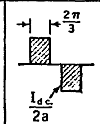
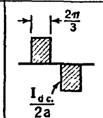
will be half maximum, the primary current takes the shape indicated in Fig. 328. This results in a different utility factor from that obtained for six-phase.

Line Currents. A Δ connection for transformer primaries requires that the line currents supplying the transformers be different from those of the primary windings. This arises from the fact that at the junction the line current is separated into components, parts going to each primary. This yields various shapes of line current differing from the primary values in all cases except where the primary winding is an extension of the line (i.e., no junction is made in connections), and hence yields utility factors for the line. A line utility factor is understood to mean the ratio of rectifier load to that of ordinary alternating current which would produce the same heat.

These unusual shapes of line current contain harmonics which may seriously interfere with adjacent communication systems unless precautionary measures are taken.

434. Ripples in the Current Wave. Distortions in the output current wave, by which it differs from a straight line, may arise from various causes, some of which are not readily corrected. Ripples may follow the variation in instantaneous voltage from the average value. Such a distortion can be removed, theoretically, by the use of suitable reactors in the output circuit, but for reasons of economy in reactor design it is sometimes more practical to permit slight current distortions on the belief that certain loads will not be greatly affected thereby. It is im-

TABLE XXVII *

Circuit	Connections	Sec. current		Sec. volts to neut.	Sec. V.a. & Util. Factor	Prim. current wave	Eff. prim. current & volts	Prim. V.a. & U.F.	Total transf. U.F.	Wave form of line current	Eff. line curr. & volts	Line V.a. & U.F.
		Wave shape	R.M.S.									
Single phase			$\frac{I_{d.c.}}{\sqrt{2}}$ $.707I_{d.c.}$	$\frac{\pi E_{d.c.}}{2\sqrt{2}}$ $1.11E_{d.c.}$	1.57 x $E_{d.c.}I_{d.c.}$ U.F.= .637		$\frac{I_{d.c.}}{a}$ $1.11aE_{d.c.}$	1.11 x $E_{d.c.}I_{d.c.}$ U.F.= .90	.746		$\frac{I_{d.c.}}{a}$ $1.11aE_{d.c.}$	U.F.= .90
Three phase			$\frac{I_{d.c.}}{\sqrt{3}}$ $.577I_{d.c.}$	$\frac{\sqrt{2}\,\pi E_{d.c.}}{3\sqrt{3}}$ $.855E_{d.c.}$	1.481 x $E_{d.c.}I_{d.c.}$ U.F.= .675		$\frac{\sqrt{2}\ \ I_{d.c.}}{3a}$ $.855aE_{d.c.}$	1.209 x $E_{d.c.}I_{d.c.}$ U.F.= .827	.743		$\frac{\sqrt{2}\,I_{d.c.}}{\sqrt{3}a}$ $.855aE_{d.c.}$	1.209 x $E_{d.c.}I_{d.c.}$ U.F.= .827
Six phase star			$\frac{I_{d.c.}}{\sqrt{6}}$ $.408I_{d.c.}$	$\frac{\pi E_{d.c.}}{3\sqrt{2}}$ $.741E_{d.c.}$	1.814 x $E_{d.c.}I_{d.c.}$ U.F.= .552		$\frac{I_{d.c.}}{\sqrt{3}a}$ $.741aE_{d.c.}$	1.283 x $E_{d.c.}I_{d.c.}$ U.F.= .780	.646		$\frac{\sqrt{2}\,I_{d.c.}}{\sqrt{3}a}$ $.741aE_{d.c.}$	1.047 x $E_{d.c.}I_{d.c.}$ U.F.= .955
Double Y			$\frac{I_{d.c.}}{2\sqrt{3}}$ $.289I_{d.c.}$	$\frac{\sqrt{2}\,\pi E_{d.c.}}{3\sqrt{3}}$ $.855E_{d.c.}$	1.481 x $E_{d.c.}I_{d.c.}$ U.F.= .675		$\frac{I_{d.c.}}{\sqrt{6}a}$ $.855aE_{d.c.}$	1.047 x $E_{d.c.}I_{d.c.}$ U.F.= .955	.792		$\frac{I_{d.c.}}{\sqrt{2}a}$ $.855aE_{d.c.}$	1.047 x $E_{d.c.}I_{d.c.}$ U.F.= .955
Y Prim. Six phase star			$\frac{I_{d.c.}}{2\sqrt{3}}$ $.289I_{d.c.}$	$\frac{\sqrt{2}\,\pi E_{d.c.}}{3\sqrt{3}}$ $.855E_{d.c.}$	1.481 x $E_{d.c.}I_{d.c.}$ U.F.= .675		$\frac{I_{d.c.}}{\sqrt{6}a}$ $.855aE_{d.c.}$	1.047 x $E_{d.c.}I_{d.c.}$ U.F.= .955	.792		$\frac{I_{d.c.}}{\sqrt{6}a}$ $1.482aE_{d.c.}$	1.047 x $E_{d.c.}I_{d.c.}$ U.F.= .955

$I_{d.c.}$ = load current $E_{d.c.}$ = average d.c. voltage output. $a = \frac{\text{Primary line potential,}}{\text{Secondary leg potential}}$

* From Mercury Arc Rectifiers and Circuits. Prince and Vogdes. Reproduced by permission of the publishers, McGraw Hill Book Co.

portant, however, in determining the size of the reactor to calculate the magnitude of ripple which it will permit, or what amounts to the same thing, to determine the inductance which will permit a definite ripple of maximum magnitude in the voltage wave.

It can be shown that the magnitude of a harmonic voltage as a fraction of the d-c potential is

$$\frac{\pm 2}{(h^2 - 1)} \qquad [680]$$

And the maximum value of pulsation of the hth harmonic is

$$V_{\text{max}} = \frac{\pm 2dc}{(h^2 - 1)} \qquad [681]$$

The minus sign applies to even values; plus, to odd values.

The order of a voltage pulsation in the output cannot be lower than the number of phases and must always be an exact multiple.

435. Example. A six-phase rectifier is to deliver 500 amperes at 550 volts. The supply transformers are connected with delta primaries and six-phase star secondaries. The arc drop is 15 volts. Supply frequency is 60 cycles. Calculate the voltage to neutral on the transformer secondaries.

From equation 673:

$$E = \frac{550 + 15}{\sqrt{2}\,\frac{6}{\pi}\sin\frac{\pi}{6}} = \frac{565}{1.35}$$

$$= 418.5 \text{ volts}$$

For what ordinary rating must the primary and secondary windings of each transformer phase be designed?

Watts per secondary phase delivered to the converter:

$$\frac{565 \times 500}{6} = 47{,}100$$

Utility factor of the secondary (from Table XXVII):

$$\text{Utility factor} = 0.552$$

Secondary volt-amperes per phase:

$$\frac{47{,}100}{0.552} = 85{,}400$$

Utility factor of the primary (from Table XXVII):

$$\text{Utility factor} = 0.780$$

Primary volt-amperes per phase (three-phase):

$$\frac{2 \times 47{,}100}{0.780} = 121{,}000$$

Suppose, instead of blocked current waves, that the inductance for the load circuit may permit a maximum current variation plus and minus not in excess of 5 per cent for any harmonic.

Calculate the required inductance.

The harmonic of the lowest possible order and hence of the greatest magnitude is the sixth.

$$V_{\max} = \frac{-2 \times (550 + 15)}{6^2 - 1} = 32.3 \text{ volts maximum}$$

32.3 volts maximum is equivalent to 22.8 volts effective.

Current variation permissible = 0.05×500, or 25 amperes.

A maximum variation of 25 amperes on a straight-line current corresponds to an alternating current with a crest value of 12.5 amperes and an effective value of 8.84 amperes. Since 8.84 amperes can be permitted to flow through an inductance due to an effective a-c component of 22.8 volts, the inductance is fixed.[3]

$$2\pi fLI = E$$

For the sixth harmonic f is 360 cycles per second.

$$\therefore L = \frac{22.8}{2\pi \times 360 \times 8.84}$$

$$= 0.00114 \text{ henry}$$

[3] F. O. Stebbins and C. W. Frick, "Output Wave Shape of Controlled Rectifiers." *Elec. Eng.*, September, 1934.

C. C. Herskind, "Grid Controlled Rectifiers and Inverters." *Elec. Eng.*, June, 1934.

CHAPTER LIV

VOLTAGE REGULATION. OVERLAP. COMMUTATION

436. Chapter Outline.

Voltage Regulation.
Angle of Overlap.
Parallel Operation.
Voltage Control.

437. Voltage Regulation. Under load the d-c voltage output of the mercury-arc rectifier drops because of several factors, viz., reactance in the transformer and anode leads, resistance, impedance drop in the source, and losses in the rectifier itself.

The reactance has the effect of causing a change in the d-c voltage wave which reduces its average value. This will be dealt with in more detail. In addition the impedance of the supply transformers causes drops which reduce the anode potential quite aside from the effect of the reactance mentioned above. The effect of resistance in the circuit causes a direct drop in voltage and also affects the wave shape. Unless the resistance is high as compared with the reactance, its influence on the wave shape is usually neglected to avoid complications. The recommended method of considering resistance drop in the rectifier circuit is to determine the total resistance losses. These are divided by the current, and the result is assumed to be the IR drop.

Theoretically, any one of these components may influence the wave shape of the rectifier and hence influence the effect of the others. The consideration of all these factors simultaneously is so complex that the practical method of determining regulation is to calculate the drop from each component and then combine to give the total regulation. Even with this approximation the problem of determining regulation, other than experimentally, is not a simple one. Some of the details are discussed in the articles to follow.

438. Effect of Load upon the Direct-current Voltage Wave. Reactance Present. The assumptions made in the preceding chapter, relating to the blocked-wave shape of current, considered the effect of inductance in the circuit only to the extent of smoothing out ripples in the output

voltage. Actually any inductance, including that of the supply transformers, would make impossible the instantaneous change of current on an anode from zero to load value. Instead, a "period of overlap" exists, during which the current gradually (instead of instantaneously) increases in the next anode. This period starts at the instant the potentials become equal on two anodes; during its continuation the current varies as shown in Fig. 329*a*. The difference in potential which would exist between e_1 and e_2 on open circuit is used in overcoming the reactance drops in the lines and supply transformers. Hence the two anode

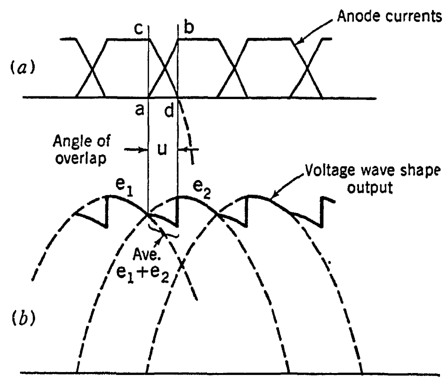


FIG. 329. Voltage and current waves showing the angle of overlap.

potentials are the same for this period, being the average of the instantaneous values of e_1 and e_2; the resultant d-c voltage wave shape is that indicated in Fig. 329*b*.

As this change results in a reduction of average d-c voltage output, and takes place under load, its effect is included with those resulting in voltage regulation. The change of wave shape influences the heating and utility factors of the supply transformers, causing less heating than the previously derived utility factors would indicate.

Method of Calculation. The percentage change in voltage brought about by this effect can be calculated after the angle of overlap is determined, or the angle can be eliminated from the equations, obtaining it as a function of IX drop and commutating voltage.

The current change during the angle of overlap [from a to b on one electrode, or from c to d on the other, (Fig. 329)] represents the initial variation of a sine wave of short-circuit current, with displaced axis, which could flow between two electrodes. The two electrodes, and that part of the arc which conducts current between them, form a short-circuited path for the difference in potential (commutating voltage),

inasmuch as they both carry current and it is therefore possible for an alternating current to flow from one to the other. This alternating short-circuit current never exceeds the value of the load current, as that would require that the current on the other anode extend below the zero axis as shown dotted in Fig. 329*a*. The valve action of the rectifier prevents any actual reversal, and the transient short-circuit current cannot take negative values below *d* nor extend above the load current at *b*. Hence the angle of overlap persists only until *i* equals zero at one of the anodes.

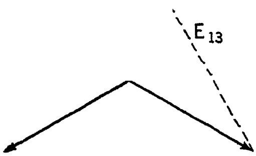


FIG. 330. On a three-phase rectifier the commutating voltage is E_{13}. It is this voltage which determines the commutating current during the period of overlap when anodes 1 and 3 each carry current.

Neglecting any resistance in the transformer winding and anode leads, the short-circuit current will have a maximum value depending upon the maximum value of the voltages between anodes; it will vary inversely with the reactance. (See Fig. 330.) The actual value which the short-circuit current attains will be fixed by the angle or period of overlap and the load current I_{dc}, since it cannot surpass this latter value. In general terms, for any number of anodes, the commutating voltage is

$$2E_{\max} \sin \frac{\pi}{p} \qquad [682]$$

where $E_{\max}$ equals the secondary voltage to neutral on the supply.

This rises, in the commutating period, to the value

$$\left(2E_{\max} \sin \frac{\pi}{p}\right)(1 - \cos u)$$

and the commutating period ends when

$$\frac{\left(2E_{\max} \sin \frac{\pi}{p}\right)(1 - \cos u)}{2X} = I_{dc} \qquad [683]$$

where X equals the reactance of the anode lead and the transformer secondary to neutral. Actually this is an equivalent reactance, including transformer primary values in secondary terms. Its computed value depends upon the type of transformer connections used.[1]

[1] Examples of such calculation applied to various types of connection can be found in the previously cited text by Prince and Vogdes, and in the paper by H. D. Brown and J. J. Smith.

Equation 683 can be used to determine the value of the angle of overlap u. It has been shown (equation 673) that the d-c voltage output is

$$E_{\max} \frac{p}{\pi} \sin \frac{\pi}{p}$$

The drop in voltage caused by the wave shape, as illustrated in Fig. 329, is

$$\left(E_{\max} \frac{p}{2\pi} \sin \frac{\pi}{p}\right) (1 - \cos u) \qquad [684]$$

From equation 683:

$$(1 - \cos u) = \frac{IX}{E_{\max} \sin \frac{\pi}{p}}$$

Substituting in equation 684, the average drop in voltage under load from this source will be

$$\frac{IX}{E_{\max} \sin \frac{\pi}{p}} E_{\max} \frac{p}{2\pi} \sin \frac{\pi}{p} = IX \frac{p}{2\pi} \qquad [685]$$

Examination of this equation shows that the drop will increase with the number of phases, giving worse voltage regulation.

Example. A three-phase rectifier is supplied by delta-star-connected transformers with an equivalent leakage reactance per Y-leg of 0.25 ohm. The effective voltage from one secondary leg is 100 volts. The load current is 100 amperes. Calculate the drop due to the effect of overlap on the wave shape, and also the angle of overlap.

$$E_{dc} \text{ at no load} = 141 \frac{3}{\pi} \sin \frac{\pi}{3} \quad \text{(from equation 673)}$$
$$= 117 \text{ volts}$$

$$\text{The drop} = 100 \times 0.25 \times \frac{3}{2\pi} \quad \text{(from equation 685)}$$
$$= 11.9 \text{ volts}$$

$$E_{dc} \text{ under load} = 117 - 11.9$$
$$= 105.1 \text{ volts}$$

The angle of overlap:

$$1 - \cos u = \frac{100 \times 0.25}{141 \times 0.866}$$
$$\cos u = 0.795$$
$$u = 37° \, 21'$$

439. Parallel Operation and Voltage Control. Rectifiers can be operated satisfactorily in parallel, either with similar units or with motor generators and synchronous converters. It is necessary to insure sufficient drop in their voltage-regulation curves as the load division between

them (as in most electrical apparatus) depends upon the relative slopes of their voltage characteristics.

Inasmuch as the arc drop remains about constant with load, the arc displays the wrong resistance characteristic to give good, stable, parallel operation. It is therefore necessary to provide the required drop by means of extra resistance or reactance in the anode leads, the former being little used because of its inefficiency.

To modify the voltage output of the rectifier, any device whereby the input voltage can be controlled will be satisfactory. Hence tap-changing equipment with the supply transformers or induction-voltage regulators in the supply lines would present satisfactory voltage control.[2]

Change in transformer connections will also vary the voltage output. Examination of Table XXVII shows that the change, say from double Y connection to six-phase, brings about an increase in secondary voltage and hence in the d-c output potential. Hence, if the rectifier is used on light loads with the former connection and either manually or automatically switched over to six-phase at heavy loads, the apparent voltage regulation can be varied, and in some cases the voltage at heavy loads made higher than the light load value. Such a "compounding" can be done automatically and gradually by the use of specially constructed "interphase" transformers.[3]

440. Summary. In the foregoing treatment we have discussed briefly the following principal topics belonging to mercury-arc rectifiers: voltage and current ratios, utility factors, voltage and current harmonics, regulation, and control.

Owing to the limitations of space a variety of topics, such as design, theoretical calculations, details of physical phenomena involved, tests, battery charging, details of auxiliary and protective circuits and apparatus, installation, operation, maintenance and repair, has had to be omitted. Many books and articles are available covering these subjects.[4]

[2] L. B. W. Jolley, "Alternating-current Rectification," Third Edition, p. 243. John Wiley & Sons.

[3] Prince and Vogdes, "Mercury Arc Rectifiers and Circuits," p. 203, McGraw-Hill Book Co.

[4] "Standard Handbook for Electrical Engineers," Sixth Edition. McGraw-Hill Book Co.

Marti and Winograd, "Mercury Arc Power Rectifiers." McGraw-Hill Book Co.

E. B. Shand, "Steel-tank Mercury Arc Rectifier." *A.I.E.E.J.*, June, 1927.

H. D. Brown, "Grid-controlled Mercury-arc Rectifiers," *Gen. Elec. Rev.*, August, 1932.

SERIES MOTORS

CHAPTER LV

THEORY AND CHARACTERISTICS. VECTOR DIAGRAMS

441. Chapter Outline.

Classification of Types.
Characteristics of the A-c Series Motor.
Commutation.
Losses and Efficiency.
Construction.
Motor Types.
Vector Diagrams.

442. Introduction. Single-phase commutator motors were developed before the advent of the polyphase induction motor. The high frequency, 133 cycles, then in use made them impracticable because of the poor commutation. The wide field of application for the induction motor centered attention upon it and arrested the development of the commutator motor, especially because of its bad commutation characteristics.

However, by 1902, 25, 50, and 60 cycles had been introduced because of their adaptability for power. The need for an a-c railway motor with series characteristics became acute at that time also, and this led to a renewal of efforts to develop the commutator type of motor. Frequencies of the order of $16\frac{2}{3}$ to 25 cycles were adopted in various countries for this class of service.

Hundreds of types and variations of such motors have been proposed and patented, but with few exceptions have not come into or remained in use. They include single and polyphase arrangements, with series, shunt, or compound characteristics, some in combination with induction-motor features. A partial list is given below, identified by type or name of the inventor.

I. Single-phase series motor.
  1. Plain.
  2. Compensated.
     Forced, induced. (Latour, Fynn.)

II. Single-phase repulsion motor. (Thomson, Arnold, Atkinson, Deri.)
  1. Compensated. (Winter-Eichberg, Latour, Wightman.)

III. Series repulsion or doubly fed. (Arnold, Alexanderson, Milch, Punga, Latour, Richter, Osnos.)

Only the above three types will be considered here, but other types of commutator motors have been developed and can be classified further. The material to follow will help clarify the types. They are:

(*a*) Single-phase shunt motors.
  Directly fed. (Behn-Eschenberg, Behrend.)
  Indirectly fed. (Fynn.)
  Doubly fed; compensated shunt motor. (Steinmetz.)
    Rotor excitation.
    Divided excitation.

(*b*) Polyphase.
  Adjustable speed series motor. (Wilson, 1888; Görges, 1891.)
  Adjustable speed shunt motor. (Arnold-La Cour, 1894.)
  Schrage motor. (See Article 246.)
  Heyland repulsion motor.

In the *series motor* the armature and field are connected in series, operating very much like the series d-c machine. This motor will be considered in detail in this section. Its chief applications are in two fields: electric railways for which the sizes used may reach 2200 hp at 200 to 400 volts, using 15 to 25 cycles; and portable domestic and industrial equipment requiring fractional or small horsepower outputs. The smaller sizes are frequently "universal" motors, operating on either a-c or d-c at 115 or 230 volts, up to 60 cycles. Speeds may be from 3500 to 16,000 rpm.

The chief problems in the development of the series a-c motor have been those of obtaining satisfactory commutation, compactness, lightweight, and good pf. The first problem is still a limiting factor which prevents the use of large motors on 60 cycles if large starting torques are demanded. Lightweight is of special importance in the portable tool industry and railways.

In the *repulsion motor*, the armature is excited *inductively* as against the *conductive* excitation for the series machine. This will be considered in a later section. The *doubly fed* machine is a combination of both, and will be described briefly.

443. Operating Characteristics. If an ordinary d-c series motor were connected to an a-c supply, it would operate, but not very satisfactorily. Since the field and armature currents both reverse every half

cycle, the torque would be exerted at double frequency in one direction. However, the alternations in field flux would cause excessive eddy-current losses in the field cores and yoke. The induced voltages and currents in the armature coils, short-circuited by the brushes when undergoing commutation, would result in vicious sparking; the inductance of the field winding would result in abnormal voltage drops and low pf; this, with the reduction and phase shift of the main flux due to core eddy currents, yields poor performance. The method of overcoming these objections is described below.

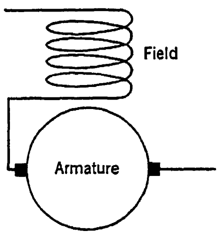


FIG. 331. Conventional representation of the plain series motor.

Plain Series Motor. The connections for such a motor are shown in Fig. 331. The poles may be either salient or non-salient with distributed windings. The pf is relatively poor unless speed is high, frequency low, air gap and field turns small, or unless

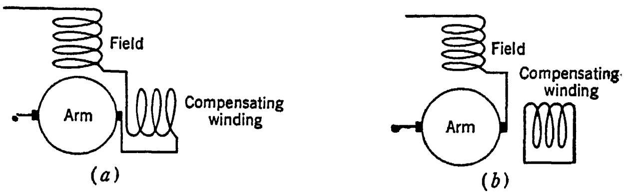


FIG. 332. Compensated series motors. (*a*) Conductively compensated. (*b*) Inductively compensated.

the brushes have a large negative lead of 30 to 50 electrical degrees.

Compensated Motor. With the brushes on the geometric neutral, the armature reaction of a d-c motor builds up "poles" of flux midway between the main poles, stationary in space and alternating in time. In other words, armature reaction results in cross-magnetization of the field. The same effect occurs in a-c series motors. It can be neutralized by a winding on the stator, so placed as to result in an mmf approximately equal and opposite to that of the armature. Such a motor is then said to be "compensated."

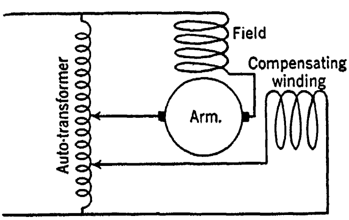


FIG. 333. One form of doubly fed series motor.

Two methods are available for exciting this compensating winding: *conductively* as shown schematically in Fig. 332*a* and *inductively* as represented in Fig. 332*b*. Inductive compensation is, of course, ineffective on

direct current. A modification of the conductively compensated motor is shown in Fig. 333, in which separate taps are used from an autotransformer supply. This type of motor is one form of the *doubly fed* series motor.

The torque in a motor depends upon the field flux, the armature current, the armature turns, the brush angle, and the cosine of the phase

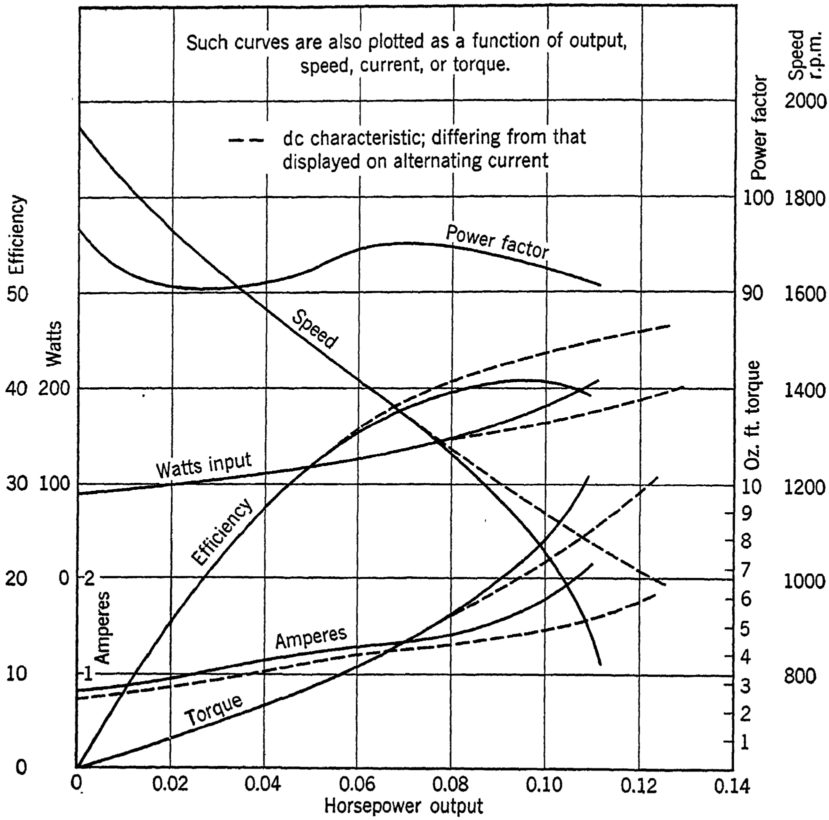


FIG. 334. Characteristic curves of a universal motor.

angle (between field flux and armature current) in time. The torque is affected also by the currents in the short-circuited coils at the brushes, which tend to reduce it at very low speeds and at zero speeds. To obtain a desired torque it is theoretically possible to have (*a*) a strong field and relatively few armature turns, or (*b*) a weak field with smaller air gap and many turns on the armature. Only the latter is used commercially. In that case, the reactance of the armature may be several times that of

the field winding. The inductive effect of the armature can be neutralized by a suitable, properly connected, compensating winding or by large negative brush lead. The net result, in designs with few field turns, is a comparatively low reactance and a high pf with higher resistance and higher losses. The field flux must not be neutralized as this would

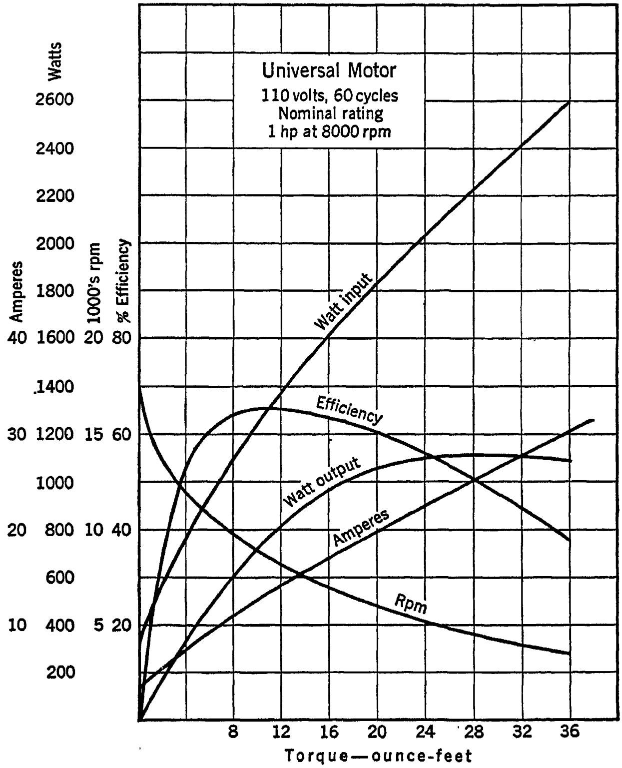


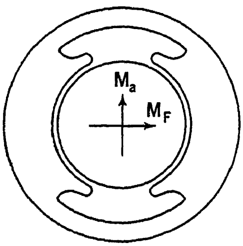
FIG. 335. Typical performance of a universal motor used by the portable tool industry.

destroy its function. However, if the motor is built with a small air gap, and a suitable cross-section of iron is provided in its magnetic circuit, the performance is generally satisfactory.

Typical performance curves for universal motors are shown in Figs. 334 and 335.

444. Compensating Windings. The directions and relative magnitudes of the field and armature mmf's are represented by vectors as shown in Fig. 336. The addition of the compensating winding with an mmf equal to that of the armature, and opposite in space phase, is the

equivalent of adding the vector M_c. These windings are distributed in slots in the field poles. They may be made theoretically stronger than the armature mmf (over-compensation), equal to it (full compensation) or weaker (under-compensation). Only full compensation and under-compensation are used in practice. Of course, the compensating winding can never completely neutralize the fluxes set up by the armature mmf because of unavoidable separation between windings, differences in distribution, and local leakages. All conductively compensated motors have their compensating windings connected in series with the armature,



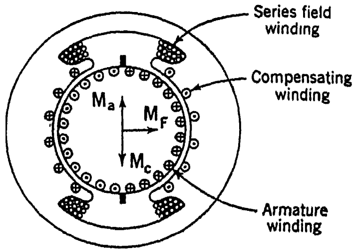


FIG. 336.

and hence tend to maintain the degree of correction established by the winding ratios, regardless of the degree of load under which they are operating. With induced compensation, the result approaches roughly a balance of armature and compensating winding mmf's, the action being that of a short-circuited transformer.

445. Commutation. In the d-c motor, the current in each armature coil reverses from $+I_c$ to $-I_c$ during the time between entering and leaving short circuit under the brushes. This may be of the order of $\frac{1}{200}$ to $\frac{1}{4000}$ sec or less. The local leakage fluxes surrounding each coil reverse with the current, and, although small, when taken in combination with the very rapid rate of change di/dt or $d\phi/dt$, become the source of emf's. They cause sparking when the circuit in which they act is broken.

The motion of these coils, through the flux set up in the commutation zone, generates an emf by speed action. This emf combines with that due to the coil inductance. With brushes in the neutral axis, both of these emf's are in the same direction. Their sum is

$$e_s + L\frac{di}{dt}$$

This emf is consumed mainly between the trailing tips of the commutator bars and brushes. A compensating winding can be provided to reduce

e_s. Full compensation reduces e_s to zero, while over-compensation in addition will neutralize $L(di/dt)$ more or less completely. An interpole serves the same purpose less effectively.

These same phenomena occur in the a-c series motor, along with another factor which tends to cause heavy local short-circuit currents and sparking at the brushes, particularly for zero or low speeds at heavy torques, or when the supply frequency is high (50 or 60 cycles). The field flux is alternating, as are also the current and cross-flux. The alternating field flux induces a transformer emf e_T in the coils, short-circuited by the brushes as in the secondary of a transformer. Such an emf is in lagging time quadrature with the field flux and supply current, while e_s is in time phase and $L(di/dt)$ may be so considered also. (The emf is large when i is large, zero when i is zero, and reverses with it.)

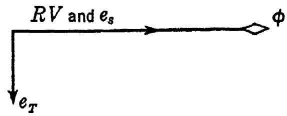


FIG. 337.

For practical purposes, it is customary to assume that e_s and e_T both appear in the proper phase relation at the coil terminals (bars), and to disregard the effect of coil and lead resistances, together with the local coil inductance, on the currents which are built up by e_s and e_T and interrupted by the motion of the commutator. Also, because of analytical difficulties, it is not customary to calculate the instantaneous value of $L(di/dt)$ at the last instant of short circuit, but it is customary to use an average value for the worst coil, called the reactance voltage (RV). When the brush covers more than one segment, a given value of RV causes nearly the same amount of sparking (assuming the same current density and brush-contact area), while the sparking due to e_s or e_T, is more nearly proportional[1] to ne_s or ne_T. The term n represents the number of bars or segments covered by the brush.

In addition, the limiting RV permissible for satisfactory commutation is of the order of 2 or 3 volts, depending on the premises used in the particular derivation; whereas that for ne_s or ne_T is about 7 volts at low speeds. Because of this, some writers limit the resultant of all three acting together.

$$\sqrt{(\pm e_s + RV)^2 + {e_T}^2}$$

This should not exceed 7 volts at low speeds, or 3 volts at high speeds. A preferred method is to weight RV so that:

$$\sqrt{\left(\pm e_s n + \frac{7}{2 \text{ to } 3} RV\right)^2 + (ne_T)^2} \leqq 7 \text{ volts at all speeds}$$

[1] No precise uniformity of calculation, nor recommended values, exists among various writers on this subject. Writers consulted were Marious LaTour, G. Ossanna, F. Punga, Arnold-LaCour, I. Döry, F. Richter, and Behn-Eschenburg.

No commutating poles are used on machines for which the above values are recommended.

Mention should be made of the fact that the current in the coils under the brushes is undergoing not only the decay and reversal process of commutation but also the usual cyclic change of the supply. But, because the period of commutation is so short, the supply current may change only a few per cent (or even less than one per cent) of its cycle, during the time the coil is under the brush. However, the supply being alternating current means that the current might be commutating when it is almost zero, or anywhere on the cycle, including maximum value. But, since the time in the cycle at which a segment commutates is continually changing, it is satisfactory to figure with effective values.

446. Interpoles. Ordinary interpoles or commutating poles, which are effective in improving the commutation of d-c machines, are of little use for a-c series motors. As previously described, when the armature coils move across the field built up by these poles, an emf is generated which will be in time phase with the flux and current. The voltage built up by transformer action in the short-circuited coil undergoing commutation is in lagging quadrature with the field flux. Hence, instead of neutralizing each other, the resultant speed and transformer voltages add in quadrature.

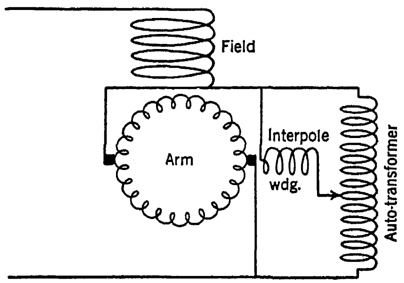


FIG. 338. Connections for the series motor with interpoles.

The proper phase relationship to cause the speed-generated emf and the voltage induced by transformer action to neutralize can be brought about through the use of interpoles excited by an autotransformer connected as shown in Fig. 338. The desirable result is not brought about automatically at each speed or load change. Instead, it is necessary to adjust the interpole leads on the supply autotransformer, varying the interpole strength as required by load and speed.

447. Shunted Interpoles. A second scheme for connecting the interpole winding into the circuit makes use of a shunt as shown in Fig. 339*a*. The non-inductive shunt causes the interpole current and flux to lag behind the current taken by the motor. One component of this flux can be considered which will have the proper phase relation to neutralize the RV; the other component, in lagging quadrature, generates a speed voltage to neutralize the transformer emf (e_T) induced in the short-circuited coils. This is an artificial division of the flux of the interpoles,

and perhaps their successful action can be better understood by the following statements:

(a) The short-circuited coils undergoing commutation have set up in them three components of voltage.

$$RV,\ e_s,\ \text{and}\ e_T$$

(b) They combine vectorially to produce a resultant.

(c) The interpolar flux which can cause e_s can be wholly or partially neutralized by a compensating winding.

(d) The current through the shunted interpole winding can take up such a position that the voltage set up in the commutating coils is oppo-

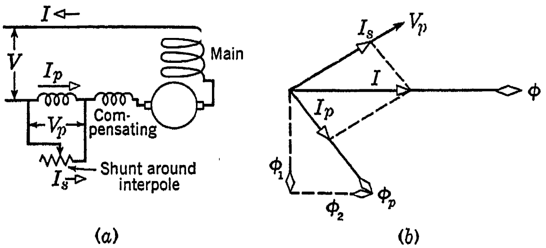


FIG. 339.

site in time phase to the resultant of both RV and e_T. RV can also be neutralized by over-compensation or a series-excited interpole.

This neutralization is good for a fixed speed with various values of currents. If the speed and load must vary over a wide range, the noninductive shunt is adjusted.

Such relatively complex construction is used only on large series motors, usually of the traction type.

448. Doubly-fed Motors. Although there are many modifications of the doubly fed motor, the principal object is to provide a simple means for controlling commutation and compensation over the operating speed range. One type will be described briefly.

A compensating winding is provided in space quadrature with the main winding and connected as shown in Fig. 340. Since the armature and compensating winding are coupled magnetically, the current in C is fixed approximately by that of the armature. Of course, C also carries an exciting current as the primary of a transformer which lags behind V_c by nearly 90°. The resulting flux from V_c also lags behind V_c by approximately a like amount.

In the coil undergoing commutation, one component of voltage is e_T, causing the short-circuited currents. One object of the compensating winding is to build up by speed action the voltage E_s which is nearly

equal and opposite to e_T. Since the speed voltage resulting from the flux of the compensating winding is in phase (or phase opposition) to the flux causing it, e_T and E_s could be opposite except for the pf angle, as indicated on Fig. 340.

Control of the compensating pole flux is brought about by adjusting the taps on the autotransformer. Good commutation can be secured at all but the lowest speeds.

Because additional power is transferred to the armature by induction from the compensating winding, the machine is classified as doubly fed.

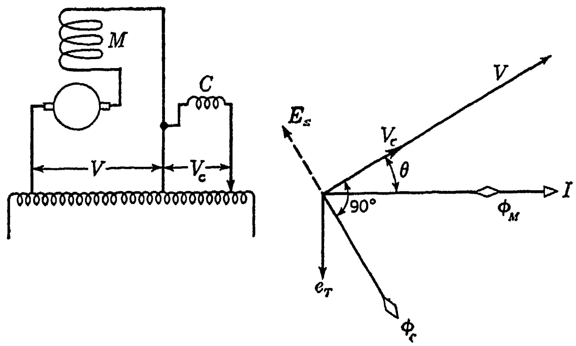


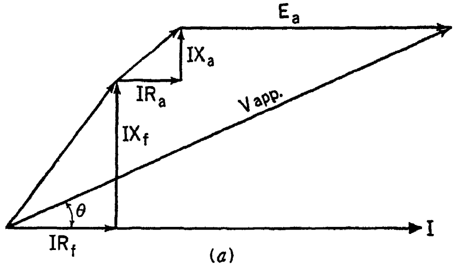
FIG. 340. A doubly fed motor, showing partial vector diagram.

Many modern railway motors use this construction, at least over some part of the operating speed range. The control equipment and connections are usually arranged so that satisfactory commutation is obtained without special attention on the part of the operator.

449. Preventive Leads. An additional scheme for improving commutation, patented by B. G. Lamme, employed resistance leads between armature winding and commutator. These leads were sometimes placed in the armature slots with the winding. Two such leads are in series with respect to the short-circuited commutator currents, since the path for this circuit would involve brush, commutator segments, leads, and coil. With respect to the supply current flowing through the brush, leads, and armature winding, such leads are in parallel.

It is possible to determine a value of lead resistance for which the resultant I^2R losses are a minimum (M. La Tour). Whereas this method of commutation improvement was largely used some years ago, it has fallen into disuse in modern designs, except for some railway motors. At present such leads are thus used to improve commutation during the starting cycle until the voltage generated by the interpole flux is built up as speed is acquired.

450. The Vector Diagram. When the plain series motor is operating, the applied voltage is absorbed in overcoming the resistance and reactance drops of its field and its armature, and a counter emf which varies with the speed. This speed emf may account for 50 to 90 per cent of the applied voltage, depending on size, speed, and frequency. The voltage



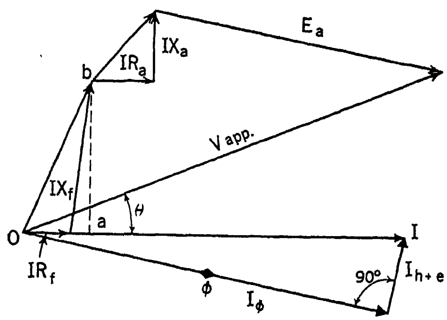


FIG. 341.

components of a plain series motor can be added vectorially as shown in Fig. 341*a*.

The actual current, which is the same in all parts of the motor, is really made up of two components, part of which is required for magnetizing the field and the other, smaller component to supply the iron losses. The flux is in time phase with the magnetizing component of the current, and the counter emf which is also in time phase with the flux is thereby swung through an angle, differing from its position in the ideal diagram. This angle is called the angle of hysteretic advance. The diagram considering these factors is shown in Fig. 341*b*. In actual practice, if the voltages across armature and field and their phase angles with respect to the current are measured, their resultant, and its phase angle, in general will be different from those of the impressed terminal emf. This is caused by the effects of saturation, hysteresis, and commutation.

The addition of a compensating winding gives an effect as shown in Fig. 342. The leakage reactance drops in both armature and compensating windings are still present. The reactance of the armature due to that part of the cross-flux built up by its mmf which would otherwise cross the air gap is reduced or made practically negligible by the compensating winding. Leakage fluxes remain, however.

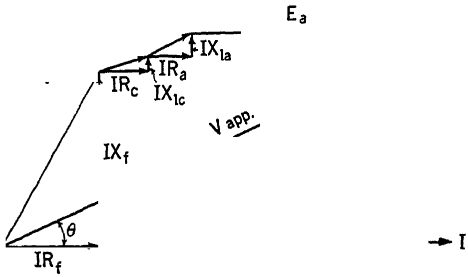


FIG. 342. Vector diagram of a compensated series motor.

IX_{lc} = the leakage reactance drop of the compensating winding.

IX_{la} = the leakage reactance drop of the armature.

451. Losses and Efficiency. The losses occurring in a series a-c motor can be classified as follows:

Copper Losses. An I^2R loss which is readily calculated occurs in each winding, armature, series field, commutating pole winding, and compensating field (if present).

The commutation losses cannot be figured and are usually ignored.

Iron Losses. If the gap and core flux distributions are approximately sinusoidal, the eddy-current loss in the stator core and teeth will be about 0.5 of that in a revolving field of equal density, and the hysteresis loss roughly 0.55. In the armature core and teeth, the eddy-current and hysteresis losses at zero speed will be as mentioned above. When in motion, with the same flux densities, the armature eddy-current losses as corrected above must also be multiplied by 1 + (frequency of rotor)^2/(frequency of supply)^2 and the hysteresis losses by unity below synchronism, and Frequency of rotor/Frequency of supply above synchronism.[2]

[2] A. F. Puchstein and Ivor S. Campbell, "Voltage Relations and Losses in Small Universal Motors," Bulletin 58, 1931.

A. F. Puchstein and E. E. Kimberly, "Universal Electric Motors," Bulletin 53, 1930.

Both published by the Engineering Experiment Station, The Ohio State University, Columbus, Ohio.

Friction and Windage. These losses include bearing friction, brush friction, and windage. At rated speed, together with the core losses, they use up from one-quarter to one-half of the total input on most series motors of fractional-horsepower rating.

452. Factors Influencing Construction. The various items discussed in the preceding pages point toward certain limitations and requirements in the design and construction of series motors.

The field poles are short and the field turns are comparatively few. The air gap is made shorter than in d-c motors but longer than in ordinary induction motors. The presence of compensating winding requires slots in the pole faces. This winding may have from 60 to 100 per cent as many ampere turns per pole as the armature and may require more copper than the main field winding. Both the armature and field cores must be laminated.

Because the voltage induced in the coils undergoing commutation is influenced by the number of turns per coil it is important that the number be kept down to secure good commutation. On the other hand, it is important that the number of armature conductors be great enough to secure the required torque. The method of satisfying these conflicting requirements is to use fewer turns per coil but more coils and hence many commutator segments, particularly in larger motors. The commutator diameters and rotative speeds are larger than in d-c practice.

Mention has been made of the difficulties involved in the design of 60-cycle series motors. Their design for application to high-voltage service (meaning above 220 volts), at low speeds, has not been satisfactorily worked out. Suppose a 60-cycle motor were to be used on 220 volts. As compared with the 110-volt motor of the same horsepower rating, the number of turns on the armature and field would have to be doubled. As the reactance varies with N_c^2, it will be increased four times. The current would be one-half of its previous value, and the IX drop will be doubled. The percentage IX drop will remain the same.

But on account of the doubling of the number of turns, the voltages in the commutating coils causing sparking are greatly increased. This can be corrected by increasing the number of commutator bars up to certain limits of commutator size and peripheral speed and by reducing the circumferential thickness of the brushes. An increase in voltage and size up to the satisfactory operating values for railway work cannot be provided on 60-cycle machines. Any reduction in frequency improves the commutation, and so all large a-c series motors are operated from 15- to 25-cycle sources and at a suitable voltage (equal to or less than 300) supplied from a tapped transformer.

As compared with d-c series motors the following points of contrast can be noted:

The a-c motor requires:

Larger diameter and length of the armature for the same speed or else a higher speed at the same output.

Larger commutator, both in length and diameter.

Compensating windings (in all except small sizes).

Shorter air gap.

More material for the same output and speed.

453. Prediction of Performance. There is no known method of calculating the performance of an a-c series motor with accuracy. Thus even though significant constants be obtained by calculation or test, the operating characteristics cannot yet be calculated with satisfactory precision. Commutation, the non-sinusoidal nature of the components, and the effects of saturation are all factors which disturb the accuracy of calculation.

Design of new motors with various performance characteristics is accomplished largely by experimental work and by the use of data obtained from past designs and tests.

CHAPTER LVI

TESTS ON SMALL SERIES MOTORS. CALCULATIONS

454. Chapter Outline.

Tests on Small Series Motors.[1]
Calculation of Performance Items.
Example.

455. Small Motor Tests. In dealing with equipment of large capacity, methods of predicting the characteristics from constants derived by calculation are of great importance because such methods save the trouble and cost of rebuilding or doing the work over. In testing small motors, several trial samples can readily be built and tested. Also, the amount of power involved is not so great as to prevent direct-loading methods from being used. For both series and repulsion motors it is desirable to make load runs with metered input and output. The test values which are obtained are commonly used to check the estimated performance. The pages which follow will deal chiefly with motors of small capacity, and the material will be presented mostly from this point of view.

Tests on series motors are used to check such items as performance by direct loading (brake), rotation, torque, commutation, distribution of potential drop around the commutator, insulation, winding connections, brush lead, mechanical balance, brush wear, temperature rise, faults, defects, and manufacturing errors.

456. Test Runs on Small Series Motors. Ordinary testing methods by which data can be obtained to determine the characteristics of motors often require considerable refinement when applied to series motors of fractional-horsepower capacity. The following paragraphs describe briefly a suggested test method and give examples of calculations and results.

The motor tested was of the 115-volt, 60-cycle, 2-pole, universal type with a full-load speed of 5550 rpm and a nominal rating of $\frac{1}{8}$ hp. Load was supplied by a brake, the torque being read on a chemical balance. (The string brake and the dynamometer are much used in practice.) Speed was read by a neon stroboscope, as any attached speed-indicating device would furnish an appreciable load to the motor.

[1] See the Ohio State Engineering Experiment Station Bulletin 53, previously cited.

In making such tests, it is sometimes desirable that the power and the voltage be read across both the armature and the field. As volt-

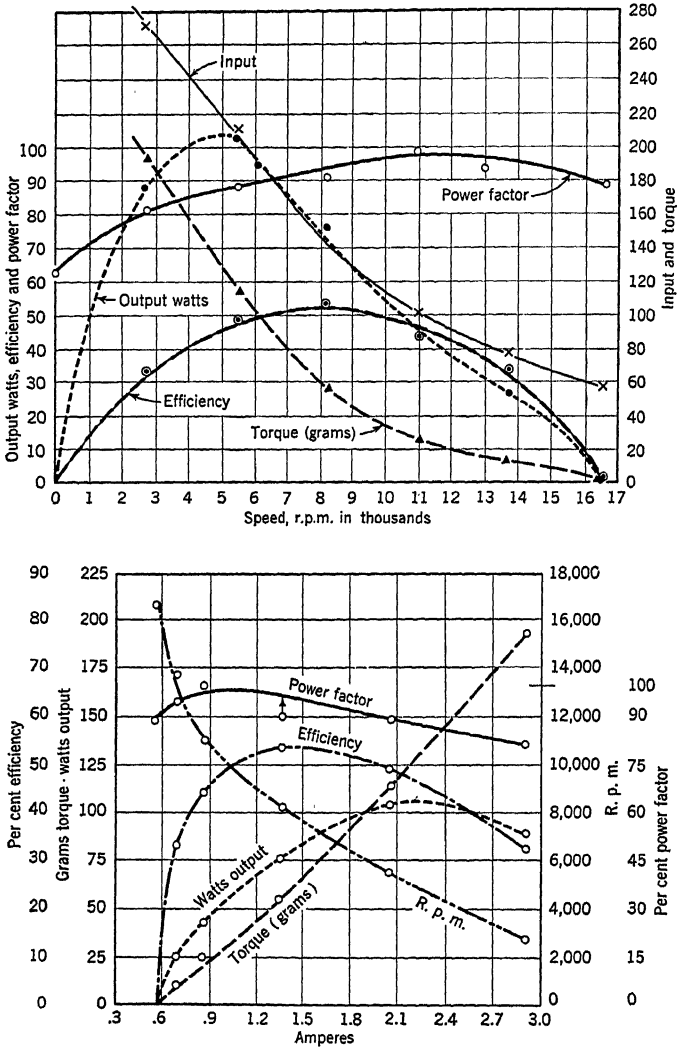


FIG. 343. Test curves on a small series motor.

meters, or voltage elements of wattmeters, provide a parallel path through which a current can flow, a considerable error is introduced unless compensating paths are provided. That is, in measuring the voltage drop across the series field, some current is by-passed around the

field so that the value and phase position of the currents and the emf distribution between armature and field are changed, unless proper precautions are taken. A high-resistance shunt around the armature, taking the same current as the voltmeter, or the use of an a-c potentiometer eliminates such error. Such a refinement is necessary only on very small motors. One set of test readings follows:

Data at rated speed:

Rpm: 5500
Volts applied: 115
Ampere input: 1.85
Frequency of supply: 60 cycles
Watt input to motor: 188
Volts across armature: 68
Watt input to armature: 119
Volt input to field: 68
Watt input to field: 58
Force on brake arm = 113 grams = 0.254 lb = 4.58 oz
Radius of brake arm = 0.533 ft = 16.25 cm
Torque = $0.533 \times 4.58 = 2.44$ oz-ft
Resistance of the armature = 6.40 ohms
Resistance of the field = 9.37 ohms

Starting conditions:

Starting current: 3.25 to 3.40 amperes, depending upon the position of the armature
Starting watts: 223–266
Terminal volts: 115

Partial design data on this motor:

Armature conductors = 1344
Area of armature conductors = 202 cir mils
Field turns per pole = 150
Area of field conductors = 404 cir mils
Number of armature slots = 12
Diameter of armature = 1.51 in.
Length of armature = 1.95 in.
Pole arc = 1.73 in.

$$\frac{\text{Pole arc}}{\text{Pole pitch}} = 0.73$$

Air-gap length = 0.020 in.

457. Calculation of Results. These data can be used to determine the efficiency, the losses, and the values for the vector diagram. In the latter case, the resultant of E_a and E_f in general does not exactly equal V because of the disturbing effect of harmonics.

(*a*) The in-phase component of the voltage drop across the armature during operation is made up of the counter emf (E_a) and the IR drop. Then

$$E_a = \frac{\text{armature watts input}}{\text{armature amperes}} - \text{armature } IR \text{ drop}$$

From the test readings:

$$E_a = \frac{118}{1.85} - 1.85 \times 6.40 \quad \text{or} \quad 52.2 \text{ volts}$$

(*b*) The power factors:

Of the entire motor:

$$\text{Pf}_m = \frac{W_m}{VI} = \frac{188}{115 \times 1.85} \quad \text{or} \quad 0.885$$

Of the armature:

$$\text{Pf}_a = \frac{W_a}{E_{\text{armature}}I} = \frac{119}{68 \times 1.85} \quad \text{or} \quad 0.945$$

Of the field:

$$\text{Pf}_f = \frac{W_f}{E_f I} = \frac{58}{68 \times 1.85} \quad \text{or} \quad 0.460$$

(*c*) The copper losses are:

$$I^2R_a = 1.85^2 \times 6.40 = 21.9 \text{ watts}$$

$$I^2R_f = 1.85^2 \times 9.37 = 32.1 \text{ watts}$$

(*d*) Output from brake reading in watts = 104.
(*e*) Input from meter reading in watts = 188.
(*f*) Losses:

$$188 - 104 = 84 \text{ watts}$$

Copper losses = 21.9 + 32.1 or 54 watts (Item *b*)
Friction, windage, and core losses = 84 − 54 or 30 watts

(*g*) Efficiency:

$$\frac{\text{Output}}{\text{Input}} 100 = \frac{104}{188} 100 \quad \text{or} \quad 55.4 \text{ per cent}$$

The vector diagram is shown in Fig. 344. By taking the current and applied voltage at the angle arc cos 0.885, obtained from item *b*, a vector diagram is obtained as indicated by Oa and OI. The component voltage drops are plotted at the angles indicated by the respective pf's of the armature and field. Thus:

$$\sqrt{(IR_f)^2 + (IX_f)^2} = \text{drop across field or 68 volts}$$

$$\sqrt{(IR_a + E_a)^2 + (IX_a)^2} = \text{drop across armature or 68 volts}$$

The vector sum of these drops indicates that the applied voltage, instead of being 115, would have to be 127 volts at an angle of θ' with the

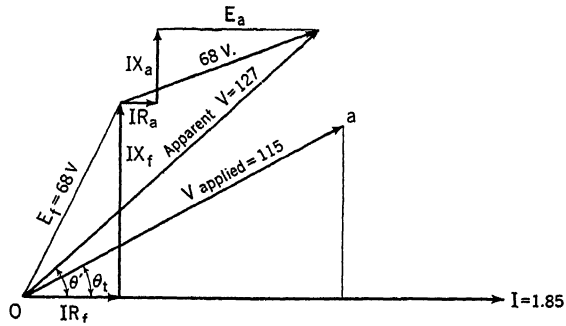


FIG. 344.

current and with a motor power factor of 0.74. In short, the vector diagram of the motor obtained from total voltage and current is only roughly in agreement with that indicated by the vector sum of the component parts. This is a frequent phenomenon in small series motors. It arises from lack of consideration of the core loss and saturation effects, tooth and commutation ripples, etc., which tend to produce the non-sinusoidal waves of current and voltage actually encountered in such machines.[2]

[2] For a discussion of non-closure of vector diagrams, see: A. F. Puchstein and Ivor S. Campbell, "Voltage Relations and Losses in Small Universal Motors," The Engineering Experiment Station, Bulletin 58, Ohio State University, Columbus, Ohio.

REPULSION MOTORS

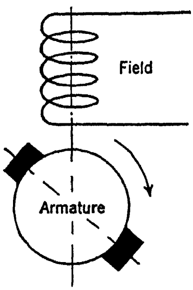
CHAPTER LVII

THEORY AND CHARACTERISTICS

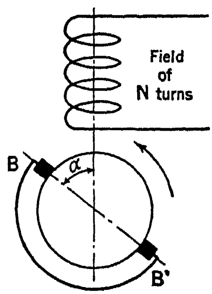
458. Chapter Outline.

Repulsion-motor Types.[1]
Theory of Operation.
Characteristics.

459. Introduction. The plain repulsion motor was developed about the year 1887, by Elihu Thomson in the United States, and by Atkinson in England (*circa* 1898). Thomson's machine was made originally as



(*a*)
Thomson's original repulsion motor.



(*b*)
Modern type of Thomson repulsion motor.

FIG. 345.

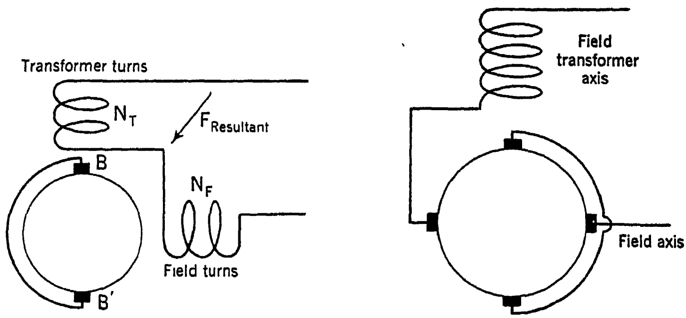
shown in Fig. 345*a*. The field winding was excited directly from the source, but the armature was excited inductively. No connection was made from positive to negative brushes, but the brushes were exceptionally wide. The direction of rotation was opposite to that of brush shift.

[1] In this treatment we will confine ourselves almost wholly to methods for making analyses of repulsion-motor action. So many modifications of repulsion-motor types have been developed that to attempt their description in this volume would be impracticable. A bibliography is given at the end of this section which will acquaint the reader with various types and developments.

This machine was superseded by the type shown in Fig. 345*b*, with more narrow, shifting brushes, which were short-circuited.

Atkinson's motor was built as shown in Fig. 346*a*. It was provided with two sets of field windings in space quadrature. Electrically this motor is equivalent to that of Fig. 345*b* except that the former displays somewhat higher copper losses in the stator, and more field copper is required in a given machine. These features militate against its present use.

The machine shown schematically in Fig. 346*b* was invented simultaneously by M. LaTour in France and Winter and Eichberg in Germany.



The Atkinson repulsion motor. Another type.

FIG. 346.

Electrically this machine has properties similar to those of the machines of Figs. 345*b* and 346*a*, except for improved pf. At synchronous speed the pf of this motor is unity. At one time this type of motor was much used for a-c railway electrification in Europe, but it has now been superseded by the series motor. As it requires four brushes per pole pair, it is not used for small motors.

Several other types of repulsion motors have been developed as modifications of these forms.

460. Simplified Statement of Repulsion-motor Action. In the series motor the armature current flows by conduction from the source. In the repulsion motor the armature is excited inductively by transformer action. If the brushes were in line with the poles, and connected to each other, the armature would act as a short-circuited transformer when excited by the primary or stator. No torque would result from this. By shifting the brushes slightly from this position, part of the field acts by transformer action to produce current in the armature. This armature current reacts with the rest of the field to produce torque in the ordinary manner.

Just as two sine waves combine to produce a resultant sine wave, the flux built up in the repulsion motor, if assumed to be sinusoidal in space, can be split up into two sine-wave components at right angles. The component in line with the brush axis is the *transformer component;* the one in quadrature is called the *field component.* These two components vary relatively with the brush shift. If the windings are not distributed in such a way as to produce a sinusoidal flux the analysis will be slightly in error. In many cases a *triangular* distribution in space is

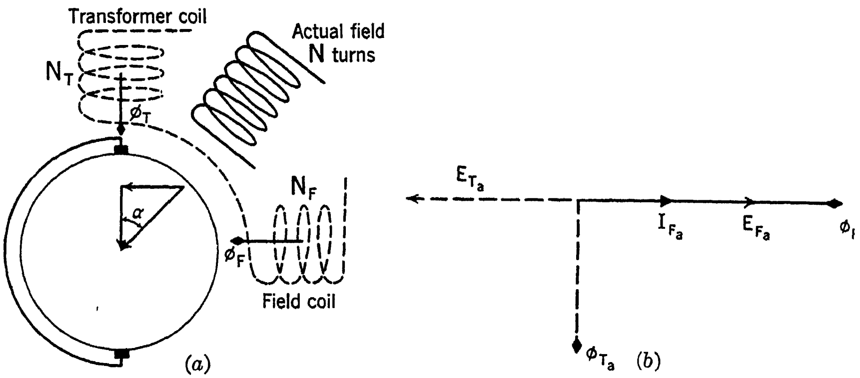


FIG. 347.

brought about; this can be analysed by a different division of the fields into their imaginary components.

Refer to Fig. 346*a*. The two coils yield a resultant mmf acting along an axis between the coils at an angle A with the brush shift. Hence the single winding N in Fig. 345*b*, with its turns distributed sinusoidally, is equivalent to the two coils N_F and N_T in Fig. 346*a*. That is, we can imagine the actual turns N replaced by two fictitious coils as shown in Fig. 347.

$$N_T = N \cos A \qquad [686]$$

$$N_F = N \sin A \qquad [687]$$

A current flowing in the stator winding sets up a flux, one component of which, ϕ_F, acts the same as the air-gap flux in a series motor. This flux links with no closed circuit except the coils short-circuited by the brushes. Neglecting this factor, the field turns N_F give only the effect of self-inductance, similar to that of any field coil. The voltage drop across this coil is therefore treated as an IX drop.

The stator current flowing through the other coil induces a current in the armature by transformer action. The armature winding is a closed circuit in that axis. The resulting armature current built up by

transformer action produces the torque as in the series motor. The difference here, of course, lies in the fact that the armature current producing torque results from induction; it depends upon the "transformer turns."

Although the turns of the N_T component are usually several times greater than those of the N_F component (the ratio depends upon the brush angle), the locus of the rotating flux vector is nearly circular at synchronism. This does not imply, however, that the motor will run with no load at a speed near synchronism. Ordinarily, the no-load speed is from 1.25 to 2.50 times synchronous speed, being higher in those machines designed for high flux densities. Because of the rotating flux, non-salient poles are nearly always used on repulsion motors.

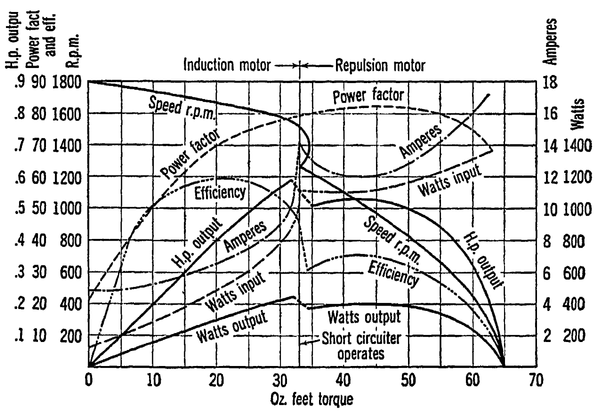


FIG. 348. Repulsion-start induction motor. Brush lead of 14 electrical degrees.

461. Performance Characteristics. The revolving field is very favorable to commutation since operation near synchronous speed gives commutation nearly as good as that found in d-c machines with the same current per brush. At starting, however, the commutation has the same poor characteristics found in the a-c series motor, and special care is necessary to prevent unduly severe sparking. At intermediate speeds the commutation is fairly satisfactory, but it becomes rapidly worse as the speed rises above synchronism.

One of the factors which modifies the performance and makes accurate prediction of the behavior at any given speed difficult arises from the coils short-circuited by the brushes. These coils produce a torque opposite to that of the remainder of the winding, but the action on the whole is to improve pf and efficiency at speeds below synchronism and to reduce them seriously at speeds above synchronism. It is this negative

torque and other rotational losses which keep the idle speed of the motor below the "run-away" speed of the series machine.

At about 75 to 100 per cent of synchronous speed the efficiency of a repulsion motor will be about the same as that of an induction motor of the same rating. Efficiency is greatly reduced by the increased losses

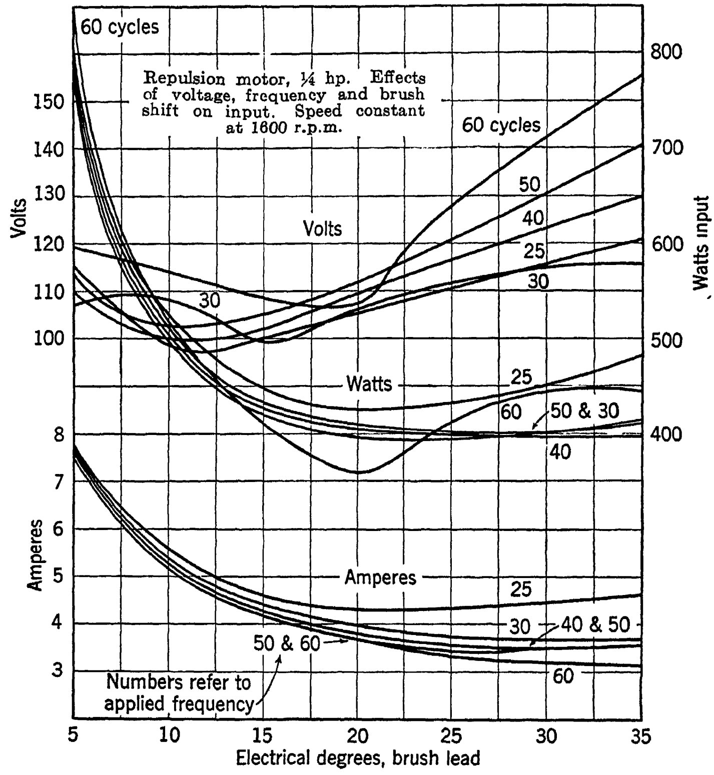


Fig. 349.

above synchronism. Because of the better performance below synchronous speeds, the repulsion motor is better suited to high-frequency than low-frequency operation. This is exactly opposite from the behavior of the series motor.

Figures 348, 349, and 350 show characteristic curves for the repulsion motor. If the brushes are shifted from 0 to 90 electrical degrees, the winding currents at zero speed will pass from the maximum possible short-circuit value progressively through smaller and smaller values. At 90° the stator current is merely the minimum possible exciting current. The armature current is zero. The torque passes progressively

from zero to a maximum and reduces again to zero for 90° shift. Beyond 90° the motor torque reverses. The normal brush angle is from 10 to 25 electrical degrees. If, for any selected speeds, readings are taken of

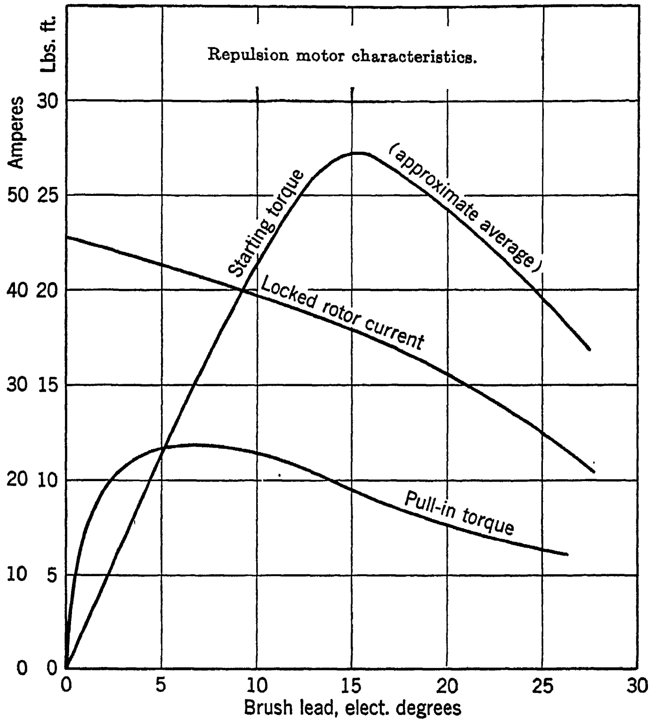


FIG. 350.

varying torque and current, versus the brush position, curves of similar shape are obtained.

The relation between brush shift and starting torque is shown in Fig. 350. This torque changes considerably for different positions of the armature. The change depends upon the number of commutator bars and rotor slots.

CHAPTER LVIII

TESTS ON REPULSION MOTORS. CALCULATIONS

462. Chapter Outline.

Repulsion-motor Tests.
- Blocked Rotor.
- Open Rotor.
- Resistance Measurements.

463. Testing Repulsion Motors. Before going any further into the theory of repulsion motors and their analyses, a method of obtaining test results will be shown.

Load Tests. The repulsion motor may be loaded by means of a brake. The brake reading and speed then indicate the output, and the meters in the supply lines measure the input. The readings usually obtained are:

Volts applied	Ampere input	Watt input
Torque	Rpm	Resistance measurements

464. No-load Tests. Several test runs are usually made to obtain the motor constants needed in checking its performance. Ordinarily, the motor constants are figured from the design data. These constants are then used to predict the operating characteristics. Test runs indicate the accuracy of the constants derived from design and the accuracy of the theoretical equations by which the performance has been checked.

Blocked-rotor Tests. With the brushes in their normal position, rated voltage is applied to the stator at rated frequency and the following readings are taken with the rotor blocked.

Volts applied	Amperes input	Watts input
Rotor amperes, when possible		Torque

Usually a series of such readings is taken for different rotor positions, observing the maximum and minimum values. When the information so obtained is needed, the test is then repeated with the brushes lifted and with the commutator wrapped with copper wire in order to short-circuit it. The machine is practically a single-phase induction motor during this test.

Open Rotor. The brushes are lifted and rated voltage at rated frequency is applied to the stator terminals. The values read are:

Volts applied Ampere input Watt input

Instead of depending upon one set of readings it is sometimes desirable to take a series of them with various values of applied voltage.

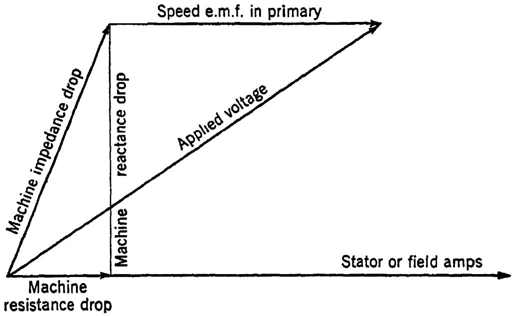


FIG. 351. The simple vector diagram of the ideal repulsion motor.

Resistance Measurements: Stator. The resistance of the stator winding should be measured by direct current and the approximate temperature at measurement should be recorded.

Rotor Resistance. The measurement of the resistance of the rotor frequently requires a knowledge of its connections. In a 4-pole motor, the

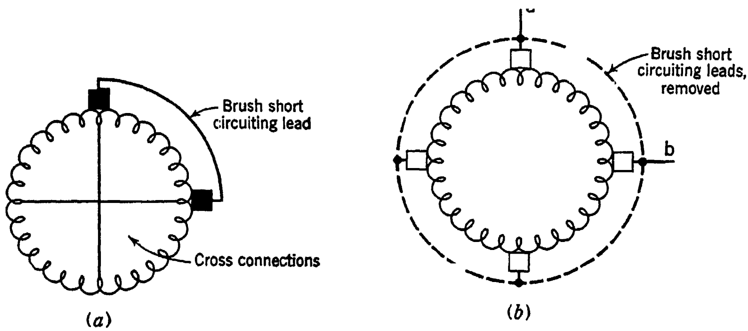


FIG. 352.

number of armature paths may be two or four, depending upon the type of winding. In such a motor, the presence of two brushes 90 mechanical degrees apart does not necessarily imply a wave winding as in the ordinary d-c machine. Lap windings with cross-connections give the effect of four brushes as indicated in Fig. 352*a*.

If all cross-connections between bars are absent and four brushes are used, the measurements made between points 90 mechanical degrees apart represent three-fourths of the resistance of one path. The resistance indicated between points a and b, Fig. 352b, is

$$R_{ab} = \frac{1}{\frac{1}{r} + \frac{1}{3r}} = \tfrac{3}{4}r \qquad [688]$$

where r equals the resistance of one path (one-fourth of the armature) only.

Hence

$$R_{ab} \text{ (as measured)} \times \tfrac{4}{3} = r \qquad [689]$$

Total armature resistance of the four paths in parallel (the condition when brushes are short-circuited) is then

$$R_{ab} \times \tfrac{4}{3} \times \tfrac{1}{4} = \tfrac{1}{3}R_{ab}$$

Such analyses must be made for other types of connections and numbers of brushes as well.

465. Test Results. The motor on which a load test was made displayed the following values:

Blocked rotor:

As repulsion motor, brushes in normal (18° shift) position.

V	I	W
40	3.6	90
110	12.4	924

As induction motor, brushes lifted, commutator segments shorted:

V	I	W
40	5.85	118

Open rotor, brushes lifted:

V	I	W
130	3.32	40
110	2.41	27
95	1.90	20

Resistances:

Stator: 1.22 ohms (cold); 1.38 ohms (hot).

Rotor: Resistance between two brushes 90 mechanical degrees apart.

Commutator bars are cross-connected. Number of bars: 48.

Resistance between bars 1 and 13: 0.87 ohm (cold); 0.98 ohm (hot).

Resistance of brush connection equals 0.08 ohm.

Calculations. A few typical calculations will be shown using the test results.

Magnetizing reactance. Neglecting the core loss and primary resistance loss an approximate value of primary reactance is

$$X_0 = \frac{110}{2.41} \quad \text{or} \quad 45.6 \text{ ohms} \qquad [690]$$

Equivalent impedance. Short-circuit data:

$$Z_e = \frac{V}{I} \qquad [691]$$

$$= \frac{40}{5.85} \quad \text{or} \quad 6.85 \text{ ohms}$$

Equivalent resistance:

$$R_e = \frac{W}{I^2} \qquad [692]$$

$$= \frac{118}{5.85^2} \quad \text{or} \quad 3.44 \text{ ohms}$$

Equivalent reactance:

$$X_e = \sqrt{Z_e^2 - R_e^2} \quad \text{or} \quad 5.93 \text{ ohms} \qquad [693]$$

Assigning one-half of this to the stator or primary gives

$$x_1 = \frac{5.93}{2} \quad \text{or} \quad 2.965 \text{ ohms}$$

466. Other Tests. In addition to the common tests already listed, a number of special tests are sometimes made on repulsion motors.

Short Circuit of Armature Coils. If the rotor is held stationary and the stator is excited it will be noticed that there is some tendency for the rotor to lock in certain positions. The presence of a short circuit between armature coils is indicated by such a locking tendency if the brushes are removed. Such faults also result in noise.

Commutation Tests. The armature current at starting can usually be measured only by cutting in to the winding. This is especially true of those small repulsion motors in which the brushes are not insulated from

the frame and the current path cannot otherwise be interrupted for the insertion of an ammeter.

The transformer emf occurring in the coils under the brushes can be measured by exciting the stator and holding the rotor stationary. If a brush is lifted and contacts are made to the commutator segments, one brush width apart, the voltage between contacts represents the transformer emf. This should be found for various positions of the armature.

Voltage distribution around the armature can also be read by means of such contacts on the commutator.

CHAPTER LIX

METHODS OF ANALYSIS

467. Chapter Outline.

Methods of Analysis.
  Factors Affecting Accuracy.
The Two-transformer Theory.
  Example of Calculations.
  Comparison of Results.

468. General Statement Regarding the Analyses of Repulsion-motor Action. There is no simple, accurate method for predicting quantitatively the characteristics of the repulsion motor. A number of methods are available, some very elaborate and unwieldy, but their authors [1] carefully indicate that great quantitative accuracy cannot be expected from their use. They are, however, sufficient for industrial requirements if the calculated results are checked by tests and needed adjustments are made.

The following factors make it difficult to obtain a high degree of accuracy in calculation:

(*a*) With the usual conductor distribution on the stator, the gap-flux distributions are neither sinusoidal nor triangular. One or the other is usually assumed in making the calculations, however.

(*b*) Stator and rotor conductor distributions are different, and they move with respect to each other, thereby changing their relative coupling.

(*c*) Magnetic saturation changes the relative flux distribution and the value of air-gap flux throughout the cycle.

(*d*) Commutation adds certain effects which improve the performance at speeds below synchronism and impair it above synchronism. Only the West analysis considers the effect of the short-circuited coils undergoing commutation as an integral part of the calculation process, although Punga's semigraphical method applies a correction.

(*e*) Differential magnetic leakage gives effects which may be, but usually are not, calculated.

[1] See: Steinmetz, "Theory and Calculation of Electrical Apparatus," pp. 327, 397.
Behrend, "The Induction Motor," p. 233.
Arnold and La Cour, "Wechselstromtechnik," Vol. V, Part 2, p. 376.

Simple Method of Attack. *a.* The actual stator winding is replaced by two windings in space quadrature as explained in Chapter LVII. These two windings have the turns $N \cos A$ and $N \sin A$. The assumption is made that the flux distribution in the air gap is sinusoidal. This assumption is much used.

b. The actual turns are replaced by two imaginary windings in space quadrature having the turns

$$N(1 - 1.5\lambda^2 + 0.5\lambda^3) \qquad [694]$$

and

$$N(2\lambda - \lambda^2) \qquad [695]$$

$$\text{where } \lambda = \frac{A^\circ}{90^\circ} \qquad [696]$$

$$= 0.2 \text{ for motors with } 18^\circ \text{ brush shift}$$

This method applies when the stator winding is uniformly distributed. It assumes that the flux distribution in the air gap is triangular, a true condition when all slots are filled.[2]

On any of these methods, special corrections can be introduced, such as saturation factors to cover the variation in saturation with flux density, the corrections of Steinmetz and Punga for commutation effects, the correction of Arnold for differential leakage, etc. These are all too highly specialized to be included here. In spite of these efforts no known method enables the designer to predict the repulsion-motor characteristics with the same accuracy and certainty possible with induction motors, excluding, of course, the (as yet) indeterminable characteristics in the latter of vibration, noise, and "crawling."

469. Ratios of Transformation. Because the resistance of the repulsion-motor armature winding may be measured directly, it will be necessary to convert it to stator terms for use in parts of the analyses. Also, if the leakage reactance of the entire motor is measured and a part assigned to the rotor, the reactance will be in stator terms. It may be required in its own terms as well. To consider these facts, it is necessary to obtain the ratio of transformation a between rotor and stator, taking account of the two fictitious parts of the stator windings. Formulas will be given, and will be applied at once to a repulsion-motor example.

The motor to be considered is rated at $\frac{1}{3}$ hp at 2200 rpm, 4 poles, 60 cycles, 110 volts, and has the following windings:

$$\text{Conductors on the stator} = 1308$$

$$\text{Stator paths } m_1 = 2$$

[2] See any text on armature reaction in d-c machines.

$$\text{Series conductors on the stator } C = 654$$

$$\text{Winding factor } k_p k_d = 0.788$$

$$\text{Rotor conductors } Z_a = 1152$$

$$\text{Rotor paths } m = 4$$

$$\text{Series conductors on the rotor} = 288$$

$$\text{Pitch factor of the rotor winding } k_p = 0.966$$

Ratios.

$$a = \frac{\text{effective series turns on stator}}{\text{effective series turns on armature}}$$

$$\text{Effective series } \textit{turns} \text{ on stator} = \frac{C}{2} \times k_p k_d$$

$$= \frac{654}{2} \times 0.788 \quad \text{or} \quad 257$$

$$\text{Effective series } \textit{turns} \text{ on armature} = \frac{Z_a}{2m} \times \frac{2}{\pi} \times k_p$$

$$= \frac{1152}{2 \times 4} \times \frac{2}{\pi} \times 0.966 \quad \text{or} \quad 88.6$$

(The factor $\frac{2}{\pi}$ is the distribution factor for sinusoidal flux distribution.)

$$a = \frac{257}{88.6} \quad \text{or} \quad 2.91$$

Component turns. Mention has been made of the fact that the space distribution of the stator and rotor windings can be considered as sinusoidal or triangular. (Arnold gives trapezoidal cases also.) The choice is not an arbitrary one, depending (especially on the stator) on the actual winding distribution. Single-phase concentric windings used on stators can be so distributed as to result in sine waves with all but two harmonics eliminated.[3]

In these analyses, sinusoidal distribution will be assumed, the correction otherwise is not often worth while, considering the comparative errors introduced in the calculations by commutation and saturation. However, coefficients, attributed to Punga, will be shown for the triangular case.

[3] W. R. Appleman, "The Cause and Elimination of Noise in Small Motors," *Elec. Eng.*, November, 1937.

(*a*) Sine waves assumed:

Effective series *turns* on stator, transformer axis:

$$\frac{Ck_pk_d}{2}\cos A = 257 \times 0.951 \quad \text{or} \quad 244.5$$

where:

$$\begin{aligned} A &= \text{angle of brush shift from neutral} \\ &= 18^\circ \text{ in the example being considered} \end{aligned}$$

Effective series *turns* on stator, field axis:

$$\frac{Ck_pk_d}{2}\sin A = 257 \times 0.309 \quad \text{or} \quad 79.1$$

Resulting ratios:

$$\text{Transformer axis to armature} = a\cos A = 2.77$$

$$\text{Field axis to armature} = a\sin A = 0.90$$

(*b*) Triangular distribution assumed:

$$\text{Effective series conductors of armature} = \frac{Z_a}{m}\left(\frac{2}{3}\right)k_p = 185$$

$$\text{Effective series turns of armature} = \frac{Z_a}{3m}k_p = 92.5$$

$$\text{Effective series turns of stator} = \frac{C}{2}\cdot\frac{2}{3} = 218$$

Transformer axis coefficient (in place of $\cos A$)

$$= (1 - 1.5\lambda^2 + 0.5\lambda^3) = 0.944$$

Field axis turn coefficient $= 2\lambda - \lambda^2 = 0.36$

where

$$\lambda = \frac{A^\circ}{90^\circ} \quad \text{or} \quad 0.2 \text{ for } 18^\circ \quad \text{(See equation 696.)}$$

Ratio, stator to armature $= a = 2.35$

470. Other Motor Constants. To complete the data on the motor to be used for illustrating the theory, the following values are required:

$r_1 = 1.34$ ohms $\quad r_a = 0.357$ ohm $\quad x_1 = 2.9$ ohms

r_2 (in stator terms) $= r_a a^2 = 3.0$ ohms $\quad x_2 = 2.2$ ohms

$$x_a = \frac{2.2}{a^2} = 0.261 \text{ ohm in rotor terms}$$

$X'_0 = X_m + x_1 = 51.9$ $\qquad$ $X''_0 = X_m + x_2 = 51.2$

$X_0 = 51.5$ (as an average) $\qquad$ Core loss = 40 watts

Friction and windage = 15 watts at 1750 rpm

471. Method of Analysis. The method of calculation which follows will deal with the transformer or generator actions occurring in the field and in the transformer axes. Because of their interrelationships, it is impossible to fix values for one and then the other, to yield a final

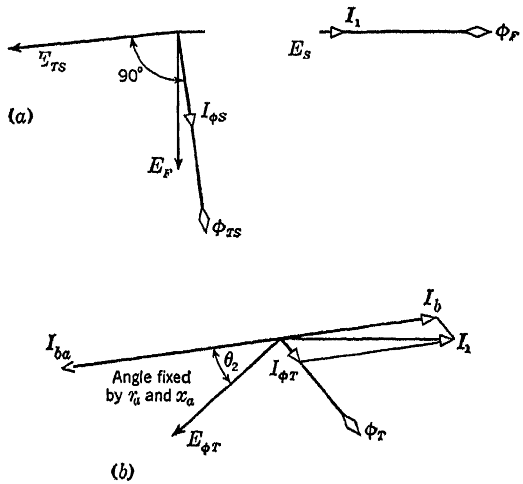


FIG. 353a. The vector relationships resulting from the field axis. (b) The vectors of the transformer axis.

solution. Our method will be to develop, step by step, the equations pertaining to each axis. Once the method is made clear, useful combinations of terms will greatly simplify the solution.

Sine waves will be assumed in both time and space. Commutation and saturation effects will be neglected.

A general statement of the actions follows:

(a) The component of stator turns, known as the field, builds up a flux ϕ_F.

(b) When running, the armature conductors cut this flux, generating in them a speed emf, E_S.

(c) E_S causes a current to flow in the armature which acts as a magnetizing current $I_{\phi S}$.

(*d*) $I_{\phi S}$ builds up an alternating flux ϕ_{TS} which induces a counter emf in the transformer axis of the stator.

(*e*) The same stator current, flowing through the transformer-axis turns, sets up a flux ϕ_T which induces an armature voltage $E_{\phi T}$ by transformer action. (See Fig. 353.)

(*f*) In the transformer axis, the armature is short-circuited, and the resulting short-circuit current I_{ba} (caused by $E_{\phi T}$) is limited by the local rotor impedance.

(*g*) As in any short-circuited transformer, the balancing component of current needed in the primary, plus the magnetizing component of the primary current, equals the total current. In this case the current drawn from the supply separates into these two components. (See Fig. 353*b*.)

(*h*) Torque results from the interaction of the in-phase components of the field flux ϕ_F and the current I_{ba}.

A negative torque results from the interaction of this field flux and the magnetizing current resulting from speed action $I_{\phi S}$. Or, in a sense, $I_{\phi S}$ causes core and copper losses which subtract from the useful output.

These values will be calculated for an assumed speed of 1200 rpm. They will be obtained in their own terms and then transferred to the stator if necessary.

(*a*)
$$S = \frac{\text{actual rpm}}{\text{synchronous rpm}} = 0.666 \qquad [697]$$

(*b*) To evaluate ϕ_F:

$$E = 4.44 f N \phi_m 10^{-8} = I_\phi X_m \qquad [698]$$

Then the flux per ampere:

$$\frac{\phi}{I} = \frac{X_m 10^8}{4.44 \times 60 \times 257} \quad \text{or} \quad 71{,}500 \text{ lines per pole}$$

$$\phi_F = \text{flux per ampere} \times (\text{stator current}) \times \sin A$$

$$= 71{,}500 \times I_1 \times 0.309 \quad \text{or} \quad 22{,}100 \times I_1 \text{ lines per pole}$$

ϕ_F will be used as the reference vector; it is in time phase with I_1.

(*c*) Rotation of the armature conductors through this field flux generates a speed voltage which, in effective values, will be

$$E_s = \frac{\text{poles} \times \phi_F \times \text{rpm} \times Z_a \times k_p}{\sqrt{2} \times m \times 60} \times 10^{-8} \qquad [699]$$

$$= \frac{4 \times (22{,}100 \times I_1) \times 1200 \times 1152 \times 0.966}{\sqrt{2} \times 4 \times 60} \times 10^{-8} \quad \text{or} \quad 3.47 I_1$$

Note that we must continue to carry the term I_1 since it determines the flux and is still unknown. This voltage E_S is in armature terms and will be in time phase with the flux causing it.

(*d*) The magnetizing reactance X_m is 49.0 ohms in stator terms, or $49/a^2 = 5.8$ ohms in rotor terms. This is X_{ma}. This reactance plus the armature leakage reactance and winding resistance forms the impedance of the armature to the speed emf.

$$I_{\phi S} = \frac{E_S}{r_a + j(x_a + X_{ma})} \qquad [700]$$

$$\frac{3.47 I_1}{0.357 + j6.061} \quad \text{or} \quad (0.0335 - j0.57)I_1$$

(*e*) The above current sets up a flux designated as ϕ_{TS}, that is, in the transformer axis but resulting from speed. Again a counter emf equals $I_\phi X_m$ or $4.44 f N \phi_m 10^{-8}$ in general, and applying this relationship:

$$\phi_{TS} = \frac{I_{\phi S} X_{ma}}{4.44 f N 10^{-8}} \qquad [701]$$

As the effective turns of the armature to use above are $(Z_a/m\pi)k_p$, or 88.6,

$$\phi_{TS} = \frac{(0.0335 - j0.57)I_1 \times 5.8}{4.44 \times 60 \times 88.6 \times 10^{-8}} = (829 - j14{,}050)I_1$$

$$\approx 14{,}050 \times I_1 \text{ lines}$$

Neglecting the iron loss component in the transformer axis for the above current, this flux will be in phase with the current causing it ($I_{\phi S}$), or nearly 90° behind E_S and ϕ_F.

(*f*) Because the flux ϕ_{TS} links with the transformer-axis stator turns, it induces a counter emf in that axis. The voltage so induced in the stator will be designated E_{TS}.

$$E_{TS} = 4.44\, \phi_{TS} f \left(\frac{C k_p k_d}{2} \cos A \right) 10^{-8} \qquad [702]$$

As a vector, and in stator terms:

$$E_{TS} = -j4.44 \times I_1(829 - j14{,}050) \times 60 \times 244.5 \times 10^{-8}$$

$$= (-9.1 - j0.54)I_1$$

Although this is in stator terms, it must have a change in sign to rotate it through 180° and place it in the stator. Then:

$$E'_{TS} = (9.1 + j0.54)I_1$$

This completes the series of reactions resulting from the armature conductors cutting the field flux. The ultimate result is a speed emf in the armature and a drop component in the transformer axis of the stator by the flux ϕ_{FS}. The influence on torque will be pointed out later.

(*g*) The field-axis flux, in addition to giving rise to the reactions mentioned above, cuts its own stator turns directly, resulting in a reactance drop or a counter emf which must be overcome by the applied voltage. This voltage is

$$E_F = 4.44f\left(\frac{Ck_pk_d}{2}\sin A\right)\phi_F 10^{-8} \qquad [703]$$

$$= 4.44 \times 60 \times 79.1 \times (22{,}100 \times I_1) \times 10^{-8} \quad \text{or} \quad (4.64I_1) \text{ volts}$$

As such a voltage is in lagging quadrature with the flux causing it, the correct vector representation is $E_F = -j4.64I_1$. Reflected in the stator: $E'_F = j4.64I_1$.

(*h*) Note that no final value can be given to any of the above components. Note, too, that the entire process has been detailed so as to point out individual values. Actually these steps are not necessary, for we can combine as will be shown.

In E_S of item *c*, use ϕ_F in general terms from item *b*. Use this value of E_S in general terms in the expression for $I_{\phi S}$, item *d*. Having $I_{\phi S}$ in general terms, substitute it in the equation for ϕ_{TS} of item *e*, doing the same in the expression for E_{TS}. Many of the terms cancel out, leaving the formula for E_{TS} in stator terms as shown below. (See equation 702.)

$$E'_{TS} = j\frac{X_m{}^2SI_1 \sin A \cos A}{r_2 + jX_0} \qquad [704]$$

In the same way the speed emf reduces to

$$E_S = SX_mI_1 \sin A \quad \text{(in stator terms)}$$

or

$$E_S = \frac{S}{a}X_mI_1 \sin A \quad \text{(in armature terms)} \qquad [705]$$

and

$$I_{\phi S} = a\frac{SX_mI_1 \sin A}{r_2 + jX''_0} \quad \text{(in armature terms)} \qquad [706]$$

Then

$$E'_F = jI_1X_m \sin^2 A \qquad [707]$$

The vectors of items *b* to *g* are drawn in Fig. 353*a*.

(*i*) Going now to the transformer axis, the same stator current flows through its turns as through the field winding. (The division is fictitious

of course, the physical winding is merely considered as being divided.) In the transformer axis the armature acts as a short-circuited secondary, and hence the stator current has two components: (a) for magnetizing this axis and (b) for balancing the short-circuited secondary current demagnetizing ampere turns. These components will be designated $I_{\phi T}$ and I_b, respectively. Hence, as vectors,

$$I_b + I_{\phi T} = I_1$$

The armature short-circuit current will be I_{ba}, equal and opposite to I_b when in the same terms, but actually differing in magnitude by the ratio of transformation between armature and transformer-axis stator turns.

Although we have not yet shown how to evaluate these vectors, the diagram can be drawn in general as shown in Fig. 353b, since this is a typical diagram for such conditions as described.

(j) The impedance of the armature offered to the short-circuit current is $r_a + jx_a$. In transformer-axis terms this is

$$(r_a + jx_a)a^2 \cos^2 A = (r_2 + jx_2) \cos^2 A$$

The voltage induced in the stator by the transformer axis flux will be

$$-E_{\phi T} = jI_{\phi T}X_m \cos^2 A \tag{708}$$

This follows from the fact that a counter emf in general is equal to the product of magnetizing reactance and magnetizing current. $I_{\phi T}$ is not yet known, but since X_m represents the magnetizing reactance of the entire stator winding, $X_m \cos^2 A$, will be the value for the transformer axis. Then:

$$I_{\phi T} = \frac{jE_{\phi T}}{X_m \cos^2 A} \tag{709}$$

The rotor short-circuit current will be in stator terms

$$\frac{-E_{\phi T}}{(r_2 + jx_2) \cos^2 A} = I_b \tag{710}$$

Then, since $I_1 = I_{\phi T} + I_b$ (as vectors):

$$I_1 = \frac{jE_{\phi T}}{X_m \cos^2 A} + \frac{-E_{\phi T}}{(r_2 + jx_2) \cos^2 A} \tag{711}$$

or

$$E_{\phi T} = -I_1 X_m \cos^2 A \frac{r_2 + jx_2}{X''_0 + jr_2} \tag{712}$$

or, rotated through 180° as reflected in the stator,

$$E'_{\phi T} = I_1 X_m \cos^2 A \frac{r_2 + jx_2}{X''_0 + jr_2} \tag{713}$$

Supplying numerical values, we have

$$E_{\phi T} = -I_1 \times 49 \times 0.951^2 \frac{3 + j2.2}{51.2 + j3}$$
$$= I_1(2.7 + j1.74)$$

or

$$E'_{\phi T} = I_1(2.7 + j1.74)$$

Also:

$$I_{\phi T} = -jI_1 \left(\frac{r_2 + jx_2}{X''_0 + jr_2}\right) \quad \text{or} \quad I_1(0.0394 - j0.0611)$$

(*k*) This completes the reactions and we now have three components of voltages reflected to the stator.

$$E'_{TS} \text{ (item } f) = (9.1 + j0.54)I_1 \quad \text{(also equation 704)}$$
$$E'_F \text{(item } g) = (0 \quad + j4.64)I_1 \quad \text{(also equation 707)}$$
$$E'_{\phi T} \text{ (item } h) = (2.7 + j1.74)I_1 \quad \text{(also equation 713)}$$

By Kirchhoff's law,

$$V = E'_{TS} + E'_F + E'_{\phi T} + I_1 z_1 \tag{714}$$

where $I_1 z_1$ is the local stator impedance drop or

$$I_1 z_1 = I_1(1.34 + j2.90)$$

Adding up all of these components, we have

$$V = (13.14 + j9.82)I_1$$

It is obvious now that the expressions in parentheses represent the impedances of the various component parts of the motor circuit. Solving for the stator current,

$$I_1 = \frac{V}{13.14 + j9.82} = 5.4 - j4.04 \quad \text{or} \quad 6.75 \text{ amperes}$$

This is the load current, taken by the motor at a speed of 1200 rpm.

If the terms of equation 714 are expanded, we obtain

$$V = I_1\left(\frac{jX_m{}^2 S \sin A \cos A}{r_2 + jX_0} + jX_m \sin^2 A + X_m \frac{r_2 + jx_2}{X''_0 + jr_2} + z_1\right) \tag{715}$$

Although many equations have been given in the previous material to show the derivation, they may be omitted in dealing with an actual problem. The first step in the performance calculation for a given motor could be the use of equation 715 to determine the current at a given speed and voltage.

An approximation which is often useful can be obtained through the factors described in Article 280 (and the note with Article 293). It will

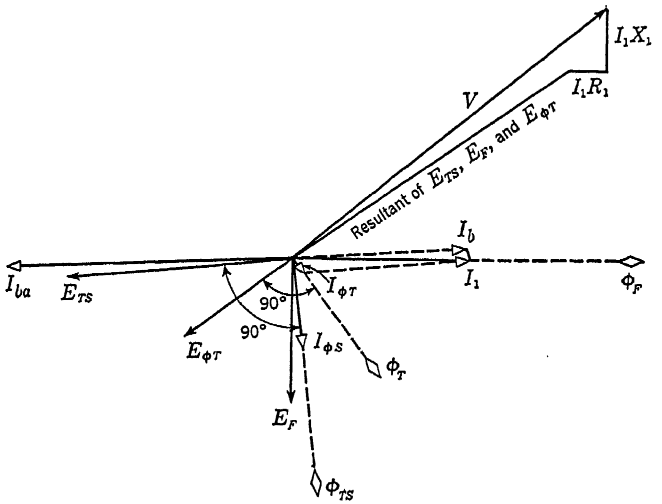


FIG. 354. Vectors of the field and transformer axis on the same diagram, located by the fact that I_1 is identical for each. Only the solid-line vectors are necessary for a final solution of the entire performance.

be recalled that K_s was used as the ratio of X_m to X''_0. Now, if we assume that r_2 is comparatively small and can be neglected when combined with X''_0, we may write

$$K_s \approx \frac{jX_m}{r_2 + jX''_0} \approx \frac{X_m}{X''_0} \qquad [716]$$

Making this substitution in equation 715, we obtain

$$V = I_1[K_sX_mS \sin A \cos A + K_s \cos^2 A(r_2 + jx_2) + jX_m \sin^2 A + z_1] \qquad [717]$$

This is a simplification of equation 715. Using these values, we obtain $I_1 = 5.8 - j3.75$, or 6.9 amperes.

472. Calculation of Performance. Other items will be calculated below, using the current obtained by the approximation of the preceding paragraph.

Stator Copper Loss.

$$I_1{}^2 r_1 = 6.9^2 \times 1.34 \quad \text{or} \quad 62.2 \text{ watts}$$

Armature Current. It has been shown that

$$I_{\phi S} = a \frac{X_m S I_1 \sin A}{r_2 + jX''_0} \text{ in armature terms}$$

Then, since

$$\frac{X_m}{r_2 + jX''_0} \approx -jK_s$$

$$I_{\phi S} = -jSK_s I_1 \tan A \text{ in transformer-axis terms}$$

The ratio of transformer short-circuit currents is also

$$\frac{I_2}{I_1} \approx -K_s \qquad [718]$$

Applied to this case,

$$I_b = -K_s I_1$$

$$\begin{aligned} I_{\text{armature}} &= I_b + I_{\phi S} \\ &= -K_s I_1 - jSK_s I_1 \tan A \\ &= -IK_s \sqrt{1 + S^2 \tan^2 A} \end{aligned} \qquad [719]$$

In armature terms,

$$I_a = (a \cos A) I_1 K_s \sqrt{1 + S^2 \tan^2 A} \qquad [720]$$

This is the final working formula for armature current as a scalar quantity. Numerically,

$$I_a = 2.77 \times 6.9 \times 0.935 \sqrt{1 + 0.666^2 \times 0.325^2} \quad \text{or} \quad 18.4 \text{ amperes}$$

Armature Copper Loss.

$$I_a{}^2 r_a = 18.4^2 \times 0.357 \quad \text{or} \quad 120.5 \text{ watts}$$

Core and Friction Losses. The former value of 40 watts will be used unchanged. Friction and windage loss was 15 watts at 1750 rpm, and will be assumed to be 11.0 watts at 1200 rpm.

Power Factor. This can be calculated from the input current.

$$\text{pf} = \frac{5.8}{6.9} \quad \text{or} \quad 0.84$$

The Input.

$$110 \times 6.9 \times 0.84 \quad \text{or} \quad 639 \text{ watts}$$

The Total Losses.

Stator copper loss	=	63.8
Rotor copper loss	=	120.5
Core loss	=	40.0
F and W	=	11.0
Sum of losses	=	235.3

Output.

639 − 235.3 or 403.7 watts (0.543 hp)

Efficiency.

$$\frac{403.7}{639} \quad \text{or} \quad 0.631$$

473. Torque. It is possible to determine the output directly from the torque, rather than from input minus losses. The torque formula will be given without derivation, although it can be identified from fundamentals.

$$T_{\text{oz-ft}} = 1.33 P \phi_F Z_a \frac{I_a}{m} 10^{-8} \qquad [721]$$

Only the in-phase portion of I_a must be used, and hence we must determine I_a by the accurate method to fix its position. When this is done, we find that 17.4 amperes are in phase with ϕ_F. ϕ_F is calculated as 148,000 lines per pole.

$$T_{\text{oz-ft}} = 1.33 \times 4 \times 148{,}000 \times 1152 \times \frac{17.4}{4} \times 10^{-8} \quad \text{or} \quad 39.5$$

This is the developed torque from which the retarding effect of core losses, friction, and windage should be subtracted. This gives 34.7 oz-ft, net torque.

Since the above equation is somewhat complicated to use, we can simplify it by the use of flux factors. For the in-phase current component, use

$$I''_a = I_1 K_s a \cos A$$

Also,

$$\phi_F = \frac{X_m I_1 \sin A}{4.44 f (0.5C) k_p k_d} 10^8$$

$$f = \frac{P \times \text{synchronous rpm}}{120}$$

$$\therefore T_{\text{oz-ft}} = \frac{112.5}{\text{synchronous rpm}} [I_1^2 X_m K_s \cos A \sin A] \qquad [722]$$

This approximate method gives a developed torque of 39.8, or a net torque of 35 oz-ft.

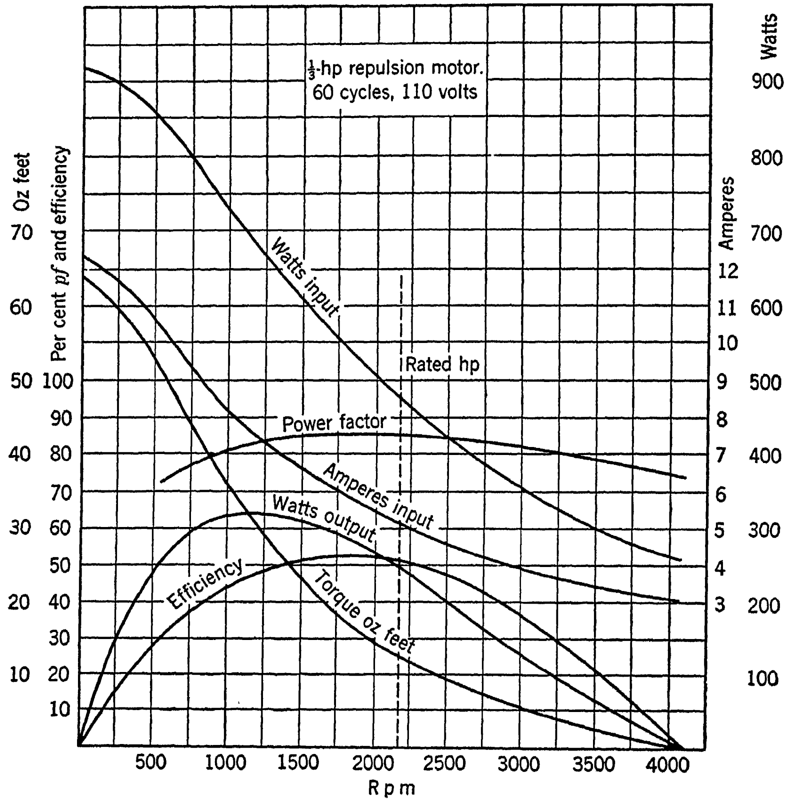


FIG. 355.

474. Discussion of Results. An examination of the previous method of analysis reveals that various refinements would be possible for theoretically greater accuracy. Unfortunately, experience indicates that test results and calculated methods are not in as close an agreement as is usually expected on other types of machines, and so further refinement will be neglected. A very worth-while improvement has been made by Steinmetz and West, however, in analyses considering the effect of the closed circuit formed by the armature coils short-circuited by the brushes. This forms an additional circuit having a resistance (r_3) including coils, brush, and brush contact. West's analysis is based on the cross-field theory, but when r_3 is increased to infinity, the equations obtained are identical with those of the foregoing pages. Examination of the method

followed indicates that the field and transformer axes are similar to the cross and main axes; in short this "two-transformer theory" could be called "cross-field" with equal propriety.

Although no other calculation methods will be demonstrated, results obtained by different processes are compared in Table XXVIII.

TABLE XXVIII

COMPARATIVE RESULTS FOR A $\frac{1}{3}$-HP, 2200-RPM, 4-POLE, 110-VOLT REPULSION MOTOR

(Investigated at a Speed of 1200 Rpm)

Item	Commutation Effects Neglected			Test Results	Commutation Effects Considered	
	"2-transformer"	Flux factors	Tri-angular		Punga diagram	Cross-field (West)
	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
Rpm	1200	1200	1200	1200	1200	1200
I_1	6.7	6.9	8.53	7.4	7.26	8.1
I_a or I_2	18.1	18.4	18.55		16.3	22.1
Input	580	639	730	680	659	800
Pf	0.787	0.84	0.775	0.835	0.825	0.900
Hp output	0.472	0.543	0.650	0.427	0.509	0.570
Efficiency	0.610	0.631	0.66	0.468	0.575	0.531
$T_{\text{oz-ft}}$	34.7	35.0	43.41	30.0	35.5	40.72

On this table, column *a* avoids the approximations of the flux factors, following the method shown; *b* tabulates the results calculated in the previous article; and *c* applies the triangular coefficients of Punga. The comparative error of the last method may not be inherent with the analysis, but may rest on the fact that the motor actually did not have a space distribution of mmf of such a shape. A complete analysis, using combination graphical and analytic methods [4] gave results as shown in column *e*, while West's cross-field theory [5] was used for *f*.

[4] F. Punga, "Das Funken von Kommutatormotoren," Verlag von Gebrüder Jänecke, Hannover.

[5] H. R. West, "The Cross-field Theory of Alternating-current Machines," *Trans. A.I.E.E.*, February, 1926.

P. H. Trickey, "Performance Calculations on Repulsion Motors," *Elec. Eng.*, February, 1941.

CHAPTER LX

REPULSION START FOR INDUCTION MOTORS

475. Chapter Outline.

Repulsion Start for Induction Motors.
Calculation of Torque and Current.
  Examples.
Repulsion-induction Motor.

476. Applications. Although the analyses of the preceding chapter have dealt with running operation, probably more motors are in industrial use depending upon repulsion-motor starting duty than upon the complete repulsion-motor principle. The cycle of operation has been described in Chapter XXXI. In recent years capacitor-start motors have gained in favor over repulsion-start, partially because of the reduced cost, complexity, and the absence of radio interference.

The chief items of importance on repulsion-start motors are good starting torque, low starting current, good "pull-up" torque, and favorable commutation.

Methods of calculating starting current and torque will be shown briefly.

477. Starting Current. All items of equation 717 involving speed or speed ratios reduce to zero at standstill, and we have for the starting current:

$$V = I_1(K_s Z_2 \cos^2 A + jX_m \sin^2 A + Z_1) \tag{723}$$

For the repulsion motor previously considered, the expression in parentheses becomes 10.25 ohms. Then:

$$I_1 = \frac{110}{10.25} \quad \text{or} \quad 10.7 \text{ amperes (versus 10.9 by test)}$$

Equation 720 for armature current becomes

$$I_a = I_1 K_s a \cos A$$

$$= 10.25 \times 0.956 \times 2.91 \times 0.951 \quad \text{or} \quad 27.1 \text{ amperes}$$

478. Starting Torque. At standstill, the torque equation is

$$T_{\text{oz-ft}} = \frac{112.5}{\text{synchronous rpm}} (I_1{}^2 K_s X_m \cos A \sin A) \qquad [724]$$

For the example this calculates 98 oz-ft against a test value of 64.

In general, values of starting torque calculated by the above method are high, requiring a correction factor of, roughly, 30 to 90 per cent to agree with test values. This is an unsatisfactory degree of accuracy.

479. Starting Current and Torque. Commutation Considered. Although the West analysis will not be derived in detail, the consideration of the effect of the short-circuited armature coils undergoing commutation will be presented. The starting current can then be calculated from the formula

$$V = I_1\left[jX_m + X_m{}^2\left(\frac{\cos^2 A}{r_2 + jX''_0} + \frac{\sin^2 A}{r_3 + jX''_0}\right) + Z_1\right] \qquad [725]$$

where r_3 equals resistance of the short-circuited coil, brush contact, and brush, in stator terms; and β equals r_3/X_0. [726]

Then:

$$I_1 = \frac{V}{X_0\sqrt{G^2 + H^2}} \qquad [727]$$

where:

$$G = K_r \sin^2 A\left(\frac{\beta}{\beta^2 + 1}\right) + \frac{r_1}{X_0} \qquad [728]$$

$$H \approx 1 - K_r\left(\cos^2 A + \frac{1}{\beta^2 + 1}\sin^2 A\right) \qquad [729]$$

The West starting torque equation is

$$T \text{ (synchronous watts)} = I_1{}^2 X_m{}^2 \frac{(r_3{}^2 - r_2{}^2)X_0 \sin A \cos A}{(r_2 r_3 - X_0{}^2)^2 + (r_2 + r_3)^2 X_0{}^2} \qquad [730]$$

By simplification, an approximate equation can be written

$$T \text{ (synchronous watts)} = I_1{}^2 K_r X_0 \left(\frac{\beta^2}{\beta^2 + 1}\right) \cos A \sin A \qquad [731]$$

Note that the chief difference between this equation and those resulting from other analyses is in the factor in parentheses, depending upon the ratio r_3/X_0. The $\frac{1}{3}$-hp motor dealt with in the example displayed an actual value of r_3 of 44 ohms in stator terms. Yet, to show the influence

of the short-circuited coils undergoing commutation on the starting current and torque, we will assume that r_3 could vary over a wide range. (A change in brush type might bring about quite a change in r_3.) Resulting influence on current and torque is shown in Fig. 356.

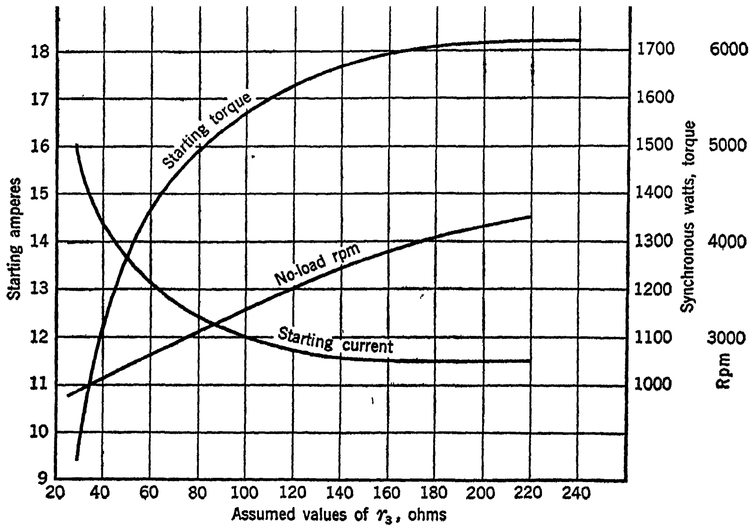


FIG. 356.

480. Example, Larger Rating. Because the principles have been illustrated by a small repulsion motor, a few numerical values will be calculated for a motor representing typical practice in the larger ratings of repulsion-start, induction-run machines

Rating: 2 hp 220 volts 4 poles 60 cycles

Full-load torque = 96.5 oz-ft

Maximum torque as induction motor = 228 oz-ft

$r_1 = 0.765\ \Omega$ $X_0 = 57.0\ \Omega$

$x_1 = 1.88\ \Omega$ $K_r = 0.935$

$x_2 = 1.88\ \Omega$ $r_2 = 1.58\ \Omega$

$a = 4.68$ C (stator conductors) = 576

$k_{w1} = 0.78$ r_c (short-circuited coils) = $0.00745\ \Omega$

a_3 = ratio of transformation stator to short-circuited coils

$a_3 = 56.25$ r_b (brush and brush contact) = $0.0140\ \Omega$

Brush-shift angle = 16°

Calculate r_3 in stator terms:

$$r_3 = (r_c + r_b) \times a_3^2$$
$$= (0.00745 + 0.014) \times 56.25^2 \quad \text{or} \quad 67.7 \text{ ohms}$$

Calculate:

$$\beta = \frac{r_3}{X_0} = 1.18$$

$$G = 0.04785$$

$$H = 0.1184$$

$$I_1 = \frac{V}{X_0\sqrt{G^2 + H^2}} \quad \text{or} \quad 30.65 \text{ amperes (32 by test)}$$

Starting torque

$$\text{(synchronous watts)} = 30.65^2 \times 0.935 \times 57.0 \left(\frac{1.39}{1.39 + 1}\right) 0.9613 \times 0.2756$$

$$= 7750$$

$$\text{Starting torque (oz-ft)} = \frac{112.5 \times 7750}{1800} \quad \text{or} \quad 486 \text{ oz-ft (472 by test)}$$

Ratio starting torque to full-load torque:

$$\frac{486}{96.5}100 \quad \text{or} \quad 504\%$$

Extensive investigation of this method on a number of different motors indicates that the starting current and torque obtained through a consideration of the effect of the short-circuited commutating coils yields a much higher degree of accuracy than can otherwise be expected.

481. The Repulsion-induction Motor. The repulsion starting principle is also utilized on a motor which shows no sudden change in characteristics by the transition from repulsion-motor to induction-motor action. The rotor of this type of machine contains two windings, a squirrel cage, deeply embedded, and a commutated winding, similar in design to that of the ordinary d-c armature. Both windings are in operation during both the starting and running periods. Their change in effectiveness is automatic. At starting, because of the comparatively higher frequency of the rotor emf's, the current which can flow in the highly inductive squirrel cage is small and produces little effect. The starting action is practically the same as that of the repulsion motor.

As the motor gains speed, the revolving air-gap flux, common to both single-phase induction motor and repulsion-motor action, induces low-frequency emf's in the rotor bars. The reduced reactance of the bars to such low frequency permits an appreciable current to flow in the squirrel cage, and the load is divided between the two windings.

The chief reasons for the low no-load speed of the repulsion motor as compared to the series type lay in the action of the coils short-circuited by the brushes. That is, these coils act as a closed rotor winding which gives a negative torque and gives less desirable performance above synchronous speed. Below synchronous speed the short-circuited coils

act, on the whole, to improve performance. We can see then, that, on the repulsion-induction motor of this type, the action of the squirrel cage is superposed upon that due to the commutated winding, plus an inductive coupling effect between them. Then, for the reason given above, this motor will run slightly above synchronism at no load, the commutated winding producing torques at higher speeds, with the squirrel-cage generating action producing a negative torque. The resultant speed may be slightly above synchronism.

At full load, the sloping speed characteristic of the repulsion motor would cause a considerable speed reduction were it not for the increased torque exerted by the squirrel-cage winding. Hence this type of motor besides displaying smooth starting characteristics has good speed regulation as well. Special means are provided for commutation improvement and for good pf.[1]

As in all repulsion motors, starting torque and starting current are quite sensitive to brush contact and brush resistance, variations as great as 50 per cent above or below calculated values being common.

[1] S. R. Bergman, "A New Type of Single-phase Motor."

H. R. West, "Theory and Calculation of the Squirrel Cage Repulsion Motor," *Trans. A.I.E.E.*, Vol. 43, 1924.

PROBLEMS

ALTERNATORS

Chapter II

1. A 4-pole alternator runs at 1200 rpm. There are 10,000 kilolines per pole. (*a*) Determine the effective voltage built up for each full pitch armature coil of one turn if the flux distribution in the air gap is sinusoidal. (*b*) Suppose the armature coil is short 30 electrical degrees of being full pitch. What will be the effective voltage per coil? (*c*) What is the pitch factor for this winding?

2. A three-phase 4-pole alternator runs at 1800 rpm. Each phase covers 2 slots under each pole. The full pitch armature coils have four turns per coil and there are two coil sides per slot. The flux per pole is 6000 kilolines distributed sinusoidally. (*a*) Determine the emf per phase. (*b*) What is the distribution factor?

3. The armature of a two-phase alternator has 8 slots per pole and 2 coil sides per slot. Draw a developed winding diagram for a coil pitch of unity.

4. Assume that the air gap in a 60-cycle alternator is such as to give a uniform flux under the poles and zero flux between. The pole arc covers 80 per cent of the pole pitch. The flux per pole is 10,000 kilolines. (*a*) If the coil pitch is unity, what will be the armature voltage induced per turn? (*b*) Calculate the armature voltage per turn and the form factor if the coil pitch is 75 per cent. (*c*) Draw the flux and emf waves for cases (*a*) and (*b*).

Chapter III

5. An alternator pole shape is such as to give a flat-topped wave which can be expressed by the equation

$$B = B_1 \sin \theta + \tfrac{1}{3} B_1 \sin 3\theta$$

If the total flux per pole from this 60-cycle alternator is 10,000 kilolines, (*a*) determine the effective armature voltage per full pitch turn. (*b*) Calculate its form factor. (*c*) Repeat (*a*) and (*b*) for a pitch of 0.66.

6. The flux in the air gap of a 60-cycle alternator is 2×10^6 lines per pole. Its equation is

$$B = B_1 \sin \theta - \tfrac{1}{3} B_1 \sin 3\theta$$

(*a*) Calculate the armature emf per turn if the coil is full pitch. (*b*) Calculate its form factor. (*c*) Repeat (*a*) and (*b*) for a pitch of 0.80.

Chapter IV

7. A three-phase Y-connected alternator has 15 slots per pole. The coil pitch is $\frac{12}{15}$. The coils are made up of 8 turns, and the windings are double layer. Calculate the pitch and distribution factors for the fundamental, the third, and the fifth harmonics in the phase voltage.

8. The flux distribution in the air gap of a certain generator can be expressed by the equation

$$B = B_1 \sin \theta + 0.3B_1 \sin 3\theta + 0.2B_1 \sin 5\theta$$

There are 5 slots per phase per pole when the connections are made for three-phase Y and the coil pitch is $\frac{12}{15}$.

The frequency is 60 cycles at 1200 rpm. There are 10^6 flux lines per pole. (*a*) Calculate the emf per turn. (*b*) If there are 10 turns per coil and 2 coil sides per slot, calculate the emf per phase. (*c*) Calculate the emf between terminals. (*d*) What is the ratio of emf per phase to emf between terminals?

9. A three-phase 60-cycle generator has 12 slots per pole. The armature is Y-connected with coils having a pitch of $\frac{5}{6}$. The flux distribution in the air gap can be expressed by the equation

$$B = B_1 \sin \theta + \tfrac{1}{3}B_1 \sin 3\theta$$

Calculate the ratio of terminal to phase voltages.

Chapter VI

10. A three-phase, Y-connected generator is rated at 100 kv-a, 60 cycles, 2300 volts. The effective resistance of the armature is 1.5 ohms per leg. The test data pertaining to the machine are given below.

Field Current	Open-circuit Terminal Volts	Short-circuit Current
10	1200	13.2
20	2100	26.0
30	2830	
40	3460	

(*a*) Calculate the synchronous impedance and the synchronous reactance per phase for this machine, using the highest point given on the saturation or open-circuit voltage curve to obtain the values.

(*b*) Repeat (*a*), using the point on the open-circuit curve corresponding to full-load short-circuited armature amperes. Account for the difference in values between X_s as obtained in (*a*) and X_s as obtained in (*b*).

(*c*) Draw the air-gap line for this generator, passing through the 3000-volt point at a field current of 25 amperes. Calculate the unsaturated synchronous reactance of this machine. Does it make any difference at what points these values are obtained?

11. (*a*) Using the data obtained in the above problem for the highest point on the saturation curve, determine the full-load regulation at unity pf. (*b*) At 0.8 pf, lagging. (*c*) At 0.8 pf, leading. (*d*) Using the value of unsaturated synchronous reactance obtained in 10*c*, calculate the regulation at 0.8 pf, lagging. Comment on the comparative results.

12. A three-phase, slow-speed, Y-connected alternator is rated at 5000 kv-a and 13,200 volts. The resistance of the armature between terminals is 0.192 ohm at

75 C. The effective resistance is 1.60 times the d-c value at 75 C. The test data on this machine are given below

Field Current	Open-circuit Terminal Volts	Short-circuit Current
90	9,800	195
135	13,000	291
180	14,900	
225	15,800	

(*a*) Calculate the regulation at a pf of 0.8 lag. (*b*) Calculate the regulation for a load of unity pf. (*c*) The air-gap line passes through the points; 135 field amperes and 16,000 volts. Calculate the unsaturated synchronous reactance.

13. A 2300-volt, three-phase, Y-connected generator is rated at 300 kw at 0.8 lagging pf. Its open-circuit saturation curve passes through points, as follows:

I_F	= 20	40	60	80	94
$E_{\text{per leg}}$	= 940	1328	1450	1530	1580

The air-gap line passes through the voltage point of 1328 at 23 field amperes. The full-load saturation, zero pf curve passes through the points

I_F	= 27.0	94.0
$E_{\text{per leg}}$	= 0	1328

(*a*) Draw these curves to scale and calculate the synchronous reactance per leg by the old A.I.E.E. method.

(*b*) Calculate the full-load field current for 0.8 lagging pf load. Neglect armature resistance.

14. Draw the Potier triangle for the generator of Problem 13 and calculate the Potier reactance. How would it be possible to determine the effect of armature reaction from this construction?

15. (*a*) Use the data of Problems 13 and 14 and determine the full-load field current of this generator at 0.8 pf by the A.S.A. method. (*b*) Calculate the voltage regulation. (Ans. 14.3 per cent.) (*c*) Calculate the regulation by using synchronous reactance (old A.I.E.E.) and compare results. From the data given and calculated, account for the small value of regulation. (Neglect armature resistance throughout.)

Chapter VII

16. (*a*) Calculate the resistance per phase of alternator A. (*b*) alternator B.

17. (*a*) Calculate the armature-reaction ampere turns per pole of alternator A, using the method of Article 56. (*b*) alternator B.

18. (*a*) Repeat 17*a*, using the step-curve method. (*b*) Using Fourier's series. (*c*) Repeat 17*b*, using the step-curve method. (*d*) Using Fourier's series.

19. The slots of alternators A and B are of uniform width, open at the outer ends. Calculate (*a*) the slot- and end-connection leakage reactance of alternator A. (Ans. 0.328 ohm.) (*b*) Repeat for alternator B.

20. (*a*) Calculate the field current required for full-load 0.8 pf lagging operation for alternator A. (*b*) Repeat for alternator B.

Chapter VIII

21. Calculate the regulation of alternator A by the Blondel two-reaction method at full-load 0.80 lagging pf. (Ans. 39.6 per cent.)

DESIGN DATA. SALIENT-POLE ALTERNATORS

	A	B
Kv-a	45,000	5
Rpm	187.5	1800
Terminal volts	12,000	110
Frequency	25	60
Phases	3	3
Poles	16	4
Diameter of armature	216 in.	8.06 in.
Gross length of armature	68 in.	4.25 in.
Ducts in armature	(25) $\frac{3}{8}$ in.	0
Slots	288	48
Size	7 by 1 in.	1.15 by 0.35 in.
Conductors	6 per slot	16 per slot
Size	24 parallel 0.16 by 0.09 in.	No. 11
Mean length of turn	280 in.	30.5 in.
Winding	4 circuit Y	YY
Pitch	$\frac{15}{18}$	$\frac{12}{12}$
Length of air gap: Minimum	0.5 in.	0.09 in.
Average	0.625 in.	0.115 in.
Pole arc to pole pitch	0.66	0.52
Pole length	67 in.	3.75 in.
Turns per pole	70	1850
Resistance of field winding	0.34	63
Percentage of friction loss	0.75	3.5
Percentage of core loss	0.86	2.5

SATURATION CURVES

A		B	
Terminal volts	*Ni* per pole	Terminal volts	*Ni* per pole
8,000	8,000	50	500
9,000	9,000	70	875
10,000	10,600	90	1350
12,000	14,000	110	2080
13,000	16,800	130	2900
15,000	24,000	150	4500
16,000	28,000		
16,700	32,000		
17,300	36,000		

22. Test data show that for alternator A, the value of X_q is 1.73 ohms, and of X_d is 2.40 ohms. Calculate (*a*) the voltage regulation at 0.8 lagging pf. (*b*) The full-load field current.

23. Calculate the percentage of X_d, percentage of X_q, and the corresponding per-unit values for alternator A.

Chapter IX

24. A three-phase alternator is rated at 6600 volts, 5000 kv-a. The Y-connected armature has an effective resistance of 0.079 ohm per leg. The regulation at 80 per cent lagging pf has been found to be 22 per cent. The resistance of the field at operating temperature is 0.481 ohm. This alternator can be driven by a rated d-c motor on which the following data are known:

Kilowatts input	38.4	56	81.4	109	137.5	168
Output in kilowatts	25	50	75	100	125	150

With the alternator running at rated speed without excitation, the d-c motor input is 43 kw. With the alternator running at rated speed and variable excitation, the following readings were taken:

Alternator-emf Open Circuit	D-c-motor Input
4750	85
6000	117
6600	133
7600	169

The open-circuit saturation curve follows:

Field Current	Terminal Volts
60	2980
100	4850
150	6550
200	7400
250	7800

(*a*) Calculate the friction and windage loss. (*b*) Calculate the core loss versus voltage. (*c*) Determine the field current at rated voltage and load, 0.80 pf, lagging. (*d*) List all of the full-load losses, neglecting stray power. (*e*) Determine the full-load efficiency at 0.80 pf, lagging.

25. (*a*) Calculate the losses at full load in alternator A, making the necessary assumptions for effective resistance of the armature circuit. (*b*) Calculate the full-load efficiency of this machine at 0.80 pf, lagging. (*c*) Calculate the full-load efficiency at unity pf.

TRANSFORMERS

Chapter XII

26. An iron-core reactor is to be connected across a 110-volt 60-cycle supply. Neglecting the resistance drop, determine the number of flux lines which must be set up in the core if the turns are 400. Determine the cross-sectional area of the core necessary for this reactor if the flux density should not exceed 60,000 lines per square inch.

27. The current taken by a reactor with 400 turns connected across a 110-volt 60-cycle line is 1.5 amperes. The power input is 65 watts. (*a*) If the hot resistance of the coil is 0.5 ohm, determine the iron loss. (*b*) Draw the vector diagram, and determine the power factor of the input. (*c*) What is the maximum flux built up in the core? (Ans. (*a*) 63.87; (*b*) 0.395; (*c*) 10.32×10^4.)

28. The primary winding on a transformer consists of two coils to be connected in either series or parallel. For series operation on 440 volts, the power input at no load is 80 watts with a current of 0.2 ampere. What will be the power input and current when the coils are connected in parallel across a 220-volt line?

29. A transformer has 800 turns on its primary winding and 160 turns on the secondary. The output is 150 kv-a at 2300 volts. Determine (*a*) the ratio of transformation; (*b*) the rated primary voltage; (*c*) the full-load secondary current; and (*d*) the full-load primary current, neglecting the no-load component. (Ans. (*a*) 5; (*b*) 11,500; (*c*) 65.2; (*d*) 13.04.)

30. A transformer is rated at 2300: 230 volts, 7.5 kv-a, 60 cycles. $r_1 = 8$ ohms; $r_2 = 0.07$ ohm. When 2300 volts are applied to the primary of 1000 turns the input with secondary loaded is 6000 watts and 3 amperes. (*a*) Assume zero leakage flux and determine the magnitude of the primary induced voltage. (*b*) Determine the maximum value of the flux. (*c*) Assume that 3 per cent of this total flux is primary leakage (i.e., it fails to link with the secondary turns). Determine the I_1X_{l1} drop. (*d*) What is the new value of primary induced voltage? (*e*) What is the secondary induced voltage if the secondary coil has 100 turns?

31. Determine the equivalent resistances and reactances of the following transformer in (*a*) primary terms, (*b*) secondary terms.

42000: 2400 volts	500 kv-a
$r_1 = 19.0$ ohms	$x_1 = 39.0$ ohms
$r_2 = 0.051$ ohm	$x_2 = 0.11$ ohm

32. A transformer is rated at 110: 10 volts with a full-load secondary current of 50 amperes. A certain load of unity pf requires 42 amperes at 7 volts. Determine (*a*) the resistance which should be connected in the secondary lines to give this voltage and (*b*) the resistance required in the primary which would produce the same result. Neglect no-load current and assume a voltage regulation of zero. (Ans. (*a*) 0.0715; (*b*) 8.65.)

Chapter XIII

33. Draw the full-load vector diagram of the following transformer to scale and determine its regulation under the conditions given.

10 kv-a	240 : 120 volts	60 cycles
$r_1 = 0.13$		$r_2 = 0.03$
$x_1 = 0.20$		$x_2 = 0.05$
Core loss = 85 watts		
$I_n = 1.0$ amperes		

(*a*) Regulation at 0.8 pf lag. (*b*) Regulation at 1.0 pf. (*c*) Regulation at 0.8 pf lead. Assume in each case that the secondary terminal voltage at full load is 110 volts.

34. Determine the change in primary voltage which is necessary in order that the secondary terminal voltage of the following transformer be maintained constant at 230 volts from full load to no load.

2300 : 230 volts	15 kv-a	60 cycles
$R_1 = 2.5$ ohms		$R_2 = 0.02$ ohm
$X_1 = 10.1$ ohms		$X_2 = 0.09$ ohm

Neglect the no-load current, and assume 80 per cent lagging pf at the secondary terminals. (Ans. 2405 to 2300.)

35. Calculate the regulation of the transformer of Problem 34 by the A.I.E.E. method for (*a*) unity pf and (*b*) 0.80 lagging pf.

(*c*) Calculate the percentage of resistance and the percentage of reactance of the above transformer.

Chapter XIV

36. Draw the theoretically exact equivalent circuit for the transformer whose characteristics are given in Problem 33. Solve this circuit for full load 0.80 pf lag, and determine the regulation.

(*b*) Determine the regulation of the transformer of Problem 35 by the approximate equivalent circuit, using the same load conditions. Contrast the values of regulation as determined by these methods.

37. A 1000 kv-a 60-cycle transformer has a voltage rating of 66,000 : 6600 volts. The constants are

$r_1 = 17.5$ ohms	$r_2 = 0.149$ ohm
$x_1 = 121.0$ ohms	$x_2 = 0.937$ ohm

The core loss is 8650 watts; the exciting current is 0.43 ampere. Determine (*a*) the magnetizing reactance X_m; (*b*) the core loss equivalent resistance r_0; (*c*) the exciting admittance Y_0.

38. Draw a theoretically exact equivalent circuit for a transformer in which the paralleled legs X_m and r_0 are replaced by an equivalent series impedance, connected between the points *b* and *b*. (See Fig. 92.) For the transformer of Problem 37, evaluate the resistance and reactance components of this equivalent series impedance.

39. Using the usual circuit theory, set up an expression for the impedance of a network such as is represented in Fig. 92. Make use of an equivalent series circuit for the magnetizing or exciting branch.

Chapter XV

TRANSFORMER TEST RESULTS

No.	Transformer Rating			No-load Test *			Short Circuit Test †		
	Kv-a	Voltages	Frequency	V_{app}	I	Watts	V_{app}	I	W
1	1.5	220 : 110	60	110	0.4	25	16.5	6.8	40
2	10.0	2,300 : 115	60	115	0.9	70	118	4.35	225
3	20.0	2,200 : 220	25	220	0.52	161	205	9.1	465
4	100.0	11,000 : 2200	50	2200	1.59	980	580	9.1	1,100
5	1000.0	66,000 : 6600	60	6600	3.7	8,700	3490	15.16	7,860
6	5000.0	14,000 : 4000	60	4000	59.0	31,000	1260	358	39,200

* Input to low-voltage side.

† Meters in high-voltage side. Low-voltage leads short-circuited.

Note: Except where otherwise specified, assume core-loss constant and neglect temperature corrections and the effect of exciting current on the regulation.

40. Calculate (*a*) the regulation and (*b*) the efficiency of transformer 3 for a load of 10 kv-a at 0.8 lagging pf.

41. Calculate (*a*) the regulation and (*b*) the efficiency of transformer 4 at full load, unity pf. (Ans. (*a*) 1.23 per cent; (*b*) 97.8 per cent.)

42. Calculate (*a*) the regulation of transformer 1 at full-load unity pf. (*b*) Determine the efficiency at full and half load, each at 80 per cent lagging pf.

43. Transformer 2 is loaded on a resistance requiring 60 amperes. (*a*) Determine the voltage applied to this load connected across the secondary leads if the primary voltage is 2400. (*b*) What is the efficiency of the transformer under this condition? Assume that the total core loss varies as the 1.7 power of the flux.

44. Determine the phase angle between the primary and secondary terminal voltages of transformer 3 at full-load unity pf.

45. Calculate the percent equivalent resistance drop and the percentage of equivalent reactance drop of transformer 2. Calculate the regulation at 0.8 pf lagging, using the percentage basis. (Ans. 2.245 per cent R; 4.60 per cent X; 4.7 per cent regulation.)

Chapter XVI

46. An iron sheet is 0.06 by 5.0 in. in cross-section and has a uniform maximum flux density for this area of 50,000 lines per square inch. The frequency of flux change is 60 cycles per second. The resistivity of the iron is 9.0 micro-ohms per centimeter cube. The length of the sheet is 8 in. Determine (*a*) the eddy-current loss per cubic centimeter and (*b*) the total eddy-current loss in the entire sheet. (Ans. (*a*) 0.092 watt; (*b*) 3.614 watts.)

47. What would be the effect on eddy-current loss, for the sheet in Problem 46, if the resistivity were doubled? Would this produce less core loss than using sheets of half the thickness with the original resistivity?

48. The following dimensions refer to a transformer having 300 turns on its primary winding: Mean length of core: 20 in. The flux density is 62,000 lines per

square inch when the impressed emf is e equals $169.7 \sin 377t$. The hysteresis loop for the core iron is given as sample A. The eddy-current loss at this density is calculated as 5 watts for the entire core. (*a*) Draw the hysteresis loop and the flux wave to the same scale for flux, and determine the shape of the magnetizing and hysteresis components of the no-load current over one-half cycle. (*b*) Draw the current wave representing the component of the no-load current which supplies the eddy-current loss. This current should be drawn to the same scale as the other components. (*c*) Determine the shape of the no-load current curve and calculate its effective value. (Ans. 0.13 ampere.) (*d*) What would be the effect on this curve of the air gaps in the joints of the laminations? Explain.

49. Suppose the voltage on the above transformer to be increased 20 per cent. (*a*) What will be the new value of flux density, neglecting any effect of IR drop in the primary? (*b*) If the hysteresis loss varies as the 1.6 power of the flux density, what will be the new area of the hysteresis loop for this condition? (*c*) What will be the new value of the eddy-current loss at this increased flux density?

Sample A

Flux density (Kilolines per sq in.)	Mmf (Ampere turns per inch of length)	
0	−0.8	+0.80
10	−0.68	+0.83
20	−0.52	+0.96
30	−0.30	+1.14
40	−0.00	+1.44
50	+0.80	+2.04
60	+2.50	+2.80
62	+3.00	+3.00

Sample B

0	−0.98	+0.98
10	−0.78	+1.05
20	−0.60	+1.15
30	−0.38	+1.30
40	−0.05	+1.70
50	+0.70	+2.20
60	+1.90	+2.90
64.5	+3.40	+3.40

50. Determine the total hysteresis loss for the transformer of Problem 49. (Ans. 4.57 watts.)

51. Determine the hysteresis loss in watts per cubic centimeter per cycle for the iron of sample B.

52. A 440-volt transformer has a core loss of 120 watts of which 36 watts represents the eddy-current loss. If the applied voltage is increased 10 per cent, what will be the new value of eddy-current and hysteresis loss? The frequency remains constant at 60 cycles per second.

53. An approximate rule, sometimes used, assumes that the entire core loss varies as the square of the density for comparatively small changes. Use this rule on Problem 52 and compare results.

54. A distribution transformer is provided with a 5 per cent tap on its high-voltage (input) winding. The supply is incorrectly connected to this tap, thereby raising the output voltage on the constant impedance load. What is the relative amount by which the total copper losses are increased? What is the relative amount by which the total core losses are increased? (Use the method of Problem 53.) If the core losses are half of the copper losses under correct operation, what is the relative change in total losses brought about through this incorrect connection?

55. A 2300-volt transformer rated at 60 cycles is to be used on a 50-cycle supply. The losses are: eddy currents, 82 watts; hysteresis, 196 watts. (*a*) Determine the total core loss at 50 cycles. (*b*) What would the losses be if the same transformer were used on a 50-cycle, 2200-volt supply?

56. What change will be produced on the no-load current and core loss of a transformer displaying the following characteristics if the voltage is increased 15 per cent? Rated voltage 11,500: 2300; 100 kv-a.

Eddy-current loss	= 180 watts
Hysteresis loss	= 390 watts
Open-circuit volt-amperes input	= 3800

Assume straight-line magnetization curve over this range of flux density.

Chapter XVII

57. Transformers with the characteristics shown below are to be connected with their primaries and secondaries in parallel, respectively. The load to be supplied is 400 amperes at a power factor of 0.80.

Voltage rating: 2300 : 230

Transformer A	Transformer B
Kv-a 75	Kv-a 25
Ratio 10 : 1	Ratio 10 : 1
In secondary terms:	In secondary terms:
$R_e = 0.0113$	$R_e = 0.0415$
$X_e = 0.0207$	$X_e = 0.0710$

(*a*) Determine the current supplied by each. (*b*) At what percentage of its rating would each transformer be operated? (Ans. (*a*) 311.0; 89.1 (*b*) 95.4 per cent; 82.2 per cent.)

58. Set up the equivalent circuit for the transformers of Problem 57, properly evaluating the constants. (*a*) Assume the load voltage is 230 and evaluate the equivalent load circuit. (*b*) Set up the expression for equivalent impedance of the transformers and load. (*c*) Supply numerical values and determine the current supplied by each transformer from circuit solution.

59. Two transformers to be paralleled display the following characteristics:

No. 1: 50 kw 13,200 : 2300 volts $R_1 = 10.3$ $X_1 = 20.1$ $R_2 = 0.32$ $X_2 = 0.66$
No. 2: 50 kw 13,200 : 2350 volts $R_1 = 10.3$ $X_1 = 21.8$ $R_2 = 0.33$ $X_2 = 0.69$

(*a*) Determine the no-load circulating current when the secondary terminal voltage equals 2300. (*b*) What is the primary applied voltage under this condition?

(*c*) When the load current is 30 amperes at a lagging pf of 0.85, what is the current and power of each transformer?

60. Two transformers are to be paralleled on both their high- and low-tension sides. Their characteristics are as follows:

No. 1:	100 kv-a	6900 : 230 volts	$\% IR = 1.25$	$\% IX = 5.49$
No. 2:	200 kv-a	6900 : 230 volts	$\% IR = 1.34$	$\% IX = 5.53$

At a total load of 300 kv-a 0.8 pf, lagging, determine the kilovolt-amperes and current supplied by each. What is the maximum total load at this power factor which will not overload either transformer?

61. Determine the smallest values of resistance and reactance to be added in the secondary leads of transformer 1 or 2, or both, in Problem 60 in order that the load supplied by each shall be proportional to the relative capacities, and both transformers shall operate in phase.

Chapter XVIII

62. An autotransformer is used for reducing the voltage from 115 to 50. The secondary output is 35 amperes. Determine the current (neglecting the magnetizing component) in each part of the winding. What is the ratio of currents in the windings? In the external leads?

63. An autotransformer has a ratio of 2 to 1. That is, the tap *b* is located halfway between the terminals *a* and *c*. The resistance of the entire winding is 0.10 ohm, and the resistance of the common part *bc* is 0.04 ohm. (*a*) If the exciting current is neglected, what is the copper loss at an output of 10 amperes? (Ans. 2.5 watts.)

The leakage reactance of the part *ab* is 0.2 ohm and that of the common part is 0.1 ohm. (*b*) What will be the input current if 20 volts is applied to the primary terminals when the winding *bc* is short-circuited? (Ans. 63.3 amperes.) (*c*) Neglecting the core loss what would be the reading of a wattmeter connected in the supply lines under the above conditions? (Ans. 400 watts.)

64. The autotransformer of Problem 63 is connected by its low-voltage leads to an a-c source. If the high-tension leads *a* and *c* are short-circuited, what will be the applied voltage necessary to cause 20 amperes to flow in the winding *ab*? Under this condition what will be the current in *bc*?

65. The following laboratory tests were made on an autotransformer rated at 25 kv-a; 440 : 220 volts; 60 cycles.

Low-tension winding short-circuited; emf applied to the hightension winding so that rated current flows through the windings.

Ammeter reading (supply)	56.8 amperes
Wattmeter	410.0 watts
Voltmeter	23.1 volts

Low-tension winding open; rated emf applied to the high-tension terminals.

Ammeter reading	1.05 amperes
Wattmeter	283 watts
Voltmeter	440 volts

D-c resistance = 0.12 ohm (*a* to *c*)

(*a*) Calculate the regulation of this autotransformer at full-load output, unity pf. (*b*) Determine the efficiency. (*c*) Determine the regulation at 0.8 lagging pf. (Ans. (*a*) 1.84 per cent; (*b*) 98.8 per cent; (*c*) 3.0 per cent.)

66. A load requiring 20 amperes at 600 volts is to be supplied from a 4000-volt source. (*a*) Determine the actual power to be transformed by an autotransformer designed for this application. How would this compare with the capacity of an ordinary two-winding transformer for the same installation? (*b*) Suppose that the supply potential was 2300. What would be the actual power to be transformed by an autotransformer for this same load?

67. A single-phase 2200-volt line is to have its voltage boosted 10 per cent by means of an autotransformer. The predicted load is 20 amperes at a pf of 0.8 lagging. What will be the current in each part of the winding and the kilovolt-ampere capacity of this autotransformer?

Chapter XIX

68. A three-phase supply operates with a potential of 230 volts between each pair of outside lines. Three transformers with ratios of 2 : 1 are to be connected across this supply as indicated below. Determine in each case the voltage across the secondaries of each transformer and the voltages across the secondary output lines. (*a*) Connection Y–Δ. (*b*) Connection Y–Y. (*c*) Connection Δ–Y. (*d*) Connection Δ–Δ.

69. A 13,200-volt three-phase generator delivers 1000 kv-a to a three-phase load operating at 2300 volts. Determine the kilovolt-ampere ratings, voltages, and ratios necessary for the three transformers needed to supply the 2300 volts if they are connected: (*a*) Y–Δ; (*b*) Y–Y; (*c*) Δ–Y; (*d*) Δ–Δ.

70. A three-phase load consisting of two 50-hp, 440-volt, three-phase motors is to be supplied by transformers connected in open delta on a 2300-volt line. Assume a motor efficiency of 90 per cent and a pf of 85 per cent. (*a*) Determine the kilovolt-ampere capacity of each of the two transformers and their ratios of transformation. (*b*) What would be the available kilovolt-amperes if a third transformer of the same rating were used with the first two?

71. (*a*) Determine the line current in Problem 70*a*. (*b*) At what pf is each transformer operating? (*c*) What is the real power supplied by each?

72. Suppose that the three-phase load of Problem 69 were to be supplied from the three-phase, 13,200-volt generator by means of T-connected transformers. What would be the voltage, kilovolt-ampere capacity, and ratio of transformation of each?

73. A three-phase load of 300 kw at a pf of 0.85 is to be supplied by a two-phase source. The two-phase lines have a potential of 2300 volts. The desired load potential is 440. Determine the ratios of transformation and the kilovolt-ampere capacity of each of the Scott-connected transformers.

74. A 2300-volt, two-phase source is available to supply a three-phase, 230-volt load. Transformers with an 86.6 per cent tap are not available, but the ones to be used have 50, 90, and 95 per cent taps. Determine the connections which come nearest to giving a balanced-voltage, three-phase supply. What will be the line voltages under this condition?

75. Power is to be transformed from a three-phase, 230-volt supply to two-phase, 4-wire 110 volts by means of T-connected autotransformers. The balanced two-phase load totals 50 kw. at unity pf. (*a*) Determine the two-phase and the three-

phase line currents. (*b*) Determine the current and voltage of each part of the two autotransformers. (*c*) What will be the kilovolt-ampere output of each autotransformer? (*d*) What will be the actual power transformed in each?

76. A three-phase load at 2300 volts is to be connected to a 4000-volt, three-phase source through the use of Δ-connected autotransformers. The rated output is 250 kv-a. (*a*) Determine the voltage ratios of the autotransformers. (*b*) What will be the actual current and voltage of each part of the winding? (*c*) What will be the kilovolt-ampere rating of each autotransformer, and how many kilovolt-amperes are actually transformed by each?

78. A three-phase, 230-volt supply is available for a two-phase load, totaling 25 kv-a at unity pf. The two-phase load is 4-wire, 115-volt, and is balanced. What will be the currents in each section of the windings of the autotransformers used for this purpose? What will be the voltage across each section?

Polyphase Induction Motors

Chapter XXII

79. The stator of a two-phase induction motor is designed with 4 slots per phase per pole. There are 10 conductors in series per slot and the conductors are connected with a pitch of 8 slots. The current in each phase is 2 amperes. Lay out a developed diagram of conductor connections and plot the distribution of the mmf along the air gap. (Phase 1 leads phase 2.) (*a*) Select the time when the current of phase 1 is zero. (*b*) Select the time when the currents in both phases have equal instantaneous values.

80. The stator of a three-phase induction motor has 2 slots per phase per pole. There are 6 conductors in series per slot, and the conductors are connected so that the coil pitch is 6 slots. The current in each phase has an effective value of 3 amperes. Lay out a developed diagram of connections and plot the distribution of the mmf along the air gap. (Phase 1 leads phase 2, etc.) (*a*) Select the instant when the current of phase 1 is zero. (*b*) Select the instant when the current of phase 1 is a maximum.

Chapter XXIII

81. A three-phase, Y-connected induction motor has a total of 300 effective turns on its stator. When a 60-cycle, 230-volt supply is connected to its terminals what will be the maximum value of the flux built up per pole?

82. A three-phase, Y-connected induction motor has 48 stator slots and 8 series conductors in each. Its stator is wound for 4 poles with a pitch of 83.3 per cent. (*a*) Determine the pitch and distribution factors. (*b*) Determine the maximum flux per pole when connected to a 60-cycle, 220-volt supply, neglecting local resistance and reactance drops.

83. The rotor for the above motor has 36 slots with 4 series conductors per slot. This winding is Y-connected with a pitch of 77.7 per cent. Determine the ratio of transformation of the motor.

84. Determine the emf generated in a conductor rotating in the air gap of the motor of Problem 82 at a speed of 1725 rpm. Assume that the entire pole flux cuts the conductor.

85. The emf induced in one phase of a rotor winding is 120 volts when the rotor is blocked. The resistance of the secondary per phase is 0.2 ohm; its reactance per

phase is 0.3 ohm. (*a*) What is the value of rotor current? (*b*) What is its power factor? (*c*) Neglecting any effect of the stator impedance drop, determine the rotor voltage per phase when the slip is 0.05. (*d*) Calculate the rotor current and its pf under this condition. (Ans. (*a*) 332 amperes; (*b*) 0.554; (*c*) 6.0 volts; (*d*) 32.9 amperes pf = 0.996.)

86. The current in each phase of a Y-connected rotor winding is 200 amperes. The resistance per leg is 0.003 ohm. Determine the power developed by the rotor at a slip of (*a*) 3 per cent, (*b*) 6 per cent. (*c*) Calculate the copper losses in the rotor in each case, and determine the power transferred across the air gap to the rotor.

87. Determine the total torque developed by the rotor for each value of slip used in Problem 86. In what units is this torque expressed?

88. A 60-cycle, three-phase, 220-volt, wound-rotor induction motor is rated at 20 hp. The synchronous speed is 1800 rpm. The ratio of transformation is 2.70. (*a*) Neglecting the stator impedance drop, determine the induced voltage per phase of the Y-connected rotor when rated voltage is applied at the stator terminals at standstill.

The leakage reactance of the rotor is 0.024 ohm per phase; the resistance is 0.007 ohm per phase. (*b*) Determine the torque at slips of 2, 4, and 6 per cent. (*c*) At what approximate value of slip is the torque a maximum? Why is this value so determined only an approximation? (*d*) What is the maximum torque developed by this rotor?

89. The motor of Problem 88 is to be started with external Y-connected resistance added to its rotor circuit. Determine the value of resistance necessary to make the maximum torque occur at standstill. If the stator impedance drop reduces the induced voltage E_1 by 25 per cent, what will be the starting current in the rotor winding?

TABLE A

Polyphase Induction-motor Constants

No.	Hp	Poles	Rpm (Nominal)	R_1	R_2	$X_1 = X_2$	I_0	Core Loss	F and W	Volts
				(Ohms per leg)				(Watts)		
1	$\frac{1}{2}$	4	1725	4.17	3.22	3.21	1.43	54.0	18	220
2	1	4	1725	2.92	1.61	2.26	1.57	61.0	40	220
3	3	4	1725	0.851	0.589	1.10	3.32	127.0	70	220
4	5	4	1725	0.457	0.675	0.795		151.0	70	220
5	$7\frac{1}{2}$	4	1725	0.26	0.23	0.482	5.87	211	75	220
6	10	4	1725	0.251	0.24	0.399	7.05	254	150	220
7	20	4	1725	0.118	0.102	0.208	11.9	407	400	220
8	30	4	1725	0.063	0.083	0.148	16.6	514	450	220

Note. The data of Tables A to D are presented as material from which problems can be prepared for Chapters XXIII, XXIV, and XXV. Note in Table A that the variations in winding constants with horsepower are clearly indicated; and, in Table C, the effect of increase in number of poles is shown on the no-load current, core loss, and leakage reactance.

Test values for these same motors are shown in Tables B and D. However, exact agreement must not be expected between calculated performance and that tabulated. The latter tables are obtained from production tests on a line of motors in which small manufacturing adjustments may have varied the winding constants as first calculated. In addition, the inaccuracies of the approximate analyses should show up quite severely on the slower-speed machines.

TABLE B

POLYPHASE INDUCTION-MOTOR TESTS

No.	Hp	Poles	T_{max} (Lb-ft)	$T_{starting}$ (Lb-ft)	$I_{starting}$	Full-load Values: Efficiency (%)	Pf (%)	I	Rpm (Actual)
1	½	4	6.4	5.25	12.8	76.6	69.7	1.86	1736
2	1	4	11.0	8.4	20.2	77.2	85.6	3.00	1730
3	3	4	27.8	17.6	47.8	83.3	88.0	8.1	1735
4	5	4	43.0	36.5	65.2	85.4	88.4	13.1	1712
5	7½	4	72.8	42.0	112.0	87.7	90.5	18.5	1727
6	10	4	84.1	58.0	136.0	84.7	92.1	25.3	1718
7	20	4	167.0	99.0	271.0	86.4	92.6	49.1	1730
8	30	4	252.0	164.0	385.0	88.0	93.0	71.4	1720

TABLE C

POLYPHASE INDUCTION-MOTOR CONSTANTS

No.	Hp	Poles	Rpm (Nominal)	R_1 (Ohms per leg)	R_2 (Ohms per leg)	$X_1 = X_2$ (Ohms per leg)	I_0	Core Loss (Watts)	F and W (Watts)	Volts
1	2	2	3500	1.57	0.948	1.46	1.98	65	75	220
2	2	4	1725	1.18	0.82	1.37	3.13	86	60	220
3	2	6	1150	1.53	1.10	1.75	4.21	94.0	60	220
4	2	8	850	1.48	0.98	1.87	5.23	95.0	70	220
5	2	10	690	0.83	1.49	2.09	5.09	131.0	60	220

TABLE D

POLYPHASE INDUCTION-MOTOR TESTS

No.	Hp	Poles	T_{max} (Lb-ft)	$T_{starting}$ (Lb-ft)	$I_{starting}$	Full-load Values: Efficiency (%)	Pf (%)	I	Rpm (Actual)
1	2	2	9.4	6.86	34.8	77.2	90.4	5.62	3430
2	2	4	21.5	14.8	37.9	85.2	81.6	5.72	1740
3	2	6	24.0	17.0	30.0	77.5	76.5	6.59	1141
4	2	8	30.7	19.4	29.1	77.0	70.2	7.24	860
5	2	10	40.5	30.8	26.9	78.5	70.7	7.06	675

90. Set up the approximate equivalent circuit of motor 1 (Tables C and D), assuming a slip of 0.047, and evaluate the constants. Assume that the friction and windage represent a portion of the mechanical load.

91. Set up the circuit as in Problem 90, except that the theoretically exact diagram should be followed.

92. For the given slip, calculate the complete performance of motor 1 (*a*) by the approximate equivalent circuit; (*b*) by the theoretically exact circuit.

93. (*a*) Using the torque formulas, calculate the starting and maximum torques of motor 1 (Table C). (*b*) Calculate the slip required for full-load output. (*c*) At what slip does the maximum torque occur? (*d*) Using these three points, draw the speed-torque curve for this motor (from standstill to synchronism).

94. Calculate the rotor-blocked current and determine the starting torque in pound-feet per ampere of starting current. What is the ratio of starting torque to rated torque; maximum torque to rated torque?

95. What would be the rotor-blocked current and its pf if full voltage were supplied under standstill conditions to motor 5 of Table C. Explain the equivalent circuit which represents the type of calculation you have made. Why is it especially important in a case like this to consider the circuit used for this calculation?

96. A 1000-hp, three-phase, 60-cycle, 16-pole induction motor displays the following characteristics:

Stator Y-connected
Rated voltage: 2200
Resistance per phase of stator: 0.063 ohm at 75 C (d-c)
Ratio of effective to d-c resistance: 1.6
Reactance of the stator at 60 cycles: 0.43 ohm per phase
Rotor Y-connected
Ratio of transformation: 2:1
Resistance per phase of rotor: 0.024 ohm at 75 C (direct current)
Ratio of effective to d-c resistance: 1.4
Reactance of the rotor at 60 cycles: 0.13 ohm per phase
No-load current: 72 amperes
No-load input (total): 23.1 kw

(*a*) Determine the slip, pf, line current, and efficiency at rated output. (*b*) Determine the slip at which the maximum power is developed and the maximum power. (*c*) Determine the resistance to be added to the rotor circuit to develop the maximum starting torque. Calculate the value of this starting torque. (*d*) Calculate the values of stator and rotor current at starting, assuming the resistance as found in (*c*) is used in the rotor. (Ans. (*a*) slip = 0.0225; pf = 0.885; I_1 = 236; efficiency = 0.935. (*b*) Slip = 0.12; 2900 hp: (*c*) 0.1046 ohm per phase; 35,800 lb-ft. (*d*) I_1 = 950; I_2 = 895.

97. A conductor carrying 150 amperes is rotating in the air gap of the motor of Problem 82. The entire pole flux cuts this conductor. Determine the pound-feet of torque exerted on the conductor if the current is a maximum when in the position of maximum flux density.

Chapter XXIV

98. A 5-hp, 4-pole, three-phase, 60-cycle, Y-connected induction motor displays the following test results:

No-load: 220 volts; 4.6 amperes; 251 watts.
Rotor-blocked: 220 volts; 67.0 amperes; 12,350 watts.

At the temperature of test, $R_1 = 0.43$ ohm. The friction loss at 1750 rpm is known to be 70 watts.

(*a*) Assume that the motor represents a simple series circuit when the rotor is blocked, and calculate X_e, R_e and R_2. They should all be in phase values. (*b*) Calculate X'_0 and, assuming $X_1 = X_2$, determine X_m. Use the method described with the footnote of Article 211, and calculate corrected values of leakage reactance and rotor resistance.

Motor A: Three-phase, 220 volts, 5 hp, 1200 synchronous rpm, 60 cycles

Blocked Rotor			No Load		
E	I	W	E	I	W
51.0	13.70	785	220	5.15	440
60.0	16.30	1110	Rated current per terminal: 13 amperes		
74.9	20.35	1860	Temperature at test: 25 C		
90.2	24.30	2600	Squirrel-cage rotor		
R_1 per leg: 0.41			Friction and windage loss = 38 watts per phase		
$R_{\text{eff}} = 1.6 \times R_{dc}$ at 25 C					

Motor B: Three-phase, 220 volts, 7.5 hp, 1800 synchronous rpm, 60 cycles

Blocked Rotor			No Load		
E	I	W	E	I	W
35.1	15.45	620	220	3.18	520
50.0	21.95	1315	Rated current per terminal: 19 amperes		
55.3	24.50	1660	Temperature at test: 25 C		
R_1 per leg: 0.33			Squirrel-cage rotor		
$R_{\text{eff}} = 1.6 \times R_{dc}$ at 25 C					

Motor C: Three-phase, 440 volts, 50 hp, 1800 synchronous rpm, 60 cycles

Blocked Rotor			No Load			
E	I	W	E	I	W	Rpm
220	202.5	31,400	440	8.78	2070	1790
R_1 per leg: 0.073 ohm at 25 C			Temperature at test: 25 C			

99. (*a*) Determine the equivalent impedance, reactance, and resistance of motor A. (*b*) What would be their values at a temperature of 75 C? (Ans. (*a*) $Z_e = 2.07$; $X_e = 1.507$; $R_e = 1.435$.)

100. (*a*) Set up the approximate equivalent circuit for motor B, dividing up the equivalent reactance between stator and rotor so that they are equal when referred to the same circuit. Determine the motor efficiency and pf at a slip of 0.06.

101. (*a*) Determine the equivalent impedance, reactance, and resistance of motor C. (*b*) Correct the resistances to a temperature of 75°, and recalculate these values. (*c*) Suppose that the effective resistances of the windings were 1.5 times the respective d-c values. Recalculate the equivalent impedance, reactance, and resistance values, and compare with those previously determined. (*d*) Calculate the torque at a slip of 5 per cent, using the values determined in (*a*), (*b*), and (*c*). Contrast the results.

Chapter XXV

102. (*a*) Determine the diameter of the circle for the circle diagram pertaining to motor A. (*b*) What is the rotor-blocked current of this motor for rated applied voltage? (*c*) What is the power lost in the motor under the condition of (*b*)? (*d*) Of this power how much represents stator copper loss and how much rotor copper loss? (Ans. (*a*) 82.5 amperes. (*b*) 61.4 amperes. (*c*) 16.2 kw.)

103. (*a*) When the rotor of motor C is blocked with rated terminal voltage applied to the stator, what is the short-circuit current in stator terms which circulates through the closed rotor windings? (*b*) Draw the circle diagram for this motor and determine the slip, efficiency, and pf at an input of 80 amperes per line. What is the horsepower at this load?

104. Refer to the motors of Table C. Select a current scale and assume that I_0 represents only the magnetizing component of current. (Neglect the no-load losses.) Calculate the circle diameter for each of the five motors, and to the same scale draw the diagram for each, showing V, I_0, I_1 (rotor-blocked), and I_2 (rotor-blocked).

Draw the current vectors for maximum pf for each motor and record the values. What effect does an increase in the number of poles have on motor pf? What one component on the diagram is chiefly responsible for the low pf for low-speed motors?

Draw in the line on each diagram which represents the maximum torque. Evaluate and compare with the test values of Table D.

Single-phase Induction Motors

Chapter XXXII

The data of Table E are presented as material from which single-phase induction motor problems can be formulated. The constants are in the usual transformer terms so that the factor 0.5 must be used with certain items when the double-revolving field theory is used.

TABLE E

DATA ON VARIOUS SINGLE-PHASE INDUCTION MOTORS

Item	1	2	3	4*	5	6†
Hp	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$
Poles	8	4	6	4	4	4
Synchronous rpm	900	1800	1200	1800	1800	1800
Volts	110	110	110	115	115	115
R_1	2.51	5.25	3.95	2.93	1.97	1.49
R_2	2.68	5.24	3.07	4.02	2.80	2.27
$X \approx (x_1 + x_2)$	7.7	9.16	9.72	7.94	4.6	3.98
K_r	0.765	0.902	0.862	0.93	0.933	0.94
X_0	33.5			113.5	69.0	67.0
Core loss	36.5	22.4	29.4	31.0	41.7	50.7
Friction	6.0	21.0	9.0	15.0	15.0	15.0
Slip						
I_1						
Input watts						
Power factor						
Output watts						
Output hp						
Efficiency						
Counter emf						

* Start-winding data for this motor: $X_S = 6.0$; $R_{1S} = 11.4$.

† Start-winding data for this motor: $X_S = 1.83$; $R_{1S} = 4.66$.

105. Set up the equivalent circuit for motor 1 of Table E and determine the items called for at a slip of 5 per cent. Neglect the core losses in the circuit but subtract them from the output. (Ans. $I_1 = 6.3$; input = 351 watts; pf = 0.506; output = 154.5 watts; efficiency = 0.44.)

106. Make similar calculations for motor 6, considering the effect of core losses throughout.

Chapter XXXIII

107. Assume a slip of 5 per cent and a counter emf of 100 volts for motor 1 of Table E. Calculate I_y, I_x, and I_ϕ. Determine the necessary voltage and correct the currents to an applied voltage of 110. Neglect the core losses.

108. (*a*) Using the final equations for the cross-field theory without core loss considered, determine the performance of motor 1 for a slip of 5 per cent. Calculate the torque also. (*b*) Calculate the maximum torque for motor 1. At what speed does this torque occur?

109. Using the $\frac{1}{6}$-hp motor of item 1, Table E, calculate the performance by the cross-field theory, considering the core losses as in Arnold's analyses. (See Art. 277.) Assume a slip of 5 per cent. (Ans. $I_1 = 5.85$; input = 311; pf = 48.5 per cent; output = 163.8; efficiency = 52.4 per cent; $E = 82.5$; $I_y = 3.59$; $I_x = 2.10$.)

Chapter XXXIV

110. Set up the equivalent circuit for motor 1, evaluating the constants. Use the cross-field theory, without core loss considered, and assume a slip of 5 per cent.

111. Set up the equivalent circuit as in Problem 110, but make use of an equivalent series impedance for core loss and magnetizing current.

112. Determine the performance items for motor 2 for a slip of 6 per cent, using the cross-field equivalent circuit without core loss being considered.

Chapter XXXV

Table F represents test data on a number of repulsion-start, induction-run motors. The brushes were lifted on the no-load tests, and the commutator was thoroughly short-circuited on the rotor-blocked tests. Hence these represent usual single-phase induction motor data.

TABLE F

SINGLE-PHASE INDUCTION MOTORS. TESTS ON REPULSION-START TYPE

Item	1	2	3	4	5
Hp	$\frac{1}{2}$	1	2	2	3
Poles	4	4	4	4	4
Synchronous rpm	1800	1800	1800	1800	1800
Volts	220	220	220	220	220
No load					
Volts	220	220	220	220	220
Amperes	2.45	4.2	7.4	7.38	10.6
Watts	133	192	287	256	340
Rotor-blocked					
Volts	80	80	96	80	80
Amperes	5.95	10.35	20.5	16.95	31.6
Watts	271	404	720	603	1600
Friction	52	50	108	78	106
R_1	2.81	1.54	0.68	0.67	0.38

113. Determine the winding constants of motor 1 (Table F), using the approximate method. Apply the corrections of methods two and three and contrast the results.

114. Prepare a suitable table of constants, obtained by the three methods using (*a*) motor 2, (*b*) motor 3, (*c*) motor 4, and (*d*) motor 5.

Chapter XXXVI

115. Refer back to the material of Chapter XXXI and determine the starting currents I_M, I_S, and I (total) for split-phase motor 4 of Table E. Calculate the starting torque.

116. Repeat as in Problem 115, using motor 6.

117. The following constants apply to a $\frac{1}{2}$-hp, 115-volt, 60-cycle, 4-pole, capacitor-start, induction-run motor.

Main:

$$R_1 = 0.94 \qquad R_2 = 2.23 \qquad X_1 = X_2 = 1.51$$

Start:

$$R_{1S} = 2.76 \qquad X_{1S} = X_{2S} = 2.34$$

Condenser (electrolytic):

$$X_c = 10.81 \qquad R_c = 0.32$$

(a) Calculate the starting current and the starting torque for this motor. (Test values obtained: $I_T = 29.8$; $T = 121.5$ oz-ft.) (b) What is the microfarad capacity of this condenser?

118. The following constants apply to a $\frac{1}{2}$-hp, 115-volt, 60-cycle, 4-pole capacitor motor, using the same capacitor for both starting and running.

Main:

$$R_1 = 1.72 \qquad R_2 = 1.98 \qquad X_1 = X_2 = 1.73$$

Start:

$$R_{1S} = 38.2 \qquad X_{1S} = X_{2S} = 16.0$$

Condenser (foil, paper, oil type):

$$X_c = 132.5 \qquad R_c = 0.4$$

(a) Calculate the starting current and the starting torque for this motor. (Test values obtained: $I_T = 21.5$; $T = 15.3$ oz-ft.) (b) What is the microfarad capacity of this condenser?

119. A special condenser motor with a rating of 1.6 hp was designed with windings for series parallel connection on either 220 or 110 volts. Operation is for a nominal speed of 1750 on a 60-cycle supply. The following tests were made with the windings in series:

Rotor-blocked:

Main:

$$V = 80 \qquad I = 12.3 \qquad W = 690$$

Auxiliary winding without condensers:

$$V = 80 \qquad I = 6.65 \qquad W = 430$$

(a) Calculate R_e and X_e for each winding by the approximate method.

Running at no-load on the 110-volt connection, the readings obtained are:

$$V = 110 \qquad I = 9.6 \qquad W = 230 \quad \text{(main winding active)}$$

$$V = 110 \qquad I = 5.5 \qquad W = 167.5 \quad \text{(auxiliary winding active)}$$

Further measurements (110-volt connection) indicate that $R_{1M} = 0.33$; $R_{1S} = 0.905$; friction loss is 60 watts.

(b) Calculate the winding constants by the more accurate method, including values of K_r, X_0, and X_m for each winding. The ratio of effective auxiliary winding to main winding turns is calculated as 1.312.

(c) Check this ratio by the individual values obtained for X_m.

To investigate possibilities of predicting the running performance of this type of motor, the following test data are given. Connection was for 110 volts, and a brake test was made with both stator windings active. A condenser with a nominal value of $80\mu f$ was connected in series in the auxiliary winding. Its resistance was 1 ohm.

Volts	110	110	110
Current in auxiliary winding	5.8	6.0	6.15
Watts, auxiliary winding	500	525	535
V across condenser	164	170	176
V across winding	134	139	143.5
Current in main winding	13.7	10.5	8.1
Watts, main winding	1360	1025	740
Rpm	1670	1690	1720
Hp output	1.9	1.6	1.3
Efficiency percentage	76.2	76.8	76.0

Synchronous Motors

Chapter XXXVIII

120. A three-phase, 60-cycle, 6-pole synchronous motor is rated at 15 kv-a, 220 volts. The armature resistance per Y leg (to neutral) is 0.11 ohm at 75 C. The effective value is 1.6 times this d-c value. The open- and short-circuit characteristics follow:

Field current	3.0	6.0	7.5	9.0	11.5
Open terminal potential	102	197	239	274	315
Short-circuit current	21	42			

If the input is 6 kw at 0.80 pf leading, determine the counter emf and the field current by the synchronous-impedance method. (Calculate the synchronous impedance at the highest value of short-circuit current measured.) (Ans. 281 volts; 9.4 amperes.)

121. Recalculate the values of Problem 120, using synchronous reactance as obtained at the point of full-load current on the short-circuit current line. What can you say as to the chief cause of discrepancy in the results?

122. Calculate the angles between the applied voltage and the counter emf for the motor of Problem 120 if the input remains at 10 kw and the pf is 0.6 lead; 0.8 lead; unity; 0.8 lag; 0.6 lag.

123. Calculate the pf of the motor of Problem 120 for a field current of 6 amperes when the armature current is 35 amperes per terminal. What is the developed power at this input? (Ans. 0.942 lag; 11.9 kw.)

124. Refer to the 100-hp synchronous motor of Article 322. What is the internal voltage and the required field current for a load requiring 105 amperes at a pf of 0.9 leading?

125. It will be assumed that three synchronous motors are available, rated at 100, 150, and 200 hp, respectively. They are Y connected for 440 volts, and each

has an efficiency of 90 per cent. Each is built so that X_d is 1.20 and X_q is 0.65 in per-unit values. (*a*) Neglect the effect of *IR* drop, and determine the internal generated voltage E_0 and the torque angle α for each motor for operation at unity pf. (*b*) Assume that the motors were designed for 0.8 pf lead. Determine the values as in question *a*.

Chapter XXXIX

126. (*a*) If a synchronous motor for unity pf operation is to have a maximum torque of at least 175 per cent of full-load torque, what is the maximum full-load torque angle at which it is should be designed to operate? Use the approximate relation resulting from synchronous-reactance theory. (*b*) Designed for 0.8 pf lead, such motors are expected to have a maximum torque of 250 per cent of the full-load value. What is the maximum full-load torque angle at which it should be designed to operate?

127. Refer to Problem 125*a* and *b*. Calculate in each case the percentage of the total power developed due to reluctance effect of the salient poles.

128. Calculate the approximate "stability factor" for each of the motors of Problem 125.

129. What are the maximum torque angles for the motors of Problem 125? Determine the ratios of maximum torque to full-load torque.

Chapter XL

130. A 60-cycle, 6600-volt, 1000 kv-a, three-phase synchronous motor displays the following constants:

R_e armature = 0.46 ohm at 75 C per Y leg
X_s armature = 12.0 ohms per Y leg
Friction, windage, and core loss, assumed constant = 38 kw

The no-load saturation curve:

I_f	50	100	150	175	200
V_{terminal}	3400	5700	7200	7900	8400

For excitations of 60, 80, 100, 120, and 140 per cent, calculate the characteristics as given in Table XIX, assuming torque angles of 5, 10, 15, 30, and 40 degrees. Plot these as curves similar to those of Figs. 270–272.

131. Draw the circle diagram for the above motor, and check at least two of the points calculated above by means of the circle diagram.

Chapter XLIII

132. A certain 900-rpm synchronous motor has a flywheel effect (WR^2) of 1700 lb-ft^2. The torque produced per mechanical radian of phase displacement is 6500 lb-ft. When the motor is operating at no load and with a negligible torque angle and negligible damping, a load of 100 hp is thrown on suddenly. (*a*) Set up the equation of torque angle variation. (*b*) What is the final torque angle after the oscillation has died out? (*c*) What is the maximum torque angle reached during the oscillation? (Ans. (*b*) 0.0898 mechanical radian. (*c*) 0.1796 mechanical radian.)

133. A damping winding is used on the above motor. The damping constant is 150 lb-ft per mechanical radian per second. (*a*) Set up the equation for torque angle for a sudden load increase of from zero to 100 hp. (*b*) What is the final torque angle? (*c*) What is the maximum torque angle? (*d*) Does the rotor oscillate during the transient period?

Chapter XLIV

134. Calculate the natural period of oscillation for the motor of Problem 132.

135. (*a*) Calculate the natural period of the above motor when the damping winding of Problem 133 is considered. (*b*) In the latter case, what would be the effect on the oscillating period of adding a weight of 350 lb with a radius of gyration of 2 ft?

136. The following data are known about a 3000-hp synchronous motor designed for 2300 volts, three-phase, 60 cycles, 225 rpm, 2350 kv-a, unity pf.

R_e (75 C) = 0.029 ohm per Y leg
X_s = 1.80 ohms per Y leg
Flywheel effect (WR^2) = 530,000 lb-ft^2
Flux per pole = 4,800,000 lines
Diameter at air gap = 100 in.
Pole pitch = 9.82 in. Pole arc = 6.75 in.
Damping effect k_1 = 36,250 lb-ft per mechanical radian per second.

Calculate (*a*) the full-load torque; (*b*) the full-load torque angle at unity pf operation, assuming an efficiency of 94 per cent; (*c*) the constant k_2; (*d*) the natural period of oscillation; (*e*) plot the torque-angle equation and determine the maximum torque angle when the initial load of 1000 hp is suddenly doubled.

Alternators in Parallel

Chapter XLVI

137. Two similar alternators are operating in parallel driven by d-c shunt motors. The speed of the first motor drops from 1200 rpm at no load to 1160 rpm at full load of 50-kw output for its alternator. The second motor has a no-load speed of 1210 which drops to 1162 for 50 kw output of its alternator. Both speed characteristics are assumed to be straight lines. (*a*) Determine the minimum load which can be supplied by both alternators operating as generators. (*b*) At what load will the alternators divide the load equally?

138. The speed characteristic of the second motor used above is adjusted parallel to its former values so that the no-load speed is 1200. (*a*) At what total output will the load be divided equally? (*b*) What should be the no-load speed of each if the rated load of 100 kw is to occur with rated frequency and for equal division of load between each alternator?

139. Two 60-cycle alternators are operated in parallel, supplying a total load of 800 kw equally divided between them. The prime movers have a speed regulation of 3.5 per cent, the speed being such as to give rated frequency at full load of 750 kw. Determine the change in frequency if one alternator is disconnected from the line.

140. Two two-phase alternators are operating in parallel. They display similar characteristics but the speed regulations of the prime movers differ. When the excitations of the two are exactly equal one delivers 50 amperes at 0.85 pf lagging,

and the other 60 amperes at 0.70 pf lagging. (*a*) What is the power factor of the load? (*b*) What percentage of the load is each supplying? In order to make the circulating current a minimum, in what way does the excitation of each have to be varied? (Ans. (*a*) 0.775; (*b*) 50.3, 49.7.)

141. Two similar alternators display the following data: 6600 volts, three-phase, 60 cycles, 1000 kv-a.

R_e armature per Y leg equals 1.02 ohms.

Saturation curve:

Field current	150	200	250	300	350	500
Terminal potential	5600	6490	7000	7400	7750	8500

X_s per Y leg equals 17 ohms.

When operating in parallel, the first alternator supplies 85 amperes at a pf of 0.85 lagging. If the load pf is 0.78 lagging and the total power is 1500 kw, determine the excitation of the second alternator. (Ans. Approximately 540 amperes.)

142. A three-phase alternator is to be paralleled with a large power system. At the instant of throwing in the switch, the phase positions are 30 electrical degrees from the ideal position. With the data given below determine: (*a*) the synchronizing torque at this instant; (*b*) the time required for the alternator to pass from its position when the switch is thrown, to the correct phase position.

The prime mover speed is such that without adjustment the alternator will float on the line. The excitation is normal. Assume that the instantaneous change in its speed does not affect the prime mover torque.

25 kv-a 60 cycles 6 poles 1200 rpm 440 volts

Field current	4	6	8	10	12
Open terminal emf	265	396	500	585	660
Short-circuit current	33	50			

R_e armature per Y leg $= 0.127$ ohm
Flywheel effect (WR^2) $= 68$ lb-ft^2
Damping $= 0$

143. A 225-rpm salient-pole alternator is rated at 2300 volts, 300 kv-a. Its per-unit values of X_d and X_q are 1.1 and 0.64, respectively. With its three-phase voltage correctly adjusted to bus potential, it is synchronized with a power system when 15° out of phase. What is the synchronizing power and the value of current inrush?

Synchronous Converters

Chapter XLVIII

144. A synchronous converter is wound with a uniformly distributed winding of $\frac{10}{12}$ pitch. The flux density is uniform under the poles and zero between them. Each pole covers 0.65 of the pole pitch. Calculate the ratio of alternating emf to unidirectional emf for single-phase operation.

145. Calculate the values of three-phase line current and potential needed to supply a converter with an output of 600 volts, 200 amperes direct current if the efficiency is 94 per cent, and the pf is 0.95 leading.

146. A 4-pole synchronous converter has 12 armature slots per pole, wound with 2 coil sides per slot, each of 6 turns. The winding is of the lap type, full pitch.

The converter is connected to a three-phase source of 310 volts, 60 cycles, and draws 400 amperes per terminal. Neglecting voltage drops and constant losses, calculate the current in the coils of each slot at the instant the applied emf in one of the phases is zero. The machine is operating at unity pf.

147. Repeat the calculations of Problem 146 with the machine operating at a pf of 0.90 leading.

148. A four-phase, 4-pole synchronous converter is operating at unity pf. Assuming ideal wave shapes, calculate the relative capacities when operating as a converter and as a d-c generator. (*b*) Repeat for operation at 0.90 pf leading current. The basis for comparison is to be equal armature copper loss.

Chapter XLIX

149. A synchronous converter is rated at 150 kw, 125 volts direct current. It is supplied by three single-phase transformers connected in Δ–Y to a nominal 440-volt supply. (*a*) Calculate the complete rating of the transformers, neglecting converter losses and regulation and assuming a pf of 0.90. (*b*) The actual supply voltage remains constant at 440 volts, and the equivalent reactance drop of the transformers is 6 per cent. Neglect resistance drop in the transformers and all drops in the converter and determine the d-c voltage output at full load, 0.9 leading pf; unity pf; and 0.9 lagging pf.

150. A 60-cycle, three-phase synchronous converter is rated at 600 volts and 400 amperes on its d-c side. It has 4 poles with 12 armature slots per pole and 8 conductors per slot; the coils are full pitch. Assume 100 per cent efficiency. (*a*) Calculate the d-c ampere turns of armature reaction per pole. (*b*) Calculate the a-c ampere turns per pole at unity pf. (*c*) What is the net effect of armature reaction in this machine?

Chapter L

Data on synchronous converter:

Capacity	300 kw	ψ	0.70
Speed	750 rpm	Poles	4
Phases	3	A-c volts	367
Conductors per slot	8	Slots	96
Frequency	25 cycles	Series turns per pole	$4\frac{1}{2}$
D-c volts	600	Shunt field turns per pole	940

Resistance of armature (d-c): 0.018 ohm at 75 C
Shunt field resistance: 42.3 ohms at 75 C
Series field resistance: 0.00083 ohm at 75 C
Friction and windage loss: 3200 watts
Core loss at 600 volts, d-c: 4700 watts
Brush shift angle: zero
Assume an IX drop in the armature of 5 per cent

No-load saturation curve: $\left(\text{no-load voltage ratio: } \frac{E_{ac}}{E_{dc}} = 0.62\right)$

Field current	2	4	6	8	10	12
Volts d-c	220	400	515	620	680	720

151. Assume for initial calculations that the pf of the above converter is 0.95 leading, and the efficiency is 0.95 per cent. Calculate (*a*) the full-load direct current and the input current per line; (*b*) the alternating current per conductor; (*c*) the IR_a drop as a converter operating at full load. (*d*) Draw the vector diagram for the above condition. (Ans. (*a*) 500 amperes, 521 amperes; (*b*) 150.5; (*c*) 7.88 volts.

152. Assume that the voltage applied at the slip rings of the above converter is 367 and the field is adjusted to give a pf of 0.95 leading at full load. Calculate (*a*) the series field NI per pole; (*b*) the demagnetizing effect of the wattless current in ampere turns; (*c*) the ampere turns required of the shunt field (see the saturation curve); (*d*) the output voltage under this condition. (Ans. (*a*) 2250; (*b*) 3110; (*c*) 7720; (*d*) 590 volts.)

153. Calculate the following losses of the above converter under the load conditions of Problem 152; (*a*) core loss; (*b*) friction and windage; (*c*) shunt field and rheostat; (*d*) series field; (*e*) armature I^2R loss. (*f*) Determine the efficiency. (Ans. (*a*) 4700; (*b*) 3200; (*c*) 4860; (*d*) 207; (*e*) 3450; (*f*) 94.9 per cent.)

Chapter LI

154. A synchronous converter is rated at 500 kw, 600 volts direct current. Its transformers are connected Y–ΔΔ (that is, the secondaries are six-phase double delta) to a three-phase, 25-cycle, 13,200-volt source. For an efficiency of 0.95 and power factor of 0.95 calculate the current in each winding of the transformers and the voltage and kilovolt-ampere rating of each. (Refer to Chapter XIX.)

155. (*a*) Determine the current in each winding, the voltage across each winding, and the kilovolt-ampere rating of each transformer needed in Problem 154 if the transformers are connected Y–YY. (*b*) Repeat for six-phase diametrical secondary connections with Y-connected primaries.

Power Rectifiers

Chapters LII and LIII

156. A single-phase rectifier is to deliver 110 volts and 25 amperes of direct current. The arc drop is 10 volts, and the supply voltage is 220. Find the following quantities: (*a*) voltage to neutral on the supply transformers (secondary); (*b*) usual a-c ratings of primary and secondary windings; (*c*) power factor and utility factor of primary and secondary windings. (Ans. (*a*) 133 volts. (*b*) Secondaries: 17.7 amperes, 2.350 kv-a each; primary: 15.1 amperes, 8.040 kv-a. (*c*) Secondary: utility factor 0.637, pf 0.637; primary: utility factor 0.90, pf 0.70.)

157. (*a*) A three-phase rectifier is to deliver 400 amperes direct current at 600 volts. The arc drop is 15 volts. Calculate the voltage to neutral for which the transformer secondaries must be designed. (*b*) A six-phase rectifier is to be used for the same load given in (*a*). Calculate the transformer secondary voltage to neutral. (*c*) An inductance is to be connected in the d-c output circuit of the rectifier of (*a*) above. Calculate the value of inductance necessary to keep the maximum current variation of any harmonic below 5 per cent. (That is, $2\frac{1}{2}$ per cent above or below the average.) The frequency is 60 cycles. (*d*) Repeat (*c*) for the six-phase rectifier of (*b*).

158. A three-phase supply is available for the rectifiers of Problem 157. Calculate the kilovolt-ampere rating of each transformer phase for (*a*) and for (*b*). Determine the utility factors for the primary and secondary windings of both the three- and the six-phase transformers.

Series Motors

Chapter LVI

159. A certain series motor has 2 poles with 95 turns per pole. The 2 field coils are connected in series and have a resistance of 3.02 ohms. The drop across the field is 62 volts when the current is 3.55 amperes at 60 cycles. (*a*) Assuming sinusoidal quantities, what is the value of the field reactance? (*b*) What is the value of the field flux per pole? (Ans. (*a*) 17.2. (*b*) 122,500.)

160. A series motor has 12 turns per armature coil. When the brushes are lifted from the commutator and 3.55 amperes flow through the field, the emf between two adjacent commutator bars with the armature at rest is 6.5 volts. How many lines of flux passed through the coil? Frequency is 60 cycles.

161. A 2-pole armature has 960 conductors. At a speed of 5200 rpm the armature terminal emf was 80 volts and the armature input 266 watts, both for a current of 3.55 amperes. The armature resistance is 3.2 ohms. Determine the following: (*a*) armature terminal pf; (*b*) effective impedance of the armature in motion; (*c*) effective reactance of the armature in motion; (*d*) armature counter emf; (*e*) useful flux per pole.

Repulsion Motors

Chapter LVIII

162. A plain repulsion motor was tested with blocked rotor. The readings were: $V = 80$; $I = 1.8$; $W = 90$. Calculate the equivalent values of R, X, and Z under this condition.

163. When the brushes on the machine of Problem 162 were lifted and the commutator short-circuited by wrapping copper wire around it, the following readings were obtained: $V = 80$; $I = 2.92$; $W = 118$. Calculate the equivalent values of R, X, and Z under this condition.

164. The machine of Problem 162 with open rotor and lifted brushes gave the following test readings: $V = 220$; $I = 1.20$; $W = 27$. Calculate the values of R, X, and Z under this condition.

165. Explain in some detail why the values of R, X, and Z, are not alike in Problems 162, 163, and 164.

166. A 4-pole armature winding with four brushes has a resistance of 0.5 ohm. (*a*) If it is a 4-circuit lap winding, what is the resistance per path? (*b*) If it is a 2-circuit wave winding? (Ans. (*a*) 2 ohms per path; (*b*) 1 ohm per path.)

167. If in Problem 166 the resistances are measured between points 90 mechanical degrees apart, what are the values to be expected in each of the following cases: (*a*) for the 4-circuit lap winding of 166*a* without cross-connections; (*b*) for the 4-circuit lap winding of 166*a* with cross-connections; (*c*) for the 2-circuit wave winding? (Ans. (*a*) 1.5 ohms; (*b*) 0.5 ohm; (*c*) 0.5 ohm.)

Chapter LIX

168. A 4-pole repulsion motor running at 1200 rpm has an air gap flux of 200,000 lines per pole, 600 conductors, and four armature paths. Calculate the effective value of the armature speed emf. (Ans. 17.0 volts.)

DATA ON REPULSION MOTOR D

(Poles 6 Frequency 25)

Armature:	
Outside diameter	19.7 in.
Inside diameter	9.85 in.
Slots	80
Core length	10.625 in.
Winding:	
Conductors	640
Size	0.571 in. by 0.0709 in.
Conductor length	26 in.
Conductors per slot	8
Pitch: coils in slots	1–14
Resistance	0.009 ohm
Stator:	
Bore	19.94 in.
Air gap	0.12 in.
External diameter	27.6 in.
Core length	10.625 in.
Slots	108
Pitch: coils in slots	1–18
Winding:	
Conductors	864
Size	0.61 in. by 0.0492 in.
Conductor length	30.1 in.
Paths	2
Conductors per slot	8.0
Resistance	0.17 ohm
Commutator:	
Diameter	16.9 in.
Length	13.4 in.
Segments	320
Brushes:	
Studs	6
Carbons per stud	6
Carbons	0.37 in. by 1.81 in.
Resistance per stud	0.0115
Resistance of short-circuited brush circuit including brushes	0.00766 ohm

169. Assuming a sinusoidal distribution, calculate (*a*) the effective field and transformer-axis turns, respectively, for the stator of motor D. The angle of brush lead is 15°.

170. Calculate the ratio of transformation, stator to armature for motor D; transformer axis to armature; field axis to armature.

171. Using the data given on the $\frac{1}{3}$-hp, 4-pole, 110-volt repulsion motor of Article 469 and those following, determine the following values for a speed of 1800 rpm: (*a*) the supply current I_1; (*b*) the armature current I_a; (*c*) the itemized losses; (*d*) the input, output, efficiency, and pf; (*e*) the torque in ounce-feet. Compare results with the test curves of Fig. 355.

172. Repeat the calculations of Problem 171 for a speed of 2200 rpm.

Chapter LX

173. A 2-hp,[1] 220-volt, 4-pole, repulsion-start induction motor displays the following values:

$R_1 = 0.75$	Total leakage reactance, $X = 4.17$
$R_2 = 1.35$	Total stator reactance, $X_0 = 56.2$

Resistance of the rotor coil short-circuited by brush, $r_c = 0.00745$
Apparent resistance of the brush contact for the probable current density used, $r_b = 0.014$
Ratio of transformation, shorted rotor coils to stator $= 56.25$
Brush shift $= 16°$.

(*a*) Calculate the flux ratio K_r. (*b*) Determine r_3 and β. (*c*) Calculate the starting current and torque. (Test values obtained: $I = 32.0$; $T = 472$ oz-ft.)

(*d*) Recalculate, using an 18° brush shift. (Ans. $I = 28.0$; $T = 445$ oz-ft.)

(*e*) Recalculate, using a 14° brush shift. (Ans. $I = 34.1$; $T = 521$ oz-ft.)

174. Recalculate the starting current and torque for the motor of Problem 173, using the analyses in which commutation effects are neglected. Compare results.

175. Assume that the brush and brush contact resistance of this motor were doubled. Recalculate the values asked for in Problem 173*b* and *c*.

[1] This represents a slightly modified design from that given in the text. Several items were changed, influencing the running action, but the starting performance gave identical tests in each case.

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